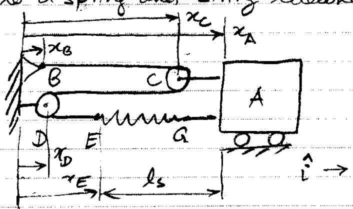
Basant Sharma, Spring 2002

① $\boxed{6.28}$

Block connected to a spring and string released from rest.



$$x_E - x_D + x_C - x_D + x_C - x_B = \text{const}$$
$$x_A - x_C = \text{const}$$
$$x_A - x_E = l_s = \text{length of spring } EG$$

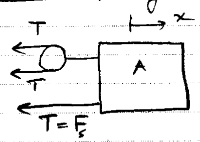
x_D, x_B are constant so

$$\Delta x_A = \Delta l_s + \Delta x_E$$
$$= \Delta l_s - 2\Delta x_C$$
$$= \Delta l_s - 2\Delta x_A \quad \text{So } \Delta l_s = 3\Delta x_A$$

Change from unstretched state

Let $x = \Delta x_A$, So force in spring, $F_s = k\Delta l_s = 3kx$.

FBD of mass and pulley C



By LMB, $m_A \ddot{x} = -3F_s = -3\times 3kx$,

or $m_A \ddot{x} + 9kx = 0$

a) when the mass is released from a distance d away from the relaxed state

By LMB, $ma = -9kd$

So $\boxed{a = -\frac{9k}{m_A}d}$

b) Total energy is conserved, So

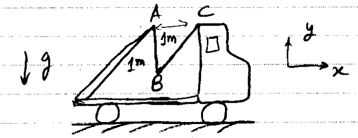
Initial $\cancel{K.E.}^{0} + P.E = K.E. + \cancel{P.E.}^{0}$ when mass is at $x=0$.

$\therefore \frac{1}{2}k(3d)^2 = \frac{1}{2}m v^2 \Rightarrow \boxed{v = 3d\sqrt{\frac{k}{m}}}$ is the speed when the mass passes through the position where the spring is relaxed, i.e. $x=0$.

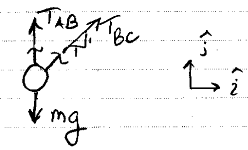
② $\boxed{6.35}$

Given: mass held by two strings in a moving truck accelerating at $3\,m/s^2$.

To find: Tension in the string AB.



FBD of mass at B



By LMB,

$ma_x = \Sigma F_x$: $m_B 3\,m/s^2 = T_{BC}\times\frac{1}{\sqrt{2}}$

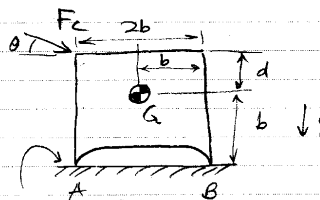
$ma_y = \Sigma F_y$: $0 = T_{BC}\times\frac{1}{\sqrt{2}} + T_{AB} - m_B g$

take $g = 10\,m/s^2$, So $T_{AB} = m_B g - m_B\times 3\,m/s^2$
$= m_B \times 7\,m/s^2$

Since $m_B = 2\,kg$, $\boxed{T_{AB} = 14\,N}$

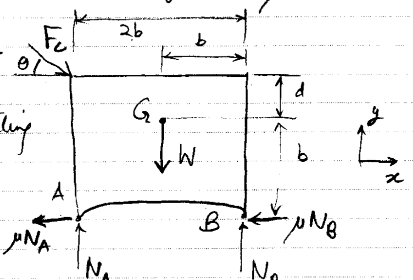
③ $\boxed{6.43}$

Find the reactions at A and B in terms of W, F_c, θ, b, d for sliding box as shown.



Coefficient of friction μ.

FBD of box



Let the box be accelerating in x-direction by a.

$\hat{k}\cdot(\Sigma \underset{\sim}{M}_{/A} = \underset{\sim}{r}\times m\underset{\sim}{a})$: $-F_c\cos\theta.(b+d) - Wb + N_B\, 2b = -\frac{W}{g}a\times b$

$\hat{k}\cdot(\Sigma \underset{\sim}{M}_{/B} = \underset{\sim}{r}\times m\underset{\sim}{a})$: $-F_c\cos\theta(b+d) + F_c\sin\theta.\,2b + Wb - N_A.\,2b = -\frac{W}{g}a\times b$

$\hat{i}\cdot(\Sigma \underset{\sim}{F} = m\underset{\sim}{a})$: $-(\mu N_A + \mu N_B) + F_c\cos\theta = ma$

Eliminating N_A and N_B and solving for a we get

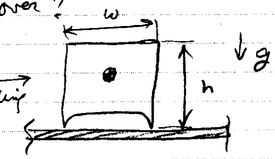
$$\boxed{a = \frac{g}{W}\left[F_c\cos\theta - \mu(W + F_c\sin\theta)\right]}$$

and then Solving for N_A and N_B

$$\boxed{N_A = \frac{W}{2}(1-\mu) + F_c\sin\theta\left(1-\frac{\mu}{2}\right) - F_c\cos\theta\frac{d}{2b}}$$
$$\boxed{N_B = \frac{W}{2}(1+\mu) + F_c\sin\theta\frac{\mu}{2} + F_c\cos\theta\frac{d}{2b}}$$

④ $\boxed{6.50}$

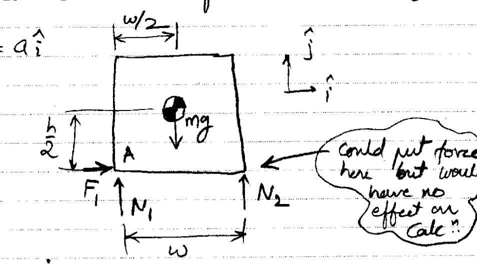
Find relation between μ, g, ω, h and m such that the box moving with a conveyor belt as shown below doesn't tip over?



If there was no slip, all boxes would tip over for sufficiently large a. Why?

Ans: Assume no slip and no rotation.

$\Rightarrow \underset{\sim}{a} = a\hat{i}$



$$\Sigma \underset{\sim}{M}_{/A} = \dot{\underset{\sim}{H}}_A$$

$$\left\{\left(-\frac{mg\omega}{2} + N_2 d\right)\hat{k} = -ma\frac{h}{2}\hat{k}\right\} \quad (1)$$

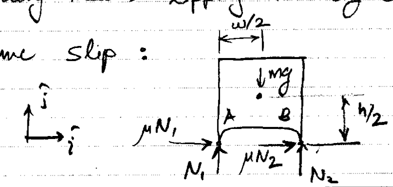
$\{\}\cdot\hat{k} \Rightarrow N_2 = \left(\frac{mg\omega}{2} - \frac{mah}{2}\right)d$

if $a > \frac{g\omega}{2} \Rightarrow N_2 < 0$

$\Rightarrow$ no tip assumption was bad!

$\Rightarrow$ no sliding means tipping for big enough a.

So assume slip:



$\{LMB\}\cdot\hat{j} \Rightarrow N_1 + N_2 = mg$

$\{LMB\}\cdot\hat{i} \Rightarrow \mu N_1 + \mu N_2 = ma$ $\Big\} \Rightarrow a = \mu g$

$\hat{k}\cdot\{AMB/_A\} \Rightarrow N_2 = \left(\frac{mg\omega}{2} - m\mu g\frac{h}{2}\right)/\omega$

like ① above

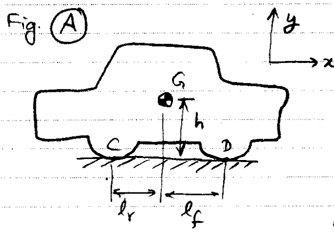
$N_2 > 0 \Rightarrow \frac{mg\omega}{2} > \frac{m\mu g h}{2}$

$\Rightarrow \boxed{\frac{\mu h}{\omega} < 1}$

This is the condition for no tipping.

⑤ $\boxed{6.52}$ (Copied from solutions for Spring 2000)

Fig. (A)



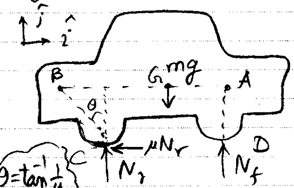
To know the deceleration, reaction forces for different case of skidding.

Given: $l_r = l_f = \frac{\omega}{2}$, $\mu = 1$.

a)

Fig. (B) FBD

Only the rear wheel skidding. (friction acts opposite to relative velocity)



$\theta = \tan^{-1}\frac{1}{\mu}$

In fig. (B) it's simplest to write AMB about A and B since contribution from N_f and N_r vanish respectively, and $\underset{\sim}{r} \times m\underset{\sim}{a}_{cm}$ has no contribution.

b) LMB for FBD in Fig. (B)

$-\mu N_r\hat{i} + N_r\hat{j} + N_f\hat{j} - mg\hat{j} = ma\hat{i}$ ___ (I)

(I)$\cdot\hat{i} \Rightarrow -\mu N_r = ma$ — (1)

(I)$\cdot\hat{j} \Rightarrow N_r + N_f = mg$ — (2)

c) $AMB/_A$ for FBD in Fig. (B)

$\left(mg\frac{\omega}{2} - N_r\omega - \mu N_r h\right)\hat{k} = -\left(\frac{\omega}{2}\hat{i} \times ma\hat{i}\right) = 0$

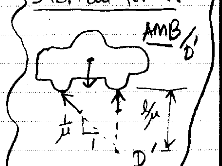
d) $\therefore mg\frac{\omega}{2} - N_r\omega - \mu N_r h = 0$

$\Rightarrow \boxed{N_r = \frac{mg\omega}{2(\omega+\mu h)}}$ — (3)

from (2) $\boxed{N_f = \frac{mg(\omega+2\mu h)}{2(\omega+\mu h)}}$ — (4)

from (1) $\boxed{a = \frac{-\mu g\omega}{2(\omega+\mu h)}}$ — (5)

Shortcut for a

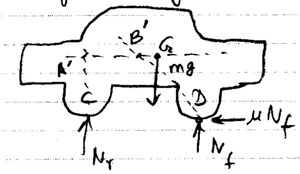


Check: if $\mu = 0$, then $a = 0$. so no friction ⇒ no deceleration.

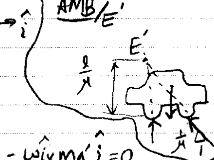
also by (5) $|a| \le \frac{\mu g}{2}$, if $\omega \gg h$ (or $h = 0$) then, we have $|a| = \frac{\mu g}{2}$.

e)

FBD of car Fig (C) Front wheel skidding



Shortcut for a'



$AMB/_{A'}: \left(-\mu N_f h - mg\frac{\omega}{2} + N_f\omega\right)\hat{k} = \frac{\omega}{2}\hat{i} \times ma'\hat{i} = 0$

$\Rightarrow \boxed{N_f = \frac{mg\omega}{2(\omega-\mu h)}}$ — (6)

LMB: $-\mu N_f\hat{i} + (N_f + N_r - mg)\hat{j} = ma'\hat{i}$ — (II)

(II)$\cdot\hat{i} \Rightarrow -\frac{\mu N_f}{m} = a' \overset{(6)}{\Rightarrow} \boxed{a' = \frac{-\mu g\omega}{2(\omega-\mu h)}}$ — (7)

(II)$\cdot\hat{j} \Rightarrow N_f + N_r = mg$

$\overset{(6)}{\Rightarrow} \boxed{N_r = \frac{mg(\omega - 2\mu h)}{2(\omega-\mu h)}}$ — (8)

Comparing equations (5) and (7), $|a'| \ge |a|$.

So the deceleration for front wheel skidding is larger and so the car comes to halt faster.

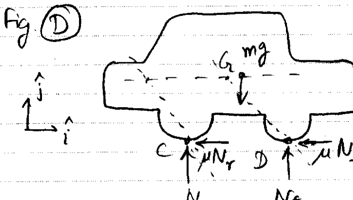
If $h = 0$ then there is no moment due to friction. And $\boxed{N_f = N_r = \frac{mg}{2}}$, $\boxed{a' = -\frac{\mu g}{2}}$ ← front wheel

$\boxed{a = \frac{-\mu g}{2}}$ ← rear wheel

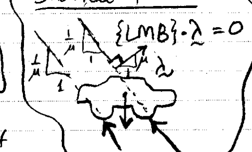
Thus the car would have same dynamics in both cases.

f) FBD for both wheels skidding

Fig (D)



Shortcut for $\bar{a}$



LMB: $(-\mu N_r - \mu N_f)\hat{i} + (N_r + N_f - mg)\hat{j} = m\bar{a}\hat{i}$ — (III)

(III)$\cdot\hat{i} \Rightarrow -\mu(N_r + N_f) = m\bar{a}$ — (9)

(III)$\cdot\hat{j} \Rightarrow (N_r + N_f) = mg$ — (10)

Using (10) in (9) $\boxed{\bar{a} = -\mu g}$ —(11)

AMB/C : $(N_f w - mg\frac{w}{2})\hat{k} = -m\bar{a}\,h\hat{k}$

$\Rightarrow$ $\boxed{N_f = \frac{mg}{w}\left(\frac{w}{2}+\mu h\right)}$ —(12)

(10) $\Rightarrow$ $\boxed{N_r = \frac{mg}{w}\left(\frac{w}{2}-\mu h\right)}$ —(13)

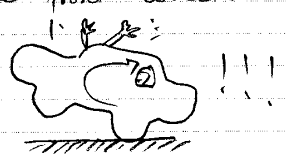
g) from (5), (7) $a + a' = -\frac{\mu g w}{2}\left(\frac{1}{w+\mu h}+\frac{1}{w-\mu h}\right) \neq \bar{a}$

So the principle of superposition doesn't work. This is because in the equation for AMB the moments added up to different values due to presence of friction at different wheels and this difference appears in the computation of normal reactions too, hence the dependence on skidding is not linear.

h) When $w=2h$, in case of front wheel skidding $N_r = 0$, ie the normal reaction at rear wheel vanishes! Since $\mu = 1$.

i) If $w < 2h$ and $\mu = 1$ then from (8) $N_r < 0$ which is physically impossible as the ground can't suck the tyre in!

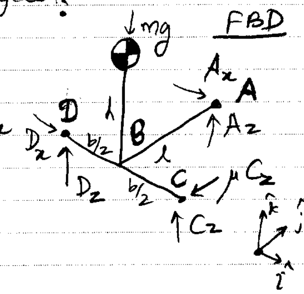
In this case, as soon as normal reaction becomes zero there is no more skidding... instead THE CAR IS TOPPLING over about the front wheel.



(6) $\boxed{6.74}$ (copied from solutions from Fall 2000)

A branch gets caught in the right rear wheel of a tricycle causing the wheel to skid in the $\hat{j}$-direction. The wheels at A and D roll without slip and are massless. What is the acceleration of the tricycle?

FBD



- A and D roll w/o slip in the $\hat{j}$-direction
- Wheel at C skids in the $\hat{j}$-direction

LMB : $\sum \underset{\sim}{F} = m\underset{\sim}{a} = ma\hat{j}$

LMB$\cdot\hat{j}$ $\Rightarrow -\mu C_z = ma$ —(1)

AMB$_{/B}\cdot\hat{j}$: $\{\sum \underset{\sim}{M}_{/B} = \dot{\underset{\sim}{H}}_{/B}\}\cdot\hat{j}$; moments about axis through B // to $\hat{j}$-direction

$\Leftrightarrow -C_z\frac{b}{2} + D_z\frac{b}{2} = 0$

$\therefore C_z = D_z$ —(2)

AMB$_{/A}\cdot\hat{i}$: $\{\sum \underset{\sim}{M}_{/A} = \dot{\underset{\sim}{H}}_{/A}\}\cdot\hat{i}$; moments about axis through A // to $\hat{i}$-direction

$\Leftrightarrow -C_z l - D_z l + mgl = -mah$ —(3)

(2) into (3) $\Rightarrow mg - 2C_z = -ma\frac{h}{l}$ (4)

(1) into (4) $\Rightarrow mg + \frac{2ma}{\mu} = -ma\frac{h}{l}$

So $a\left(\frac{2}{\mu}+\frac{h}{l}\right) = -g \Rightarrow a = \left(\frac{-\mu g/2}{1+\mu h/2l}\right)$

$$\boxed{\therefore \underset{\sim}{a} = \frac{-\mu g}{2}\left(\frac{1}{1+\frac{\mu h}{2l}}\right)\hat{j}}$$

Check: (1) when $\mu = 0$, $\underset{\sim}{a} = \underset{\sim}{0}$ (no forces in $\hat{j}$-direction $\Rightarrow$ constant $\underset{\sim}{v}$).

(2) when $h = 0$, $\underset{\sim}{a} = -\frac{\mu g}{2}\hat{j}$ (Half of the rider's weight is supported at wheel C).