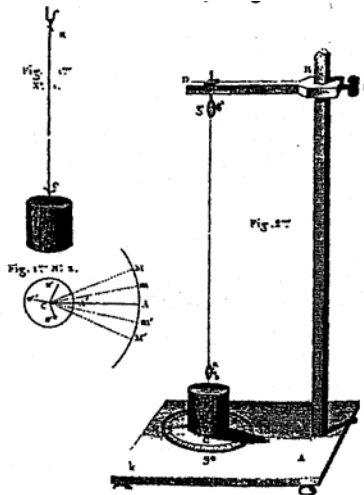# ENGINEERING 203
LABORATORY MANUAL

DYNAMICS

(Read before attending lab.)

This manual has evolved over the years. Contributors in the past two decades include: Kenneth Bhalla, Jason Cortell, Drew Eisenberg, Jill Evensizer, Richard Lance, Jamie Manos, Francis Moon, Dan Mittler, James Rice, Andy Ruina, Bhaskar Viswanadham, and Alan Zehnder.

Coulomb's Drawing of his Torsion Vibration Apparatus of 1784.

REVISED AUGUST 2005

DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS
CORNELL UNIVERSITY
ITHACA, NEW YORK

THEORETICAL AND APPLIED MECHANICS 203 DYNAMICS

Revised: August 2005

INTRODUCTION

PURPOSE

The laboratories in dynamics are designed to complement the lectures, text, and homework. They should help you gain a physical feel for some of the basic and derived concepts in dynamics: force, velocity, acceleration, natural frequency, resonance, normal modes, and angular momentum. You will also get exposure to equipment and computers which you may use in the future. Some mathematics from courses you have taken recently or are now taking will be used, which we hope will help you make the tie between mathematics and physical reality that is essential to much of engineering. The labs may come either before or after you cover the relevant material in lecture. Thus, they can be either a motivation for the lecture material or an application of what you have learned, depending on the timing.

Lab groups are small enough (2 or 3 people) that you can get direct experience with the instruments and equipment. Both you and your lab partners should learn how to do all aspects of the lab. The laboratory teaching assistants will have scheduled office hours in the laboratories so that you can return to use the facilities independently, or ask questions. These hours will be posted on the door of the laboratory. You may also ask the lab TA about other course material if time allows.

CONTENT

There are four labs:

       1) Forced Mass-Spring-Dashpot System
       2) Multiple Degree of Freedom Oscillator
       3) Slider Crank Mechanism
       4) Gyroscope

These labs will be done with physical equipment and some will also involve simulations on microcomputers. It is essential that you read through the lab (especially the procedure section) and answer the pre-lab questions before coming to lab. It is not necessary that you understand all of the material perfectly before the lab period. In addition to the regular labs, there are some demonstrations set up for you to experiment with, if enough time is available. Some of the labs may also include extra topics for you to research, or ask you to develop a small experiment of your own.

LOCATION

The laboratories are in Thurston 101.

SCHEDULING

Each of the four labs is taught for two weeks. You will be scheduled to attend lab during one of the two weeks. The dates for your laboratory section will be posted outside room 101 Thurston. In general, you will have a lab once every three weeks, but be aware that this may vary due to exam and break schedules. (Summer session lab schedules will differ.)

See the Secretary in Kimball 212 if you have any problems with your lab schedule. You'll need to get her approval for any changes, so that the lab sections do not become overly full. Turning in a course change form to the registrar is not enough.

TURNING IN YOUR LAB REPORT

Lab reports are due one week from the day you performed the lab, at 8:00 in the morning, unless your TA specifies another time.

Turn in reports in the boxes in Don Conway room on the first floor of Thurston Hall. Be sure to put your report in the correct box corresponding to the TA in charge of your lab section.

MISSED LAB AND LATE REPORT POLICY

All make-up labs must be arranged with your TA and the Secretary, in 212 Kimball Hall. If you know in advance that you'll be gone, you should sign up with her at least one week prior to the scheduled lab. This gives you a better chance to sign up for a convenient time, and there's no point penalty. If you miss your lab without arranging a make-up lab in advance, you should still try to arrange a new time. However, a one-point lab report penalty will be imposed, unless you were ill, etc. You should arrange to make up a missed lab as soon as possible, since the lab setups are changed after a lab is finished. In special circumstances, labs may be made up at the end of the semester; sign up with your TA and the Secretary in 212 Kimball Hall.

If you show up for lab after it is under way, your lab instructor may ask you to leave, and to perform the lab another time. The ENGRD 203 lab equipment should not be used without proper training. Note also that answers to pre-lab questions are due at the beginning of lab, and will not be accepted for credit later.

Reports turned in late will be marked down 2 points, and 4 points if they are more than a week late. Maximum late penalty is 4 points. Late reports may be handed in until one week after the last regularly scheduled lab of the semester.

Exceptions to the above policies may be made in the case of documented illness or other emergency. Talk to your lab TA about late reports; see your TA and the Secretary in 212 Kimball Hall about making up a lab.

PRE-LAB QUESTIONS

Each lab has pre-lab questions which should be answered *before* you come to lab. These questions encourage you to read through the laboratory procedure prior to attending the lab, and gain an understanding of what will be done during the laboratory period.

For this reason, your answers are due at the start of the lab.

ACADEMIC INTEGRITY

Your pre-lab answers and lab reports should be in your own words, based on your own understanding and your own calculations. You are encouraged to discuss the material with other students, friends, TAs, or faculty. Any help you receive from such discussion must be acknowledged on the cover of your lab report, including the name of the person or persons and the exact nature of the help. Violations of this policy will be reported to the academic integrity board.

You may, however, do a joint report with your lab partners (turn in one report for your lab group). All partners get the same grade on the report but separate grades on pre-lab questions.

When you are done in the lab, you must have your TA sign one of your data sheets. This sheet must include the name of your lab partners and the time and date the lab was performed. The TA will not sign this sheet until your work station is clean and all equipment is accounted for. No lab reports will be accepted without this signed sheet.

CREDIT AND GRADING

Each lab is graded from 0-15. At the end of the semester the grades will be rescaled so that the average grade given by each TA is the same. This grade will be given to your recitation TA.

Grading:

You must attend the lab and turn in a report to get credit for the lab.

> +2 points for pre-lab questions.
> +5 points for attendance, competent in lab performance, and a minimally passable report.
> +5 points for clear neat report that is mostly correct.
> +3 points for perfectly correct report.
> +? points for observations that go beyond the direct questions.
> -? penalties (see the missed lab and late report policy).

PROBLEMS AND COMPLAINTS

1)   Your teaching assistant. The lab TA's job is to help you. See your TA if you have problems with the pre-lab questions, lab, or lab report. In-lab office hours will be available if you need to redo some part of the lab, or want to collect additional data (For this you can see any of the lab TAs.)

2)   Dan Mittler, 218 Kimball, (5-9172), dm68@cornell.edu. See Dan if you have problems with equipment operation. You may also arrange with him to redo part of a lab, or perform additional lab work.

3)   Alan Zehnder, 317 Kimball, (5-9181). See Professor Zehnder about problems with laboratory content or policy.

4)   Secretary, 212 Kimball (5-5062), pmm4@cornell.edu. See her if you have problems with your lab schedule, or need to make up a missed lab (read the policy, above).

LABORATORY NOTES

A rule of laboratory work is to keep a neat, complete record of what has been done, why it was done, how it was done, and what the result was.

The success or failure of an experiment in a research laboratory often depends critically upon the record made of the experiment. The outcome of a poorly documented experiment becomes a matter of personal recollection, which is not reliable enough to serve as a basis for further work, especially by someone else. You should take copious notes. If in doubt, WRITE IT DOWN. One can ignore what is written, but one can not resurrect that which was never recorded. Similarly, NEVER erase in your lab notes. If an erroneous reading was made, strike it out with a single line and record the new data. You may later decide that it was not in error.

All lab notes, in their original form, must be submitted with your report.

THE LAB REPORT

Your laboratory report should be neatly printed or typed. Do not crowd your writing. Make sure there is room for comments by your TA. This report should communicate clearly and convincingly what was demonstrated or suggested by the lab work. Your TA is looking for evidence of thought and understanding on your part. Your logic and methods are as important as results or "correct" answers. It is essential that you provide information and calculations which indicate how you arrived at your conclusions. It is permissible (and a good idea if you want a very good grade) *to discuss observations and material relevant to the lab which is not specifically asked about in the questions.*

Format:

The lab reports should contain the following material in the order specified.

I)   *Cover page*: A plain sheet, firmly attached to the rest of the report with staples or another binder. The cover should contain the following (with appropriate substitutions for the words in quotes):

        "**NAME OF THE LAB**"
        TAM 203
        By:
            "Your name and your signature (both partners if a joint report)"
        Performed:
            "Date"
        Performed with:
            "Name of person(s) with whom you performed the lab"
        Discussed lab with:
            "Names of people with whom you discussed the lab, and nature of the discussions"
        TA:
            "Laboratory Teaching Assistant's name"
        TA signed the data on page:
            "Page #"

II)   *Procedure*: (1/2 page maximum) This section should be included only when you deviate from the procedure specified in the lab manual. This section will be needed when there are problems with malfunctioning equipment, or if you develop your own procedure.

III)   *Answers to questions*: Concisely answer the questions that are asked and number them as they are numbered in the lab manual. Include any necessary plots, data or calculations. Your answers should be self-contained and presented in an orderly fashion (i.e. the reader of the report should not have to refer back to the questions that are asked, nor should he or she have to hunt through the report to find your answers). While many questions require that you perform calculations, written explanations of what you are doing and sketches can be very helpful. Show all calculations that you perform in arriving at your answers. If you are performing repetitive calculations, you need show only one sample calculation.

IV)   *Observations and conclusions*: If you did anything or observed anything in the lab which was not covered in your answers to questions, this is the place to discuss it. This is optional.

V)   *Supplemental procedures and questions*: If you choose, or are required, to do additional lab work, repeat sections II, III, and IV for each topic.

VI)   *Mistakes and suggestions:* This is an optional section. Point out errors in any of the documentation or oral information you were given. Make suggestions for changes in the lab procedure, instructions, content, etc. Please put this section on a separate page, so that it may be kept by the TA for future reference.

VII)   *Appendix:* Append ALL notes and records taken in the laboratory. (Including data sheets signed by your TA.) If you have used an *x-y* plot or data table in your previous answers to questions you need not include it here again.

DATA ANALYSIS AND PRESENTATION
[To be read carefully]

I)   *Significant figures*: When reporting numerical data, an appropriate number of significant figures should always be used. Large numbers should be written in scientific notation, so that the number of significant figures is not ambiguous.

The numbers 3.840, 0.003840, and 3.840×10^5 each have four significant figures.

When multiplying two numbers together, the general rule of thumb is to write the answer using the same number of digits as the multipliers. When the multipliers have different numbers of significant digits the smallest is used. Thus $0.3526 \times 1.2 = 0.42$ (not 0.42312). This same method should be used for division.

Addition is different. Consider the example: $0.2056 + 14.25 + 576.1 = 593.1$. An answer of 593.1276 is not appropriate because the last three digits (.0276) add nothing to the accuracy of the results, since one of the numbers being added (576.1) is accurate only to tenths. Subtraction should be done in a similar manner.

II)   *Percentage difference calculation*: Percentage difference calculations can be used to quantify how well experimental results agree with theoretical or expected values. Rather than writing "the experimental results agree very well with the theoretical calculations," this phrase can be changed to make a quantifiable statement; "the experimental results are within 5 percent of the theoretical calculations." Percentage difference is calculated as:

100% * (Value being compared - Reference value) / (Reference value)

Although formal error analysis should only be used if it is necessary to make a point, your answers should include some discussion of the types and relative sizes of errors in your data.

III)   *Units*: The dimensions of all physical quantities should be clearly presented in all calculations, tables and figures.

IV)   *Graphs*: Figure i.1 is an example of how your graphs should appear in lab reports. The following is a checklist of the items your graph should include.

- Use graph paper or computer.

- Curves should be drawn with rulers or French curves (not sketched).

- Graph title.

- Both axes should be titled, with the appropriate units listed in parentheses.

- Numerical values on the axes should be at reasonable intervals and scales be chosen so that all of the data points can be displayed on the graphs.

- On graphs with more than one curve a legend should be used to identify the curve. Data points can be enclosed by some symbol (*i.e.* circle, rectangle, etc.) to distinguish different data sets.

- The independent variable should be placed on the horizontal axis.

- All labels, symbols, etc., should be neat and readable.

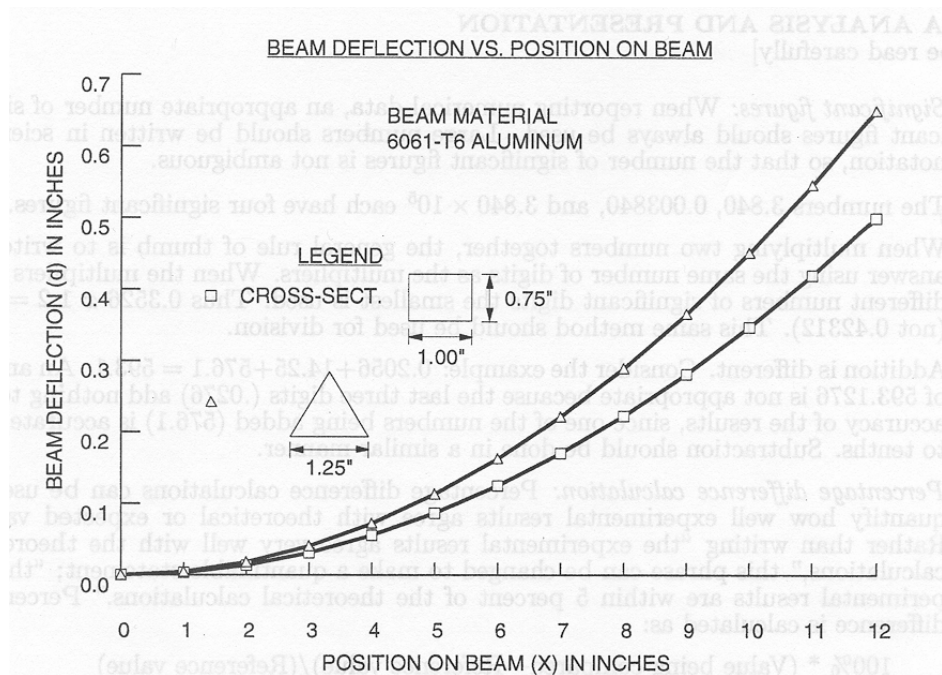- When plotting with a computer, the considerations of labeling axes, etc., still apply.

Figure i.1

V)   *Linear, semi-log and log-log plot interpretation*:

- Linear: a straight-line plot on linear graph paper indicates a relationship of the general form $y = mx + b$, where m is the slope and b is the y-axis intercept. Choose two points along the line (x_1, y_1), (x_2, y_2), preferably well separated. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Since $y_1 = mx_1 + b$,

$$b = y_1 - mx_1$$

Sometimes it is useful to plot a function of x and/or y, instead of plotting x and y directly. For example, if y^2 is proportional to x, you could plot y^2 vs. x, and obtain a straight line. The slope of the line then gives you the constant of proportionality. If you don't have (or don't want to use) semi-log or log-log graph paper, you can plot ln y vs. x or ln y vs. ln x, respectively. (Logs to other bases will also work.) This is often very useful with computer-generated graphs. The procedure to obtain an equation from the graph is similar to the one described above. For a graph of ln y vs. ln x which forms a straight line,

$$\ln y = m(\ln x) + b.$$

Find two well-separated points on the line, and write

$$m = \frac{(\ln y)_2 - (\ln y)_1}{(\ln x)_2 - (\ln x)_1}$$

since $(\ln y)_1 = m(\ln x)_1 + b$,

$$b = (\ln y)_1 - m(\ln x)_1.$$

Then

$$e^{\ln y} = e^{m(\ln x) + b} = e^b e^{\ln(x^m)},$$

and therefore

$$y = e^b x^m,$$

a power-law relationship (e^b is just a constant).

- Semi-log: a straight-line plot on semi-log paper indicates an exponential relationship of the general form $y = ae^{cx}$. Choose two points along the line, as above, reading the values from the graph paper scales. Then

$$c = \frac{\ln(y_2) - \ln(y_1)}{x_2 - x_1}.$$

Since $y_1 = ae^{cx_1}$,

$$a = \frac{y_1}{e^{cx_1}}$$

If you want to plot your data on a computer, use a log scale for the y axis, or plot $\ln y$ vs. x with linear scales, as described earlier.

- Log-log: a straight-line plot on log-log paper indicates a power-law relationship of the general form $y = ax^n$. Choose two points along the line, as above, reading the values from the graph paper scales. Then

$$n = \frac{\ln(y_2) - \ln(y_1)}{\ln(x_2) - \ln(x_1)}.$$

Since $y_1 = ax_1^n$,

$$a = \frac{y_1}{x_1^n}$$

If you want to plot your data on a computer, use log scales for both axes, or plot $\ln y$ vs. $\ln x$ with linear scales, as described earlier.

LABORATORY I
MASS-SPRING-DASHPOT
ONE DEGREE OF FREEDOM OSCILLATOR
WITH FORCING

Revised: August 2005

PRELAB QUESTIONS
[Due at the beginning of the lab period]

Read through the laboratory instructions and then answer these questions:

1. Derive the equation of motion for a forced mass, linear spring, linear dashpot system.

2. Solve or look up the solution if the forcing is given by $x_s(t) = A$.

3. Define in words natural frequency, damping coefficient, critical damping coefficient, underdamped, overdamped, resonance, phase shift, amplitude ratio.

4. What will you calculate and measure during the lab period? Are you prepared to do this?

5. Find the phase shift between the curves below, using the formula provided. Report an answer $-\pi < \phi \le \pi$ in radians. The values t_1 and t_2 refer to adjacent points of the same type on the two curves (e.g., two zero crossings or two peaks).
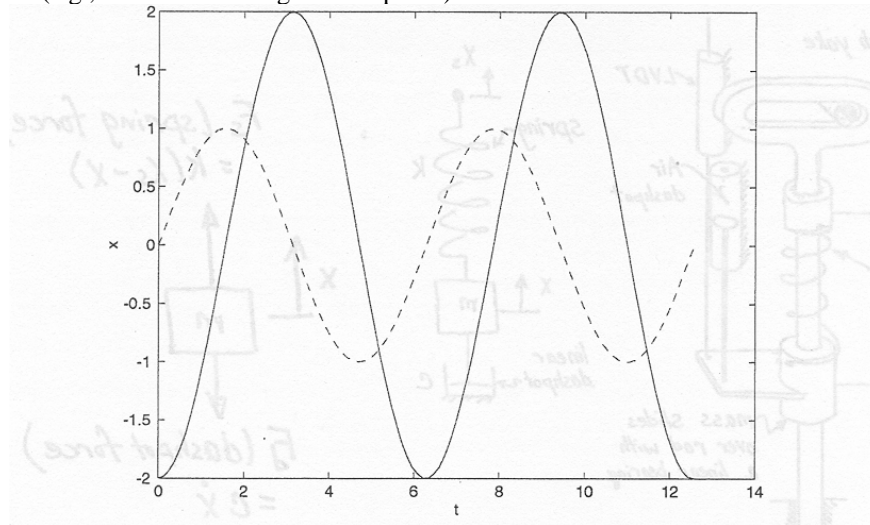

Fig. II.1

6. Suppose that you are measuring two sinusoidal waveforms of equal amplitude, $x_1(t)$ and $x_2(t)$, with a phase difference of $\pi/2$. What would be the shape of the curve if you plotted $x_1(t)$ vs. $x_2(t)$? What if the phase difference is zero? π? If you have trouble visualizing the situation, try calculating a few points and plotting them.

INTRODUCTION

The mass-spring-dashpot is the prototype of all vibrating or oscillating systems. With varying degrees of approximation, car suspensions, violin strings, buildings responding to earthquakes, earthquake faults themselves, and vibrating machines are modeled as mass-spring dashpot systems. This laboratory is aimed at demonstrating some of the basic concepts of the mass-spring-dashpot system. Additionally, the computer solution of the governing differential equations will be demonstrated with a computer simulation program. Phrases connected with some of the key ideas are: 'natural frequency', 'resonance', 'forcing function', and 'frequency response'.

MASS-SPRING-DASHPOT

The picture in Figure II.2a shows (crudely) the laboratory mass-spring-dashpot or one degree of freedom oscillator. A mass is supported by a spring and is also constrained to slide on a rod. In this lab you will record the vertical motion of the mass both with the support still and with the support oscillating vertically. The middle picture shows an idealization of the laboratory apparatus. The spring is modeled as linear (the force it applies is proportional to its increase in length) with proportionality constant k. The damping is produced by a linear air dashpot. The force transmitted by a linear dashpot is proportional to the rate at which it is being stretched with proportionality constant c. The vertical displacement of the mass is x and the vertical displacement of the support is x_s.
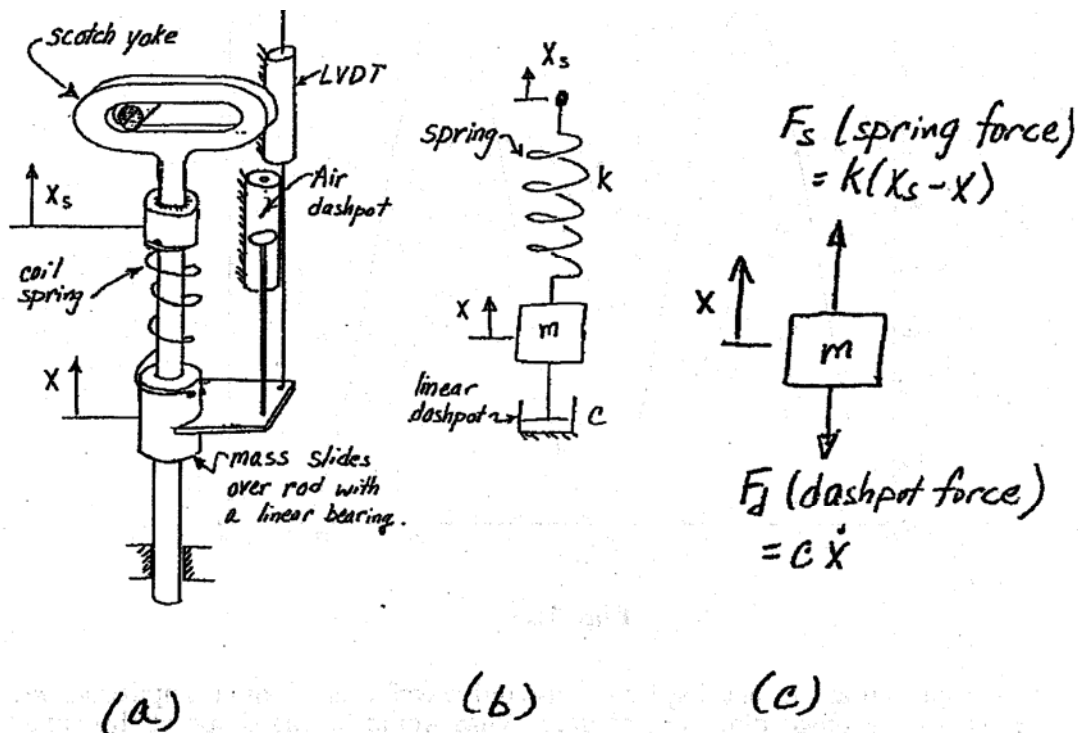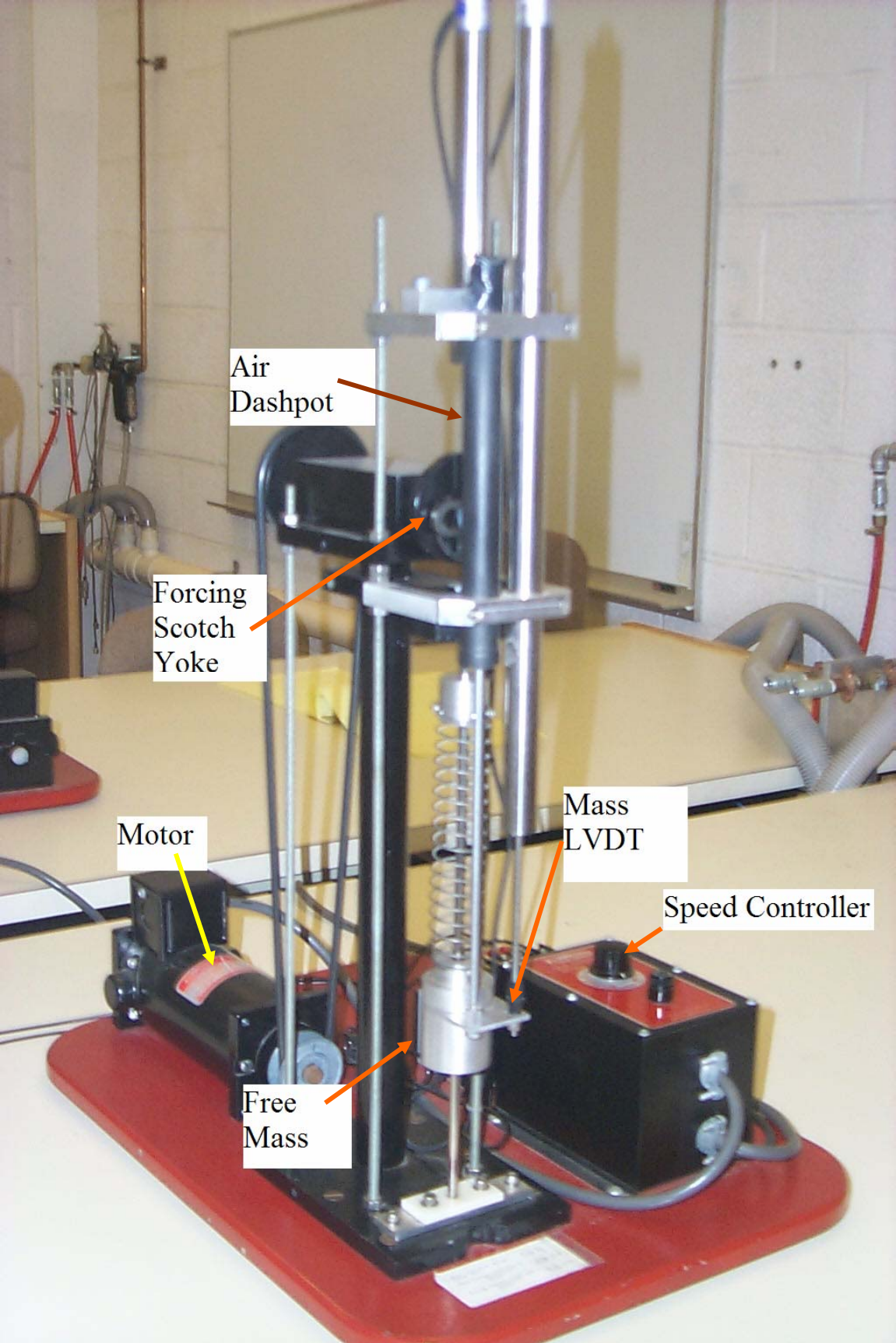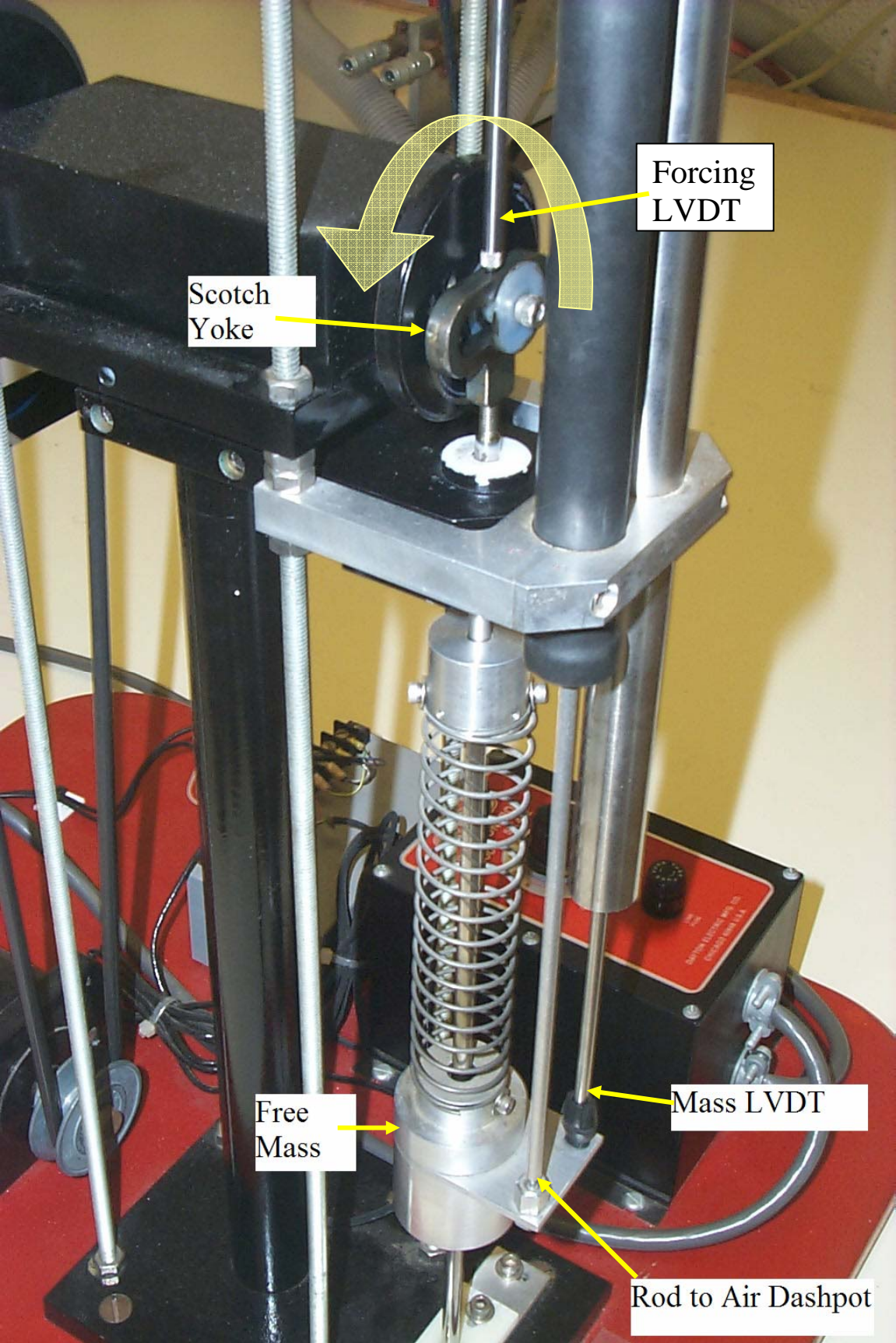

Fig. II.2

Air
Dashpot

Forcing
Scotch
Yoke

Motor

Mass
LVDT

Speed Controller

Free
Mass

Forcing LVDT

Scotch Yoke

Free Mass

Mass LVDT

Rod to Air Dashpot

The picture on the right is a free body diagram of the mass. It has two forces acting on it, neglecting gravity (When can gravity be neglected?):

F_s = The spring force = $k(x_s - x)$, (assuming a linear spring)

F_d = The dashpot force = $c \; dx/dt$, (assuming a linear dashpot).

The system is a one degree of freedom system since a single coordinate is sufficient to describe the complete motion of the system. (The support displacement x_s does not count as a degree of freedom since it is specified by the motor position, over which we assume we have complete control.)

The equation of motion for this system is

$$m\ddot{x} = -F_d + F_s \text{ where } \ddot{x} = d^2x/dt^2 .$$

Assuming a linear spring and a linear dashpot this becomes

(1) $$m\ddot{x} + c\dot{x} + kx = F(t) \text{ with } F(t) = kx_s .$$

where $F(t)$ is the (presumably specified) excitation 'forcing function'. In this case the forcing function is the position of the end of the spring as a function of time, multiplied by the spring stiffness.

The air dashpot provides resistance to motion by drawing air in and out of the cylinder through a small opening at the top of the cylinder. Due to the small, but nonzero viscosity of air, a pressure drop is created across the opening that is linearly proportional to the speed of the air flowing through. This produces linear damping. Nonlinearities are introduced due to the friction between the piston and the cylinder. Note also that the compressibility of the air in the dashpot introduces some springiness to the system in addition to the coil spring. The compressibility of the air may be thought of as a spring in series with the dashpot.

In the slider-crank part of this experiment you will attempt to determine the value of the viscous damping constant c by measuring the rate at which oscillations decay towards zero. In addition, the system response to both free vibration and 'forced' motion will be observed experimentally and through computer simulation.

REAL-WORLD EXAMPLE: LOUDSPEAKER

A speaker, similar to the ones used in many home and auto speaker systems, is one of many devices which may be conveniently modeled as a one-degree-of-freedom mass-spring-dashpot system. The one you will observe in this lab is typical. It has a plastic cone, supported at the edges by a roll of plastic foam (the surround), and guided at the center by a cloth bellows (the spider). It has a large magnet structure, and (not visible from outside) a coil of wire attached to the point of the cone, which can slide up and down inside the magnet. (The device described above is, strictly speaking, the speaker driver. A complete speaker system includes an enclosure, one or more drivers, and various electronic components.) When you turn on your stereo, it forces a current through the coil in time with the music, causing the coil to alternately attract and repel the magnet. This results in the vibration of the cone which you hear as sound.

In the speaker, the primary mass is comprised of the coil, cone, and (in this case) LVDT core. The 'spring' and 'dashpot' effects in the system are due to the foam and cloth supporting the cone, and perhaps to
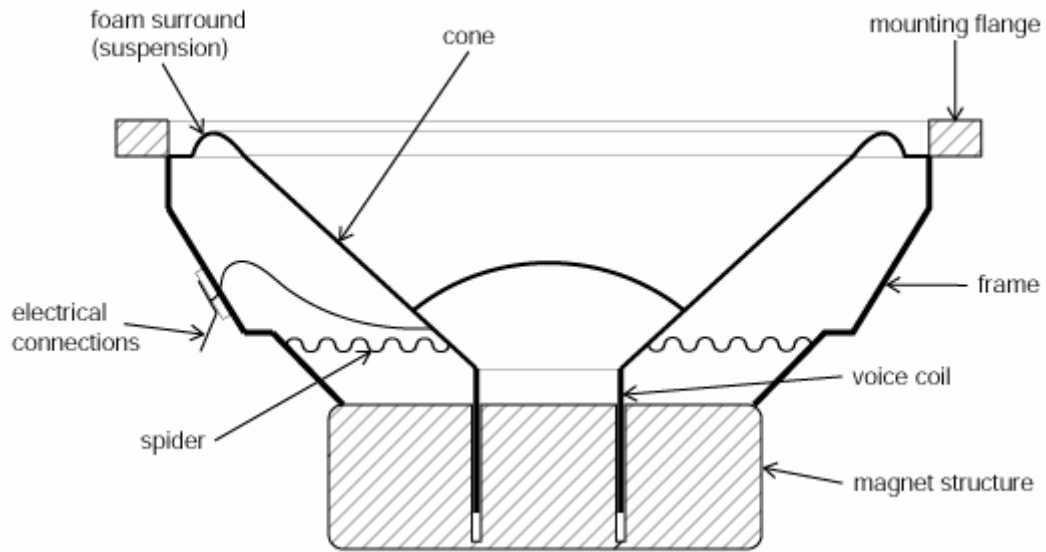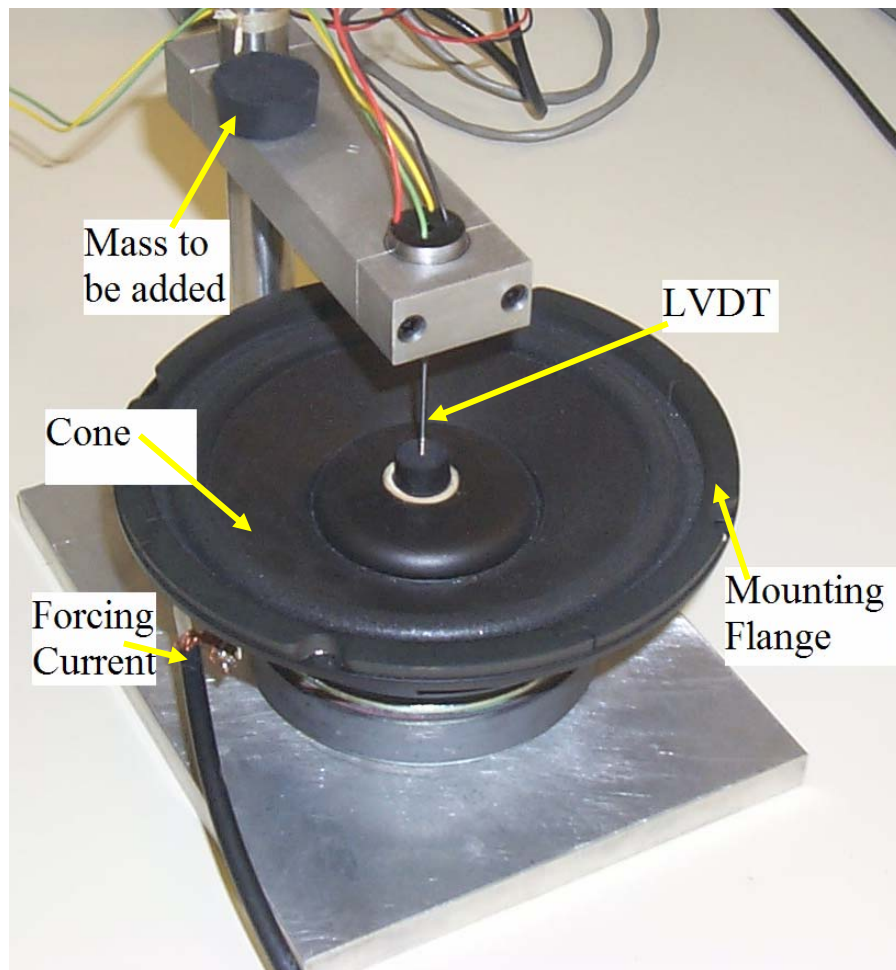
Fig. II.3 Cross-sectional view of a speaker.

various magnetic effects. Speaker system design is greatly complicated by the fact that the air surrounding the speaker must also be taken into account. Changing the shape of the speaker enclosure can change the effective values of all three mass-spring-dashpot parameters. (You may be able to observe this by cupping your hands over the speaker (gently, without touching the moving parts), and observing amplitude or phase changes.) Nevertheless, knowledge of the basic characteristics of a speaker (e.g., resonant frequency), is essential in speaker system design.

The equation of motion for the speaker is similar to that of the laboratory mass-spring-dashpot above, except the forcing function is electrical, rather than mechanical:

$$m\ddot{x} + c\dot{x} + kx = F(t) \text{ with } F(t) = ai(t)$$

where $i(t)$ is the electrical current flow through the coil in amps, and a is the electromechanical coupling coefficient, in newtons per amp. In the second part of this experiment, the current flow through the speaker will be generated, controlled, and measured using a waveform generation and a data acquisition program. Using this data, the effective mass, damping coefficient, and spring constant for the speaker will be calculated.

SOLUTION OF THE GOVERNING EQUATIONS

We wish to know the motion of the mass $x(t)$ for a given forcing $F(t)$. Two cases are of particular interest:

(2) $$F(t) = 0 \text{ (unforced or 'free')}$$

and

(3) $$F(t) = kx_s(t) = \text{sinusoidal oscillation}$$

Equation (1) is a linear, second order ordinary differential equation with constant coefficients. This solution with F(t) given by either (2) (homogeneous) or (3) (inhomogeneous) is discussed in every freshman and sophomore math text.

Briefly, the solution can be found as follows:

Assume a solution of the form $x(t) = \text{Real part of} [Ae^{(\lambda t)}]$. If $F(t) = 0$ we must find λ. If $F(t)$ is sinusoidal in time with angular frequency ω (e.g. $F(t) = \sin(\omega t)$) then $\lambda = \omega i$ is used and the amplitude A of the motion is of primary interest. (This procedure breaks down in the special case of undamped resonance, *i.e.* when $c = 0$ and forcing frequency $\omega = [k/m]^{1/2}$) The 'homogeneous' solution, although still part of the solution, is often ignored when looking at the forced response, since it decays towards zero with time while the 'inhomogeneous' solution remains finite.

Damped Free Vibration

In the case F(t) = 0, the application of the assumed form of the solution leads to a quadratic equation for λ :

$$\lambda^2 + (c/m)\lambda + (k/m) = 0$$

This characteristic equation has two roots

$$\lambda_{1,2} = -c/2m \pm [(c/2m)^2 - k/m]^{1/2}$$

The general solution of the differential equation is then

$$x(t) = \text{Re}[A_1 e^{(\lambda_1 t)} + A_2 e^{(\lambda_2 t)}]$$

where A_1 and A_2 are constants determined by the initial displacements and velocity of the mass.

In general, damping of some kind is present in all systems $(c > 0)$ Depending on the amount of damping, the system has one of three types of behaviors:

1) undercritically damped (little dissipation, c is 'small')
2) critically damped
3) overcritically damped (a lot of dissipation, c is 'large')

(See your math text for a mathematical analysis or further explanation.)

In this laboratory the system is undercritically damped so that the mass oscillates as it comes to rest.

The damping constant of a mass-spring system, being zero, is the extreme limit of undercritical damping. Though all systems have some dissipation this approximation is frequently employed if frictional effects seem small. The equation of motion then becomes:

$$m\ddot{x} + kx = 0$$

This equation has the solution

$$x(t) = B_1 \sin[(k/m)^{1/2} t] + B_2 \cos[(k/m)^{1/2} t].$$

The system oscillates at its undamped natural frequency $\omega = (k/m)^{1/2}$. The amplitude of the oscillation is constant with time and this motion is called undamped free vibration. You cannot graduate from college (with a clean conscience) without knowing the above equation and solution quite well.
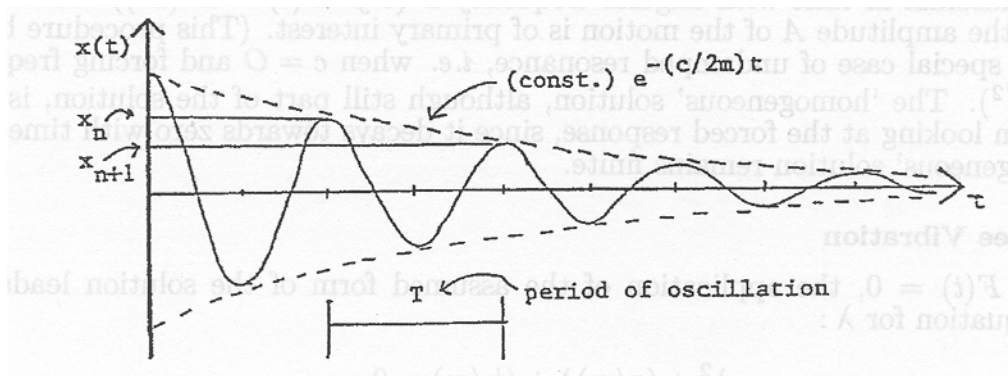

Fig. II.4

Logarithmic Decrement Method

The viscous damping constant, c, may be determined experimentally by measuring the rate of decay of unforced oscillation. The logarithmic decrement, which is the natural logarithm of the ratio of any two successive amplitudes, is used. The larger the damping, the greater will be the rate of decay of oscillations and the bigger the logarithmic decrement:

$$\text{logarithmic decrement } D = \ln(x_n/x_{n+1})$$

where x_n and x_{n+1} are the heights of two successive peaks in the decaying oscillation pictured above.

Because of the exponential envelope that this curve has, $x_n = (const.)e^{-(\frac{c}{2m})t_1}$ and

$x_{n+1} = (const.)e^{-(\frac{c}{2m})(t_1+T)}$. Then

$$D = \ln[(e^{-(\frac{c}{2m})t_1})\Big/ e^{-(\frac{e}{2m})(t_1+T)}]$$

and simplifying,

$$D = \frac{cT}{2m}$$

where T is the period of oscillation. Thus, the damping constant c is given by

$$c = \frac{2mD}{T}$$

Forced Vibration

When the forcing function is sinusoidal with frequency ω, steady state solutions, $x(t)$, which are sinusoidal in time with frequency ω can be found. The amplitude of these sine waves depends, of course, on both the amplitude A and the frequency ω of the forcing. In particular, $x(t)$ has quite a large amplitude when the forcing frequency ω is close to the natural vibration frequency, ω_n, of the system.

Other Useful Formulae

$$\omega_n^2 = k/m$$
$$\omega = 2\pi f$$
$$f = 1/T$$
$$\text{phase shift (rad)} = 2\pi[\frac{t_2-t_1}{T}]$$
$$c_c = 2m\omega_n$$
$$\omega_d = \omega_n\sqrt{1-(\frac{c}{c_c})^2}$$

A) LAB MASS-SPRING-DASHPOT AND SIMULATION
[Four lab groups should start with this part, while two groups start with the speaker lab.]

Procedure

1) *Review the equipment*: The apparatus consists of a laboratory-model mass-springdashpot system with displacement transducers (LVDTs or Linear Variable Differential Transformers) for measuring $x(t)$ and $x_s(t)$. The output from the LVDTs goes to the data acquisition board. An electric motor and controller, acting through a scotch yoke, allow a sinusoidal forcing function to be applied to the system. Note that the controller dial readings are arbitrary; frequency and period data must be obtained from your computer plots.

2) *Safety precautions*: Keep long hair and loose clothing well away from the electric motor, pulleys, and other moving parts.

3) *Using the data acquisition programs*: The four programs you will be using for part A are: "FreeAcq," (Fig. II.5) for making measurements of the unforced system; "FreeSim," (Fig. II.6) for simulation of same; "ForcedAcq," (Fig. II.7) for measurements of the system with a sinusoidal forcing function; and "ForcedSim," (Fig. II.8) which may be used for simulation of the forced system. Although somewhat different in appearance and function, the programs share the following features:

   a) *Set-up*: The data acquisition programs automatically convert the voltage output of the LVDTs to meters. To do this, they need a set of conversion factors, which are on a label on the mass-spring-dashpot base board. After starting a measurement program, make sure that the sensitivity and offset values on the left hand side of the window match the values listed on a small sheet of paper in front of the apparatus. Also please enter your name in the box provided.

   b) *Running the Program*: To run the program, you must hit the white arrow in the top left of the screen. If this arrow is black, that means that the program is already running. For the data 8

   c) acquisition programs, a green box on top will define the amount of time the program will record the motion of the mass-spring after hitting the arrow. To reset the data acquisition, press on "STOP without Saving" and then press the white arrow to begin again.

   d) *Saving data*: After getting data pressing the "Save and STOP" button stores your current data on disk, for comparison later to the simulation. Any previous data is erased. The data file is only used by the simulation programs FreeSim and ForcedSim; it is not available to the data acquisition programs.

   e) *Cursors*: You may find it convenient to obtain numerical data from your plots using the cursors, rather than using a ruler. Two cursors are available, one indicated by a circle and one by a square. To use a cursor, use the mouse to drag it to the point you want to measure. The x and y values of the point you have chosen will appear above the graph, in the row corresponding to that cursor. For best accuracy, you should utilize every full cycle on the screen. For example, if three cycles are displayed, you should measure the time elapsed from the start of the first cycle to the end of the third, and then divide Δx by three to obtain the period.
   If your cursor has vanished off the screen, you can enter an on-screen position for it into the x and y display boxes, and it will reappear in the desired location. You can also move the cursors around using the little arrow "buttons" on the screen in the cursor control box. If a cursor turns a darker color, it is locked on to a data point, and will trace the curve point by point if the left or right arrow is pressed. Zoom and other features are available for the cursors and graphs; see the LabView manual for details.

4) *Free Vibration, Mass-Spring-Dashpot*:
   a) Here you will be recording the motion of the mass after it has been pulled out of equilibrium and then released. First start up the FreeAcq program (Fig. II.5) check the

calibration. Start with a data acquisition time of 3 seconds. Give the system initial conditions $x(0) \neq 0, v(0) = 0$ (i.e., pull down the mass), press the white run arrow in the top left, and then immediately release the mass. Repeat this procedure until you have a nice curve develop over the three seconds. Note that you will not be able to see the curve until you save it and then load it in FreeSim. Note that the zero position is somewhat arbitrary and will depend on the position of the scotch yoke when the mass is released. Also, the zero level for the scotch yoke and for the mass may not agree exactly. You may need to take data long enough for the mass to stop oscillating, so that you have a good zero reference. Save one of the curves on disk for later comparison to simulation. (Press *"Save and STOP"* button).

b) Next, start the program FreeSim (Fig. II.6) to simulate free vibration of the massspring-dashpot system. If you want to compare the simulation data to measured data you had saved earlier, turn the measured-data switch on; otherwise, turn it off. To change a simulation parameter, you can either type in the value you want and press enter, or click on the 'up' or 'down' arrows. The following parameters can be changed in the simulation:

k is the spring constant, in newtons/meter.
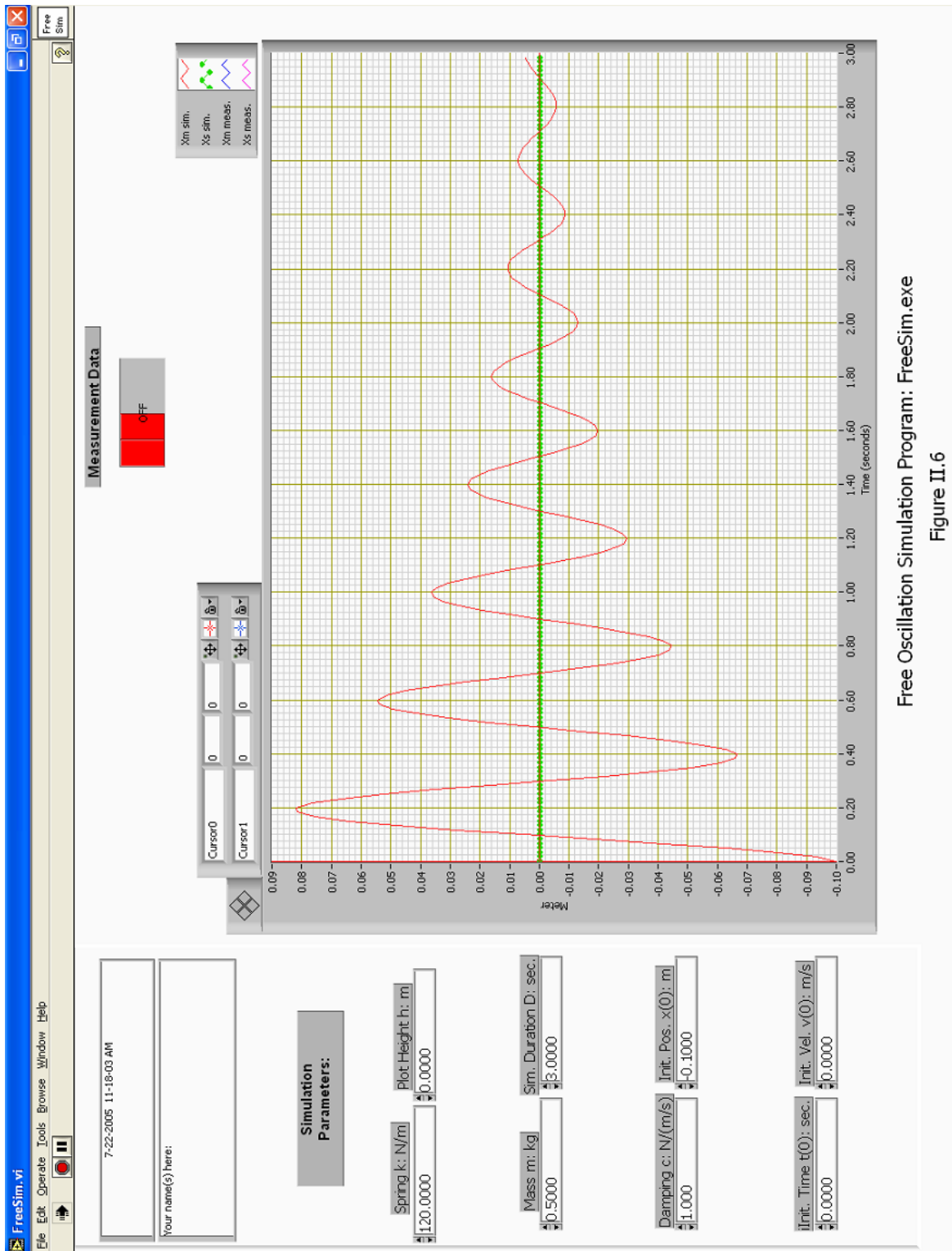
m is the mass, in kilograms. (In looking at a real system you may need to think about what should be included here.)
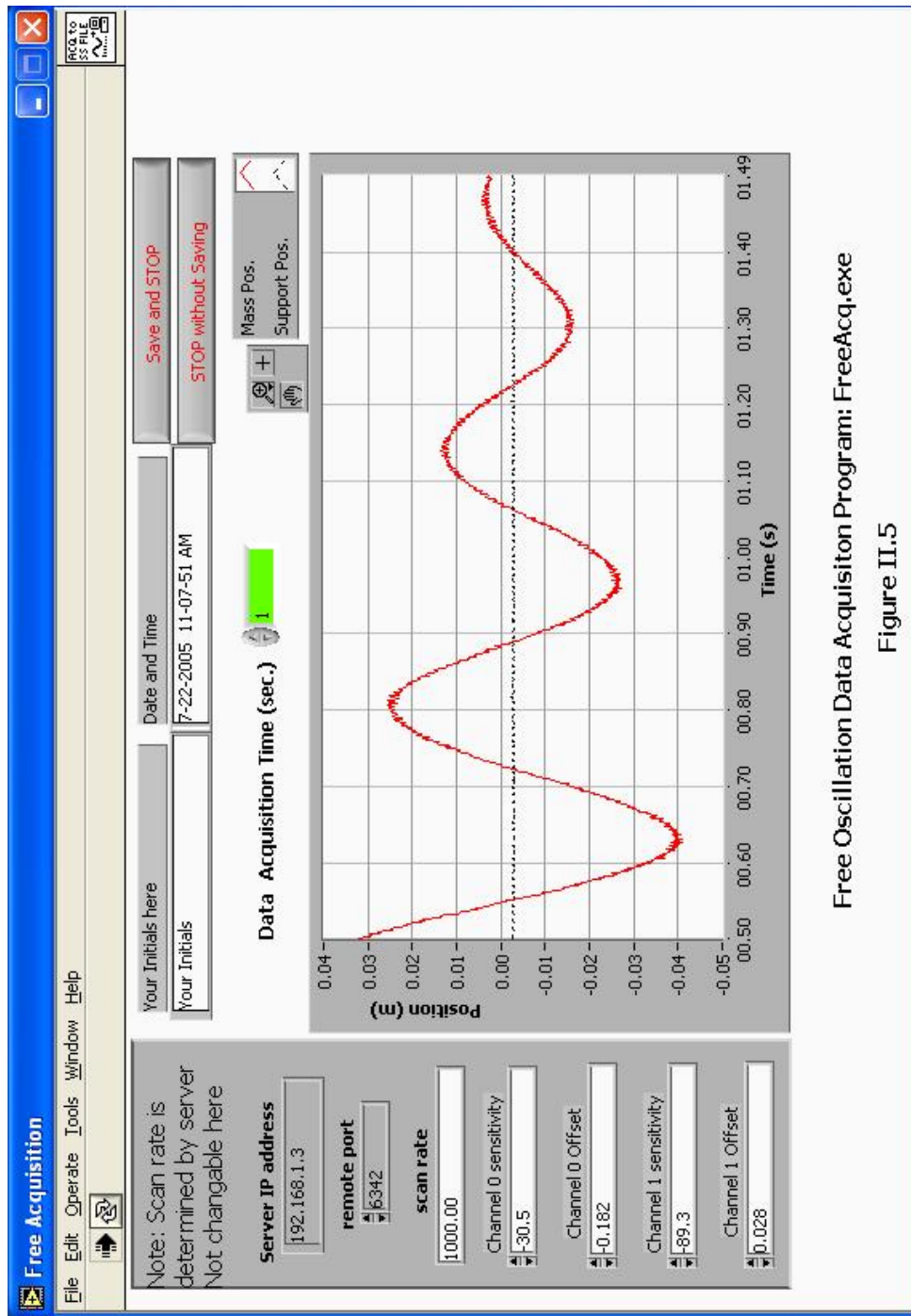
Fig. II.5

Fig. II.6

Forced Oscillation Data Acquisition Program: ForcedAcq.exe

Figure II.7

Fig. II.7

Fig. II.8

c is the damping coefficient, in newtons/meter/second.

$t(0)$ (in seconds) lets you adjust the relative starting point of the simulation plot. Essentially, it lets you move the plot left or right, as necessary, making comparison with the measured data easier.

h (in meters) lets you adjust the simulation plot up or down.

D (in seconds) is the numbers of seconds worth of simulation data plotted. Set it equal to the duration of the measured data set, if you are comparing them.

$x(0)$ is the initial position of the mass, in meters (probably the distance you pulled it down).

$v(0)$ is the initial velocity of the mass, in meters/second (probably zero).

Add the measured data to the graph by pressing on the switch above the graph. Using the cursors, measure the logarithmic decrement of the measured data to determine the damping coefficient, c, and measure the frequency of oscillation. Check if c is constant by measuring the logarithmic decrement for several separate cycles. Determine k, the spring stiffness. (See the *Useful Formulae* section). Make a print-out of one of your curves.

Simulate unforced motion using the values of m, k and c that you just determined in the FreeSim parameters. Obtain $x(0)$ from your measured data. How far down did you pull the mass? Compare your simulation with your measured data. If agreement is not good, adjust k and c until you have good agreement. Make a print-out. How do changes in k and c affect the results?

5) *Forced Vibration, Mass-Spring-Dashpot*:

a) Start up the ForcedAcq program (Fig. II.7), set the acquisition time to 30 seconds and turn on the data acquisition, and turn on the motor. Try various speeds and play with the program. Two graphs are displayed. The left one is a plot of the mass position and spring support position (forcing) vs. time. The right one plots mass position vs. support position, and helps show phase relationships. Observe how different driving frequencies affect the motion of the mass. Observe resonance (*i.e.* large amplitudes) when the forcing frequency is just right. Observe the plot of $x(t)$ vs. $x_s(t)$. How does this change as one goes through resonance?

b) For at least five different forcing speeds with a large range of values get nice screen plots of several cycles of motion. The forcing speeds should include:

- The slowest speed for which the motor runs smoothly.
- A very fast speed.
- Resonance (the speed that maximizes the block motion).
- A speed just less than resonance.
- A speed just greater than resonance.

You may make a printout of one or two of these plots, if desired, but save each plot to analyze them in ForcedSim. Once finished saving each of these plots, open ForcedSim.exe and then look at your saved plots by turning on the measured data switch. To change the measured data to one of your other plots, you must reopen ForcedSim.exe. Once looking at the measured data, make the measurements you need using the computer screen cursors ForcedSim.exe. You will need to make sufficient measurements to make graphs of amplitude ratio and phase shift versus frequency. In particular, you should make measurements of the period, forcing function amplitude, mass motion amplitude, and phase lag between the forcing function and the resulting mass motion, for each of

your five speeds. You can then enter in your calculated values into ForcedSim and see how well it models the measured data. The parameters in ForcedSim include most of the ones described for the FreeSim program, plus the following:

x_s is the amplitude (in meters) of the motion of the spring support, which is moved up and down by the motor and scotch yoke. This motion supplies the forcing of the system.

ω is the angular velocity of the spring support motion, in radians per second.

You may also want to save the data to a USB storage device or write it to a CD for later analysis. To do this just copy the text files of the desired data onto your storage device.

6)   (Optional) Put the scotch yoke in the middle of its range. Use the manual mode of the data acquisition system and quickly turn the motor knob up to a fairly high speed. The complicated motion which results is a sum of the free vibration and steady-state forced vibration observed above. You can also save this data and store it on USB or CD for later analysis.


Questions: (Part A)
[Remember to follow the format given in the Introduction]

A.1)   What are k and c for your mass-spring-dashpot setup? Show how you calculated these values. Is c constant? What does this say about the air dashpot acting linearly?

A.2)   Discuss the comparison of simulated to measured unforced motion. Comment on any similarities or differences of interest. Comment on any adjustments you made in k or c to better fit the data. Show the best corrected plot if you made one.

A.3)   Make a plot of the amplitude ratio (peak mass displacement divided by peak forcing displacement) versus forcing frequencies.

A.4)   Make a plot of phase angle between $x(t)$ and $x_s(t)$ versus forcing frequency.

A.5)   For a typical value of c that you measured, what is the percent difference between the natural frequency (ω_n) and the damped natural frequency (ω_d). Does the addition of a dashpot to a mass-spring system speed up its oscillation frequency, or slow it down?

A.6)   Discuss the plots from (A.3) and (A.4). Do they look like what you expect based on textbook solutions to the damped, one degree of freedom oscillator? Relate the phase angle plot to the $x(t)$ vs. $x_s(t)$ plots. Why do the ellipses change shape and rotate as you go through resonance? State your observations about the behavior of the mass as the forcing frequency is varied in words without using numbers, angles, variables, graphs, or equations.

B) VIBRATION OF A SPEAKER
Procedure:

1)   *Look over the apparatus*: Speaker on a stand, with one LVDT to measure cone displacement. Waveforms are generated by the computer, amplified, and sent through a resistor to the speaker (thus approximating a current source). The computer is also used to measure current flow through the speaker and displacement of its cone (using the attached LVDT).

2)   First, open the folder named speaker on the desktop and then open Speaker.exe. Initially, set up the waveform generator controls as follows, to obtain a maximum-amplitude square wave at about 5 hz: Set the 'Waveform' control to 'Square', the 'Frequency' control to 5, and the 'Amplitude' control to 2. Leave the 'DC Offset' control at zero. Note that when changing waveforms or frequencies, you must wait a few seconds for the computer to equilibrate and output correct data.
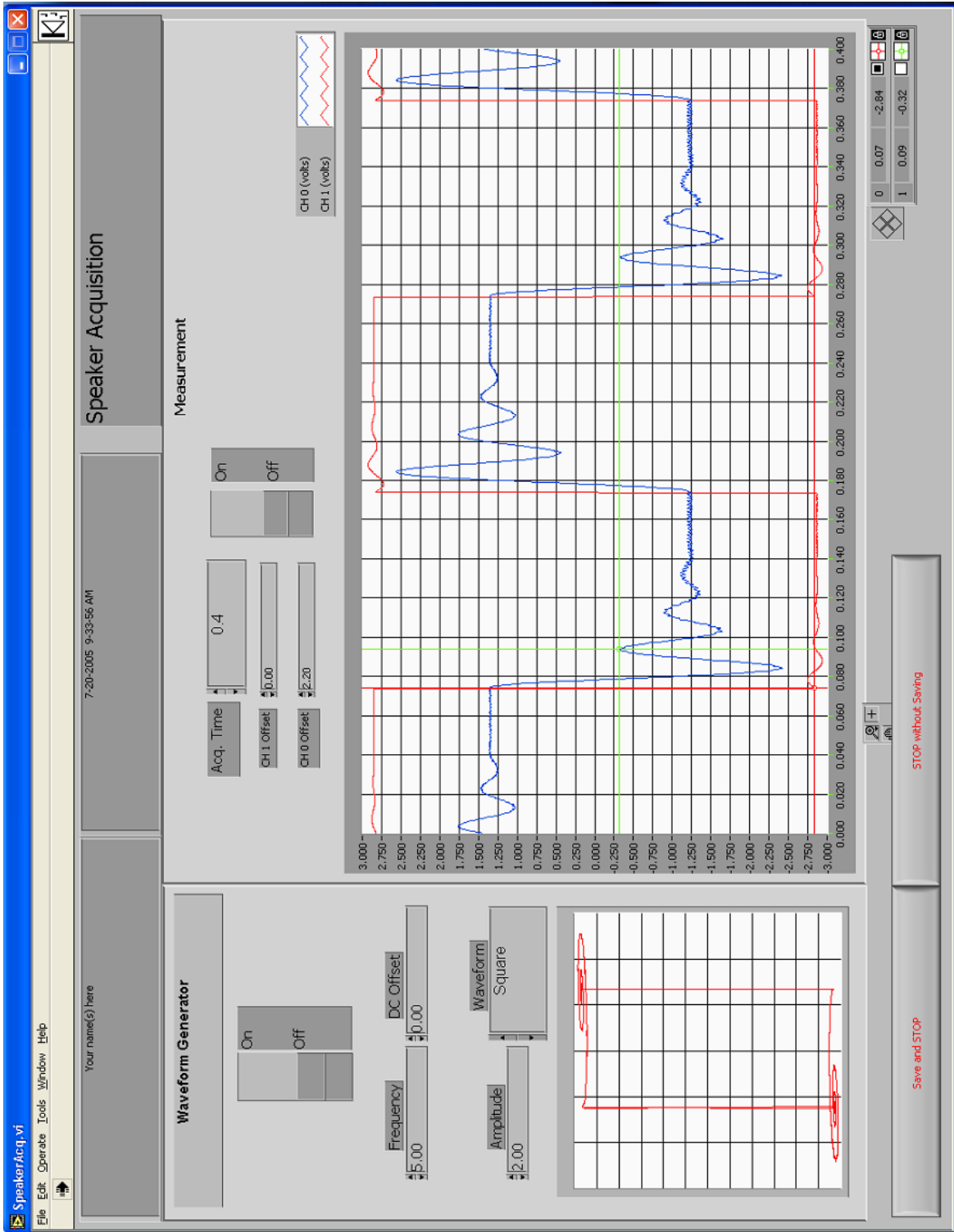
Speaker with a Square Wave
Fig. II.9

Set the data acquisition time control to about 0.4 seconds. The displacement of the cone is displayed as channel 0 and as a blue line, and the current flow through the speaker coil (the forcing function) is displayed as channel 1 and a red line. What you want to see is a square wave with 'ringing' wiggles after each shift in level that gradually die out. Use a low enough frequency for the square wave that the ringing dies out completely before the square wave again changes level. Be careful not to shake the table during the experiment as small vibrations can cause errors. This is the step response of the speaker, approximately equivalent (in this lab) to the free response you obtained by pulling down the mass and releasing it. When you have a good display of the ringing, turn off the data acquisition.

3)   Using the green cursor, measure the logarithmic decrement. Try to measure the coordinates of at least three successive peaks of the blue curve, yielding at least two values of the logarithmic decrement. Remember to measure to the equilibrium level (the level your exponential decay curve ends up at), not to the zero level of the plot. Measure the period of oscillation of the ringing.

4)   Next, set the waveform generator controls to put out a 2 volt sine wave. To do this, set the 'Waveform' control to 'Sine' and the 'Amplitude' control to 2. Leave the 'DC Offset' control at zero. Leave the data acquisition time to 0.1 seconds. The 'CH 0 Offset' and 'CH 1 Offset' controls may be used to move the plots up and down, if necessary. Turn on the waveform generator and data acquisition "switches," and adjust the 'Frequency' control value until you observe resonance of the speaker cone. To change the frequency, you must press "STOP without Saving" then enter the desired frequency into the numerical box and then press "run"(the white arrow in the top left) to observe the new frequency. (hint: it helps to watch the displacement vs. forcing function plot in the bottom left of your screen; i.e., CH 0 vs. CH 1).

4)   Neither the spring constant nor the mass of the speaker is easily measured at resonance. However, you can observe what happens when the mass is changed a known amount, and then derive the approximate mass and spring constant. Carefully press a rubber weight onto the LVDT shaft. The best way is to spread the weight open, position it, and release it. If you don't think you can put it on gently enough, ask your lab partner or TA do it. Find the new resonant frequency, as above. Record the mass of the rubber weight.

Questions:

B.1)   Calculate k and m for the speaker, using the resonant frequencies and mass you measured in lab.

B.2)   Calculate k, the damping coefficient, for the speaker. Is the speaker overdamped or underdamped? How linear was the speaker damping?

B.3)   Find another real-world vibrating system which could be reasonably modeled as a mass-spring-dashpot. Give the system a "push" and observe its response. Try applying a forcing function of various frequencies, and look for resonance.

    a) Describe how you modeled your vibrating system as a mass-spring-dashpot. That is, what does the mass represent, what is the spring, and what is the dashpot? Be as specific as possible.

    b) Is this system typically overdamped? Underdamped? If applicable, what was the resonant frequency (approximately)?

    C) In what ways does the system you found most significantly differ from an ideal linear mass-spring-dashpot system?

LABORATORY II
TWO-DEGREE-OF-FREEDOM OSCILLATOR

Revised: August 2005

PRE-LAB QUESTIONS

1)   Are the number of degrees of freedom of a system and the number of its normal modes related? If so, how?

2)   How can a normal mode be recognized physically?

3)   What data will you record during the lab?

4)   What observations will you make?

5)   What do you expect to happen when you drive a system at one of its natural frequencies?

6)   Draw a free body diagram and derive the equations of motion for a three-degree-of-freedom system, with three different masses and four equal springs. Put them in matrix form. (See the derivation for a two-degree-of-freedom system in the lab manual. Your result should resemble equation (4).) Substitute in the normal mode solution (1), to get an eigenvalue problem similar to (5).
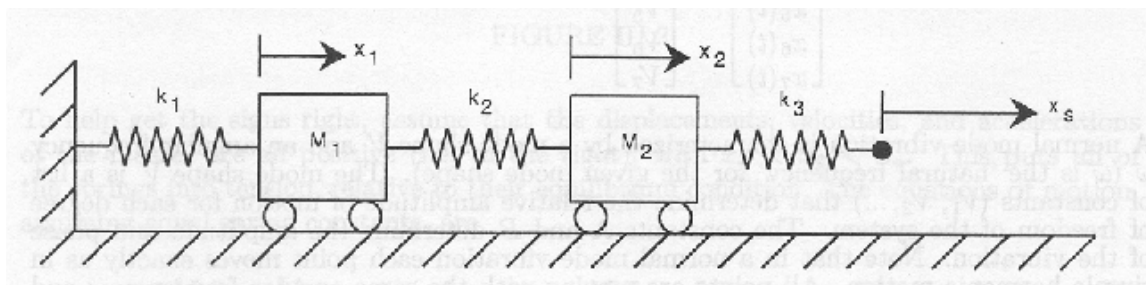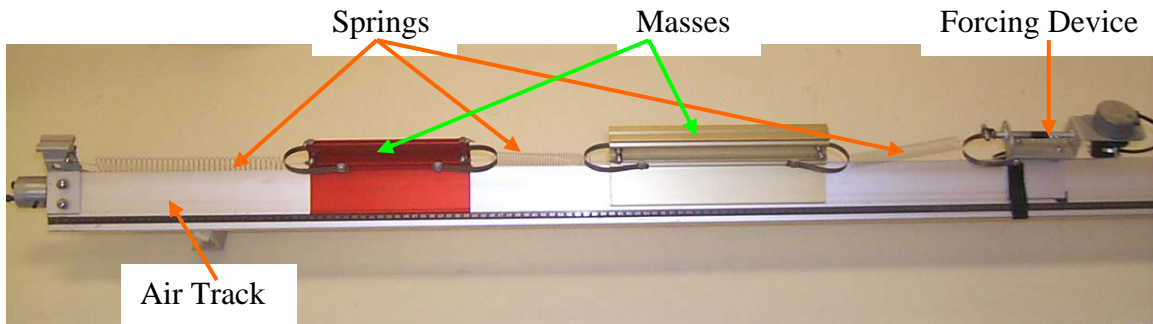
INTRODUCTION

FIGURE III.1

Springs   Masses   Forcing Device

Air Track

The system illustrated above has two degrees of freedom. This means that two is the minimum number of coordinates necessary to uniquely specify the position of the system. The purpose of this laboratory is to introduce you to some of the properties of linear vibrating systems with two or more degrees of freedom. You have already seen a one-degree-of-freedom vibrating system (the spring-mass-dashpot) and should have some familiarity with the ideas of natural frequency and resonance. These ideas still apply to an undamped linear system with two or more degrees of freedom.

The new idea for many degrees of freedom systems is the concept of 'modes' (also called 'normal modes'). Each 'mode shape' has its own natural frequency and will resonate if forced at that frequency. The number of modes a system has is equal to the number of degrees of freedom. Thus the system above has two modes and two natural frequencies. The primary goal of this laboratory is for you to learn the concept of normal modes in a two degree of freedom system – the simplest system which exhibits such modes. You will learn this by experimentation and calculation.

NORMAL MODES – GENERAL

The concept of normal modes can be expressed mathematically in the following way. Say the position of an n degree of freedom system can be described by the n numbers $x_1, x_2, x_3...x_n$ (this is, in fact, the definition of an n degree of freedom system). Since the system is dynamic, each of these variables is a function of time $x_1(t)$, $x_2(t)$ etc. A motion of the system corresponds to a specified list of these functions. In general these functions of time can be quite complicated. However, for linear undamped systems there turn out to be many solutions that are, in some sense, simple. In fact, there are n such simple solutions called normal mode vibrations. A fortunate and often used fact is that every possible solution of the system can be written as a sum of these solutions. (In the language of linear algebra one can say that the normal mode solutions span the space of all solutions.) A normal mode solution for a seven-degree-of-freedom system looks like this:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} \left[A\cos\omega t + B\sin\omega t \right] \tag{1}$$

A normal mode vibration is characterized by a mode shape V and an angular frequency ω (ω is the 'natural frequency' for the given mode shape). The mode shape V is a list of constants $(V_1, V_2,...)$ that determine the relative amplitude of motion for each degree of freedom of the system. The constants A and B determine the amplitude and phase of the vibration. Note that in a normal mode vibration each point moves exactly as in simple harmonic motion. All points are moving with the same angular frequency ω and are exactly in phase or exactly out of phase, depending on the signs of the appropriate elements of V.

The general motion of an n degree of freedom undamped linear vibrating system can be written as the sum of normal mode solutions:

$$\underline{x}(t) = \sum_{i=1}^{n} [A_i \cos(\omega_i t) + B_i \sin(\omega_i)][\underline{V}]_i \tag{2}$$

The system is characterized by its natural frequencies ω_i and mode shapes $[\underline{V}]_i$. The constants A_i and B_i are determined by the initial conditions, and specify the amplitude and phase of the i-th normal mode. The mathematics involved in the discussion above is very similar to the mathematics for a set of first order differential equations. (The governing equations for an n degree of freedom vibrating system can, in fact, be written as a set of $2n$ first order equations.)

NORMAL MODES -- TWO DEGREES OF FREEDOM

We will derive here the equations of motion for the two-degree-of-freedom air track experiment. The variables and physical setup are shown in Fig. III.1. We will draw the free body diagram for each mass and work out its equation of motion.
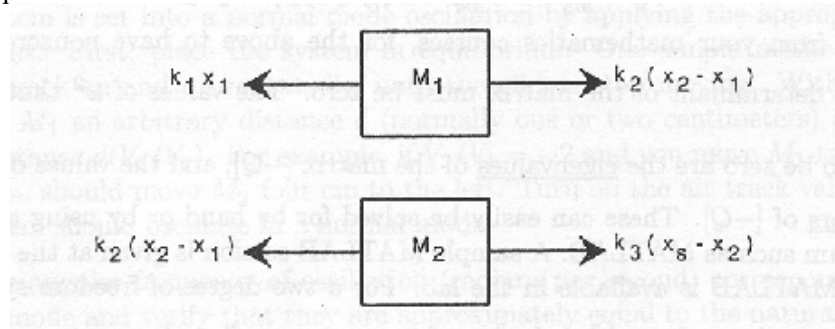
FIGURE III.2

To help get the signs right, assume that the displacements, velocities, and accelerations of the masses are all positive (i.e., to the right), with $x_1 < x_2 < x_s$. This puts all of the springs into tension, relative to their equilibrium condition. The equations of motion, assuming equal spring constants, are

$$M_1\ddot{x}_1 = k(x_2 - x_1) - kx_1, \qquad M_2\ddot{x}_2 = k(x_s - x_2) - k(x_2 - x_1) \tag{3}$$

We can rewrite this in matrix form as

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-2k}{M_1} & \dfrac{k}{M_1} \\ \dfrac{k}{M_2} & \dfrac{-2k}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ x_s(t)\,k/m \end{bmatrix} \tag{4}$$

$$\text{or } \underline{\ddot{x}} = [Q]\underline{x} \ ,$$

$$\text{where } [Q] = \begin{bmatrix} \dfrac{-2k}{M_1} & \dfrac{k}{M_1} \\ \dfrac{k}{M_2} & \dfrac{-2k}{M_2} \end{bmatrix}$$

$$\text{and } \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

We now look for normal modes of this system. As discussed above the normal modes are expressed

as $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_i = (A_i \cos \omega_i t + B_i \sin \omega_i t)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_i$. Thus, taking $x_s(t) = 0$ for now,

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}_i = -\omega_i^2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_i = -\omega_i^2 (A_i \cos \omega_i t + B_i \sin \omega_i t)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_i \tag{5}$$

Substituting for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_i$ and $\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}_i$ in the equation of motion, we get

$$-\omega_i^2 \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_i = \begin{bmatrix} \dfrac{-2k}{M_1} & \dfrac{k}{M_1} \\ \dfrac{k}{M_2} & \dfrac{-2k}{M_2} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_i , \tag{6}$$

$$\text{or } \begin{bmatrix} \dfrac{2k}{M_1} - \omega_i^2 & \dfrac{-k}{M_1} \\ \dfrac{-k}{M_2} & \dfrac{+2k}{M_2} - \omega_i^2 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

As you know from your mathematics courses, for the above to have nonzero solutions for $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$, the

determinant of the matrix must be zero. The values of ω^2 that cause the determinant to be zero are the

<u>eigenvalues</u> of the matrix $[-Q]$, and the values of $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ are the <u>eigenvectors</u> of $[-Q]$. These can easily

be solved for by hand or by using a computer algebra program such as SciLab. A sample SciLab session is
given below. SciLab is available in the lab. For a two-degree-of-freedom system, you should obtain two

eigenvalues, ω_1^2 and ω_2^2, and two corresponding eigenvectors, $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_1$ and $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_2$. General motion, with both normal modes present, is given by the sum

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = (A_1 \cos \omega_1 t + B_1 \sin \omega_1 t) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_1 + (A_2 \cos \omega_2 t + B_2 \sin \omega_2 t) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_2$$

THE PHYSICAL LAB

Apparatus

- Air track with four plug-in springs, four or more air track gliders, a mechanical oscillator, a photogate timer, and a digital stopwatch. *Note:* there are two somewhat incompatible styles of glider, which should only be used on the appropriate air tracks. Each glider has a label listing its mass (including spring) and the air tracks on which it will work.

Procedure

1) Play with the air track, gliders, and timer. Adjust the mechanical oscillator left or right so that each spring, at equilibrium, has a total length of about 20 cm (the oscillator is attached with a Velcro strap). Have the TA turn on the main air supply, if it is not already on, and turn on the valve at the end of the airtrack.

2) Find the spring constant for your springs. Attach a small weight (40 to 50 grams) to one end of the spring and hold the other end solidly against the tabletop. Pull the weight down a few centimeters and release it, and then measure the period of oscillation. Use $\omega = \sqrt{k/M}$ to find k. Remember to include part of the spring and plug mass in M -- half is a good approximation in this case. Check several springs, to determine the variability in k.

3) Choose two gliders of different sizes, and calculate the eigenvalues and eigenvectors for the two normal modes. The eigenvalues are the squares of the natural frequencies of the normal modes (in radians/sec.), and the eigenvectors describe the relative amplitudes of the mass motions. You may do the calculations by hand, or use SciLab on the computer. Weigh the gliders if necessary; remember to include the mass of the plug-in springs.

4) The system is set into a normal mode oscillation by applying the appropriate initial conditions. First, place the system in equilibrium. One simple method is to turn the air track on and off repeatedly, until the gliders stop moving. With the air off, displace M_1 an arbitrary distance d (normally one or two centimeters) and displace M_2 a distance $d(V_2/V_1)$. For example, if $V_2/V_1 = -2$ and you move M_1 two cm to the right, you should move M_2 four cm to the left. Turn on the air track valve abruptly; the system should oscillate in a normal mode.

   Find the angular frequency of oscillation (radians per second) corresponding to each normal mode and verify that they are approximately equal to the natural frequencies calculated. Note the phase difference between the two masses at each normal mode.

   The angular frequency of the masses is found by timing a number of oscillations (*e.g.*, ten), and then converting the resulting period to ω. Digital stopwatches are available at the air track.

5) Use some arbitrary initial conditions and set the system into a non-normal mode oscillation. Observe the motion. (It is difficult to see that it is the sum of normal mode vibrations.)

6)   Attempt to obtain normal mode vibrations by driving the system at each natural frequency. The frequency of oscillation is obtained by timing the motion of the driving rod connected to the motor, using either a stopwatch or a photogate timer. Set the driving frequency to one of the natural frequencies you have calculated, with the air off. Does the system resonate when you turn the air on? Be patient. Start the system from rest every time you change the motor speed. Time the frequency at resonance and compare to the natural frequencies. Observe, as best you can, the relative phase between the scotch yoke and the masses at resonance.

7)   Set up the airtrack with three (approximately) equal masses and four (approximately) equal springs. Adjust the mechanical oscillator to give an equilibrium spring length of about 20 cm. Verify by observation that (1,-1.414, 1) is approximately a normal mode for this system.

8)   Find another normal mode for this system by observation. Find another still. Are there any more? Use SciLab to find the normal modes and natural frequencies.

9)   Using SciLab, find the normal modes and natural frequencies for a system with three unequal masses and four equal springs, and test them on the air track (free vibration only).

HOW TO USE SCILAB

1) Open SciLab by clicking on it on the desktop of your computer. This program is a freeware program similar to MatLab and should look quite similar.

2) To find the eigenvalues and eigenvectors you must use the function "spec()" as shown below. Send the matrix [Q] as the function parameter and returned with be the eigenvalues along the diagonal of a square matrix and the eigenvectors as the columns of the second returned matrix.

```
scilab-3.1.1 (0)

File  Edit  Preferences  Control  Editor  Applications  ?

-->k1=5 ; k2=5 ; k3=5 ; m1=2 ; m2=2
 m2   =

    2.

-->Q=[-(k1+k2)/m1 , k2/m1 ; k2/m2 , -(k2+k3)/m2]
 Q   =

! - 5.     2.5 !
!   2.5  - 5.  !

-->[eigenvectors,eigenvalues]=spec(Q)
 eigenvalues   =

! - 7.5    0.  !
!   0.   - 2.5 !
 eigenvectors   =

!   0.7071068    0.7071068 !
! - 0.7071068    0.7071068 !

-->_
```

3) For the lab, however, you must find Q for a three degree of freedom system. This example is for the two-degree of freedom simulation. If you wish to try this function on MatLab, everything is the same except for the function name, in MatLab you must enter "eig(Q)" to find the eigenvalues and eigenvectors.

QUESTIONS [Remember to follow the format given in the Introduction]

1)   List your values of k for the springs, and a sample calculation. What is the average value of k, and what was the largest variation from the average (in percent)?

2)   Did you obtain normal mode oscillations using initial conditions based on your eigenvectors? How could you tell?

3)   How close were your experimental frequencies to those calculated? How does this experiment deviate from theory?

4)   In what way did the block motions look like normal mode vibrations when you forced the system? In what ways did they not look like normal mode vibrations? Consider three cases:

      a)   forcing frequency = a natural frequency

      b)   forcing frequency close to a natural frequency (Was the amplitude of the oscillations constant in this case? If not, how did it vary?)

      c)   forcing frequency far from a natural frequency.

5)   a)   Write down the equations of motion for the system with two equal masses and three equal, massless, linear springs, as derived previously. Assume x_s (see Fig. III.1) is a given function of time: e.g., $x_s = \sin(t)$.

      b)   Derive the equations again, this time with x_s fixed at zero but with a known force F acting on M_2, in addition to the two spring forces.

      c)   Suppose that $F = k\sin(t)$. Show that the systems in a) and in b) are mathematically equivalent.

      d)   If the spring forces were given by the equation $F_s = kx^2$, and the force F in b) was given by $F = k\sin^2(t)$, would the two systems still be equivalent?

6)   How many normal modes are there in the three equal mass system? What are they and how did you recognize them as normal modes? How many were you able to find experimentally? How do they compare with those you calculated?

7)   What were your calculated normal modes and natural frequencies for the system with three unequal masses? Did normal mode oscillations occur with these ratios and frequencies on the air track?

LABORATORY III
SLIDER CRANK LABORATORY

Revised: August 2005

PRELAB QUESTIONS

1)   What data will you collect from the lawn-mower engine?
2)   What will you simulate on the computer?
3)   Which parameter(s) can be varied on the adjustable slider crank? Which are fixed?
4)   Derive the equation relating the piston displacement to the crankshaft speed, w, time, t, connecting rod length, L, and crank radius R. (Hint: Use your basic trigonometry).
5)   What other mechanisms for converting rotary to linear motion will be studied?

INTRODUCTION

In this laboratory we will investigate the kinematics of some simple mechanisms used to convert rotary motion into oscillating linear motion, and vice versa. The first of these is the slider crank, a mechanism widely used in engines to convert the linear thrust of the pistons into useful rotary motion (see Figure I.1). Here, you will measure the acceleration of the piston of a lawn mower engine at various speeds. The results exemplify a simple relation between speed and acceleration for kinematically restricted motions, which you should discover. An adjustable slider crank apparatus and a computer simulation will show you some effects of changing the proportions of the slider-crank mechanism on piston velocity and acceleration. Other linkages and cam mechanisms may also be used for linear-rotary motion conversion, and some of these will be included in the lab.

Because $F = ma$, knowledge of the acceleration permits analysis of the forces which occur in an engine or other machine. Knowledge of these forces is crucial if one is to choose the right material, proportions, and operating conditions for a new design.

SLIDER CRANK KINEMATICS AND INTERNAL COMBUSTION ENGINES

Figure I.1 shows a sketch of the slider crank mechanism. The point A is on the piston, line AB (with length L) is the connecting rod, line BC (with length R) is the crank, and point C is on the crankshaft. In an engine, a mixture of gasoline and air in the cylinder is ignited in an exothermic (heat producing) reaction. As a result, the pressure in the cylinder rises, forcing the piston out. The force transmitted through the connecting rod has a moment about the center of the crankshaft, causing the shaft to rotate. An exhaust valve releases the gas pressure once the piston is extended. Inertia of machinery (often a flywheel) connected to the crankshaft (as well as forcing from other pistons in multi-cylinder engines) forces the piston back up the cylinder. In a standard 'four-cycle' engine the crankshaft makes another full revolution before another ignition (to bring in fresh air and compress it before ignition). Though the piston is only being forced one fourth of the time, the crankshaft rotates at a more or less constant rate.

In this experiment, the crankshaft is driven by an electric motor. The piston is driven by this crankshaft rotation at a more or less constant rate. The same motion results as when the combustion process takes place. As the crankshaft rotates, the piston moves in the positive and negative x direction. The basic measurements in this lab are the position and velocity of the piston in the x direction (which happens to be vertical in the laboratory). These measurements can be compared to those calculated by hand (if you are energetic), or to the results of a computer simulation; the simulation and the adjustable crank allow you to see some of the effects of varying the ratio of connecting rod length L to crank length R.
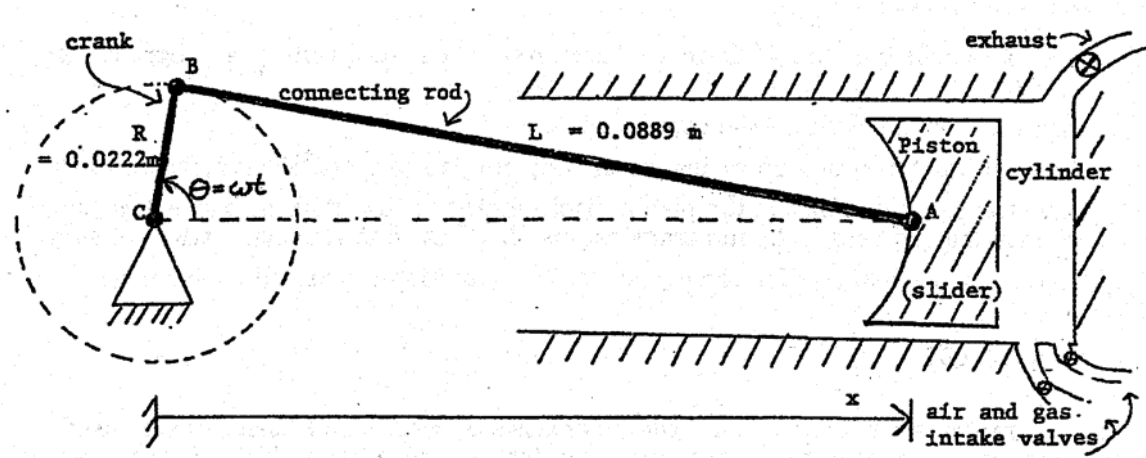
Figure I.1

**PROCEDURE** [Read the procedure all the way through before you do anything else.]

1) *Slider crank--lawn mower engine:* A stripped-down lawn mower engine is driven by a variable-speed electric motor. Sensors are installed on the engine's piston to measure displacement and velocity. A data acquisition program is used to measure, analyze, and record the piston data.

- *Engine:* Look at the engine, and see how its various parts fit together. It may help to look at Figure I.1, and at the various demonstration slider-cranks available to look at in the lab. Identify the piston, connecting rod, and crankshaft (the connecting rod won't be visible at your lab set-up, but you can see it in the demonstration slider-cranks). The cylinder head has been removed, exposing the top of the piston and allowing sensors to be attached.

- *Electric motor and controller:* The speed and direction of the electric motor are controlled by a knob and switch on the motor controller. The numbers on the speed controller are arbitrary; do not write them down as r.p.m. or radians per second. (Obtain angular velocity information from the data acquisition program.) Does the direction of motor rotation affect the slider crank kinematics?

- *Displacement Transducer (LVDT):* The larger cylindrical metal device attached to the piston is an LVDT (Linear Variable Differential Transformer) for measuring $x(t)$. An LVDT is an electrical transformer which produces an output voltage proportional to the linear displacement of the ferromagnetic core of the transformer. The LVDT can be used for static, as well as dynamic experiments.

- *Velocity transducer:* The smaller cylindrical device attached to the top of the piston measures velocity. The velocity transducer consists of a coil of wire in which a magnetic core moves linearly. The transducer generates a voltage proportional to the velocity of the core.

- *Data acquisition card and LabView software:* The displacement and velocity data are collected and plotted with the computer, which also measures and displays the peak values (see Figure I.2). Acceleration is calculated by the computer through numerical differentiation of the velocity data. The computer also measures and displays the angular frequency, by timing successive crossings of the zero line and converting to radians per second. A simple simulation program lets you compare your data to theoretical values, and look at the effects of different slider-crank geometries.

2) *Safety precautions:* The electric motor driving the lawn mower engine is powerful enough to cause serious injury if you get in its way. Keep long hair and loose clothing well away from the belt and pulleys at the back of the engine. If you need to touch the pulley, piston, or LVDT for some reason, check first that the electric motor power is off, and that the speed control is set to zero. Make sure your lab partner knows what you are doing.

3*) Using the data acquisition program:*

a) *LabView Crank:* Open up the Engrd203Lab account and then open the folder "Crank" on the desktop then open the program "Crank". As soon as the program is running, it will ask you to move the piston to the top of its travel, press *Ready* after you have done this and wait until the next pop-up comes before moving the piston again. Then once prompted move the piston to the bottom of its travel, and press *Ready* again and allow the computer a few seconds to calibrate. This calibration procedure allows the computer to convert the output of the LVDT (in volts) into displacement (in meters). Do this carefully; it may help to rock the pulley back and forth slightly as you try to home in on the highest (or lowest) piston position. If you make a mistake, you can redo the procedure by clicking on the *SET-UP* button. The *Crank* program has a box for the initials of your lab group. Click on the box with the mouse, type, and then press the enter key, not the return key. Your initials will then appear on your plots, making it easier to identify them as they emerge from the laser printer.

b) When the data acquisition 'switch' on the screen is turned on, the computer acquires and displays a new set of data every ten seconds or so; allow ten or twenty seconds for the data plot to stabilize after changing the motor speed. If you have a plot that you want to keep, turn the data acquisition off. Also, turn the motor off promptly when you are not acquiring data, to save wear and tear on the lab set-ups, and on the nerves of other students.

   The legend and scale factors for the plots are displayed in the top right corner. Multiply the y-axis reading (between -1 and 1) by the appropriate scale factor to obtain the actual measured value, in the units given in the legend. For example, if the velocity plot has a y-value of 0.5 at a particular time, and a scale factor of 4 m/s, the measured velocity at that time would then be 2 m/s.

c) *Printing:* Before printing, check that data acquisition is off. Otherwise, one plot can take 20 minutes or more. Also, be sure your initials are on the graph, so you can distinguish it from other lab groups' graphs. To print, pull down the *File* menu and select *Print*. Each new graph takes a minute or two, so only print one out if you really need it. However, you can get a copy for your lab partner in just a few additional seconds by setting *Number of Copies* equal to two. You can continue working while plots are being printed.
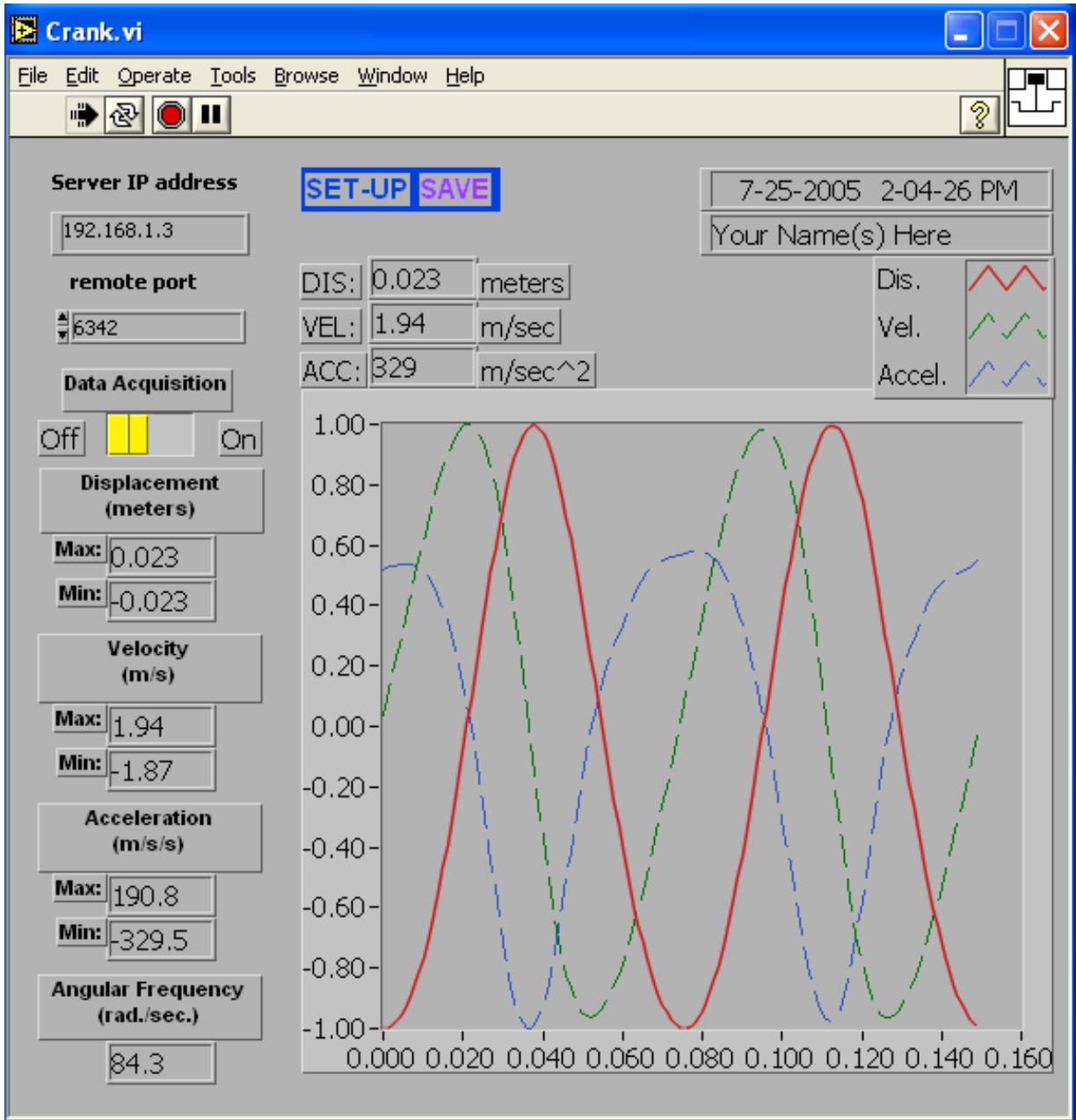
Crank.exe Data Acquisition Labview Program
Figure I.2

d) *Saving data*: The *SAVE* button stores your data on the hard disk, but the file created this way can only be used by the simulation program (*CrankSim*).

e) *Quitting the program*: To exit from the program, click the 'close box' in the top right corner of the window. To leave LabView completely, at any time, pull down the *File* menu and select *Quit*. If the program tells you that "Quitting now will stop all active VIs," select *OK*.

4) *Collect data*: Here you will record and analyze $x(t), v(t),$ and $a(t)$ while spinning the lawn mower engine at various speeds.

   a) Check that the electric motor power switch is off, the speed control knob is at zero, and the data acquisition is on. Twist the pulley back and forth by hand, and look at the resulting plot of piston position, velocity and acceleration. If the piston moves upwards, in what direction does the plotted curve move? Allow several seconds for the data to be displayed.

   b) Put a penny on top of the piston, turn on the motor, and adjust the motor speed so that the penny just barely starts to bounce on top of the piston. You should be able to hear a faint clinking sound. Wait until you have a good graph of the data, and then turn off first the data acquisition and then the motor. Record the angular velocity, and the minimum and maximum values for the displacement, the velocity, and the acceleration. Check that the displacement plot makes sense, given that the crank length is known to be 0.0223 m.

   c) Remove the penny and repeat the procedure above for at least four additional speeds. Try to get as wide a variety of speeds as possible. At very slow speeds the motor does not turn smoothly, and the data is drowned out by noise. When using very high speeds, try to acquire data quickly, turn off the data acquisition 'switch,' and shut the motor off immediately. Record your data in a table, including the penny data. It is good practice to make at least a rough plot of your data as you go along, so you will know what additional data points are needed while you are still in the lab.

5) *Slider Crank Simulation(CrankSim2):* A CrankSim2 (Figure I.3) will be used to compare the theoretical values for displacement, velocity, and acceleration with the values measured above. The effects of changing the crank length, connecting rod length, and angular velocity of the crankshaft may also be observed.

   a) *CrankSim2:* To start up the simulation program, double-click on 'CrankSim' in the Crank Lab folder. If you want to compare your simulation to your most recently saved data, turn the measured-data "switch" on; otherwise, turn it off, to eliminate the clutter of all the extra graphs. Described below are the parameters you can change in the simulation:

   R is the crank length in meters.
   L is the connecting rod length.
   w is the angular velocity in radians per second.

   As with the data acquisition program, the maximum and minimum values are displayed. These are the <u>simulation</u> maxima and minima. Note that the displacement shown is the value x in Figure I.1, minus the connecting rod length L. This makes it more easily comparable to the measured data. The $x = 0$ point is thus defined to be halfway between the piston's top and bottom positions, instead of at the center of the crankshaft.

Figure I.3   Crank Simulation Program: CrankSim2
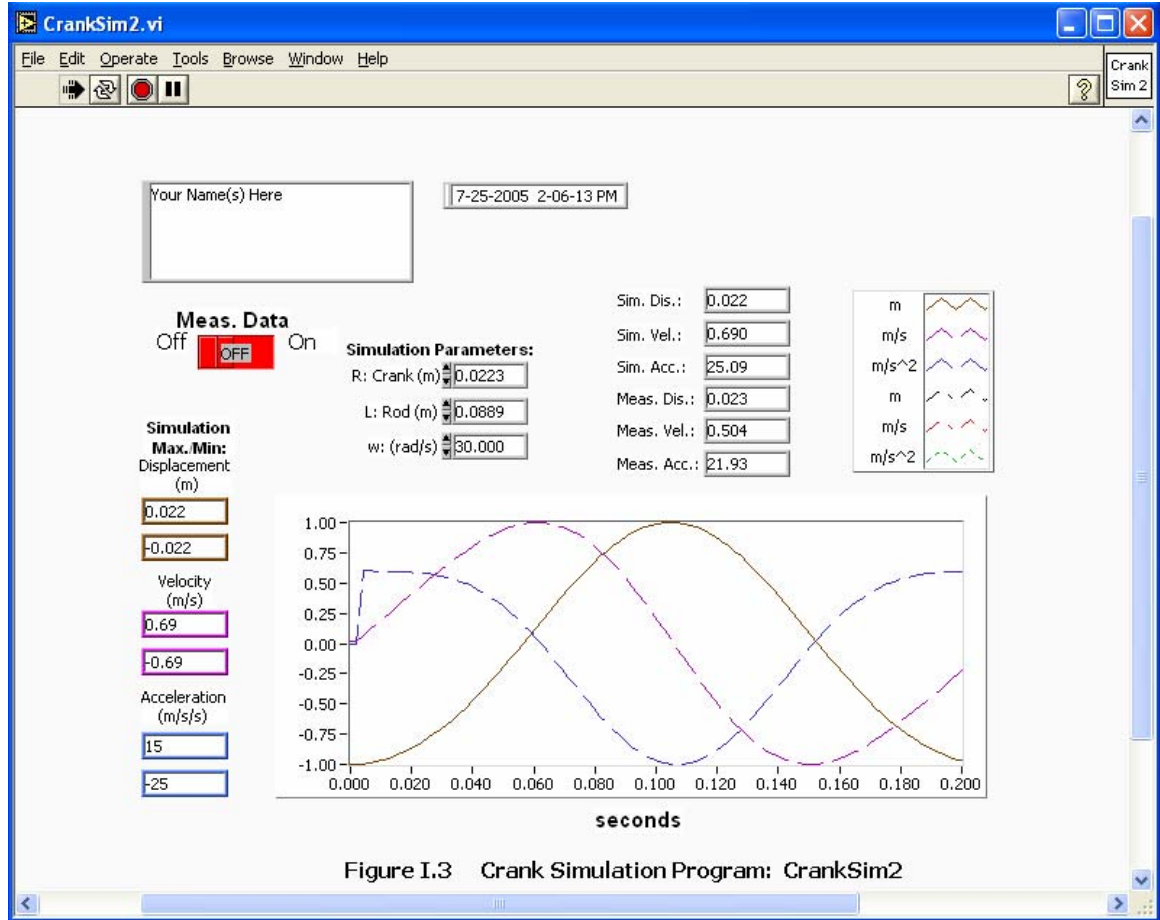
b)   Set up the simulation with the crank length and connecting rod length of the lawnmower engine. Enter the angular velocity used in the data you 'SAVEd' previously, and turn the 'Measured Data' switch on. Adjust the simulation curve up or down for best alignment, and compare the two sets of plots. You will need to print your data.

c)   Switch off the measured-data curve. Now, simulate slider-cranks with different geometries by varying the crank length R and the connecting rod length L. Observe and record velocities and accelerations when L is much greater than R (e.g., L of 10 m, with R of 0.0223 m); when R is increased, but still much smaller than L (e.g., L of 10 m and R of 0.223 m); when L is decreased, but still much larger than R (e.g., L of 1 m and R of 0.0223 m); when R and L equal the values for the lawn mower engine (L of 0.089 m and R of 0.0223 m); and when L is only slightly greater than R (e.g., L of 0.0224 m and R of 0.0223 m). What happens physically when R is greater than L? Make one or two print-outs if necessary to support your observations or conclusions.

6) *Adjustable slider crank:* This device allows you to adjust its R/L ratio from zero to slightly more than one, using an adjustment knob which changes the effective crank length. A handle is located underneath, to rotate the apparatus by hand. Please be gentle with it! Large forces can be generated

7) with even a small input torque, when the R/L ratio is close to 1. If you see things bending, back off. When turning the hand crank, do it slowly. You can also push and pull on the masses at the end of the 'piston,' to look at the way it converts linear to rotary motion. Be sure you can identify the crank, connecting rod, and piston on the adjustable crank apparatus; first appearances may be misleading. Here's a hint: the long thin rod with a weight on each end is the piston. Compare the shapes of the curves you saw in the simulation above to what you observe and feel with the adjustable crank.

8) *Other rotary-linear conversion mechanisms:* The slider-crank is just one of many devices that have been invented to convert linear to rotational motion, or vice-versa; the scotch yoke, the cam, and the four-bar linkage are some others.

   a)   *Scotch yoke:* Look over the scotch yoke mechanism, which is driven by an electric motor and gearbox. Try it at different speeds, and (with the motor off) push and pull on its various parts. Rotate the pulley by hand, watching the motion of the rod. Take measurements, or make a drawing, if you wish. Be prepared to find a kinematical equation relating disk rotation to yoke displacement, and think about the advantages and disadvantages of the scotch yoke relative to the slider crank.

   b)   *Cam and follower:* Cam-and-follower mechanisms are a particularly versatile way to convert rotary to linear motion, because you can select the type of motion you want by changing the shape of the cam. For example, cams are used in an internal combustion engine to open and close the intake and exhaust valves. Cam shapes are chosen to optimize fuel economy, power, and emission control. The cam in this lab is a simple eccentric disk. Try out the cam mechanism at various speeds with the motor on, and (with the motor off) turn it by hand. Feel the output from the follower as the cam is rotated, and then try rotating the cam by pushing and pulling on the follower. As with the scotch yoke, be prepared to relate the angle of cam rotation to follower displacement, and think about the mechanism's advantages and disadvantages.

QUESTIONS [Remember to follow the instructions in the Introduction]

1)   Plot peak piston acceleration vs. crankshaft angular velocity on linear and log-log paper. From these graphs, find an appropriate equation relating the two variables.

2)   How does the peak piston velocity depend on the angular velocity of the crankshaft? Plot your experimental data, and find an approximate formula relating the two variables.

3)   Examine your plot comparing the measured data and the corresponding simulation data. What explanations can you give of the similarities or differences in the graphs?

4)   From your experimental data, what is the crankshaft angular velocity for which an ant standing on the top of the piston would start to need sticky feet in order to not lose contact with the piston?

5)   Using your simulation data, how does the length of the connecting rod, relative to the crank length, affect the shape of the displacement, velocity, and acceleration curves?

6)   The lawn mower engine piston weighs 0.175 kg. Using your simulation data, what are the maximum velocity, acceleration, and force on the piston, approximately, for a connecting rod length only slightly longer than the crank length? For a connecting rod length extremely long compared to the crank length? For the connecting rod length actually used in the engine? Use the same crank angular velocity and length in each case.

7)   Argue for or against one of the following points. Back up your arguments with either real or simulated data and/or any other appropriate analysis and logic.

   a)   For all slider cranks, the peak velocity occurs at the midpoint of the stroke.

   b)   There is an optimum L/R ratio for a lawn mower engine. [Clearly state what is being optimized.]

   c)   Argue an issue of your own choosing related to this lab.

8)   For the scotch yoke, work out the equation relating rotation of the pulley to linear motion of the rod.

9)   Why is the slider-crank, and not a scotch yoke, used in an engine? Also, what special advantages does the scotch yoke have in some applications?

10)   How does the cam-follower mechanism you saw in lab compare kinematically to the scotch yoke? What reasons might a designer have for choosing one over the other?

LABORATORY IV
GYROSCOPIC MOTION OF A RIGID BODY

Revised: August 2005

PRELAB QUESTIONS
(Due at the beginning of the lab period)

Read through the laboratory instructions and then answer these questions.

1)   What is a gyroscope?

2)   Where is the fixed point of the lab gyroscope?

3)   How will moments (torques) be applied to the lab gyroscope?

4)   What are you going to do with the electric gyroscope?

INTRODUCTION

Gyroscope is a word used to describe a rigid body, usually with symmetry about an axis, that has a comparatively large angular velocity of spin about its spin axis. Some examples are a flywheel, top, football, navigational gyroscopes, and the Earth. The gyroscope differs in some significant ways from the linear one and two degree-of-freedom systems with which you have experimented so far. The governing equations are the 3-dimensional equations of motion. Mathematical analysis of the gyroscope involves use of 3-dimensional geometry. The governing equations for the general motion of a gyroscope are non-linear. Non-linear equations are in general hard (or impossible) to solve. In this laboratory you will experiment with some simple motions of a simple gyroscope. The purpose of the lab is for you to learn the relation between torque, angular momentum, and rate of change of angular momentum. You will learn this relation qualitatively by moving and feeling the gyroscope with your hands and quantitatively by experiments on the precession of the spin axis.

AXISYMMETRIC GYROSCOPE ROTATING ABOUT A FIXED POINT

Our experiment uses a rotating sphere mounted on an air bearing so that the center of the sphere remains fixed in space (at least relative to the laboratory room). This is called a gyroscope with one fixed point.

As the gyroscope rotates about its spin axis it is basically stable. That is, the spin axis remains fixed in space and resists any externally applied force that would tend to alter its direction. As you should see in the experiment, the larger the spin rate the larger the moment needed to change the direction of the spin axis. When a moment is applied to a gyroscope, the spin axis will itself rotate about a new axis which is perpendicular to both the spin axis and to the axis of the applied moment. This motion of the spin axis is called precession.

THE PHYSICAL LAB

Apparatus
- *Steel ball on an air bearing (see Figure IV.1)*: On one side of the ball a rod is mounted for reference and for touching. This side of the ball has also been bored out, so that the rod side is lighter and the center of mass can be adjusted to either side of the center of the sphere by sliding a balance weight in or out. The balance weight is black, with reflective tape, to make rotation rate measurements easier. The sphere is supported in a spherical cup into which high pressure air is supplied so that the sphere is actually supported by a thin layer of air (similar to the air track).

- *Tachometer*: for the timing of rotations, in rpm (rev. per min.)

- *Bicycle wheel and rotating platform*: for hands-on experience.

- *Electric Gyroscope*: (classroom demo) for more of the same.

- *Electric Gyroscope*: (the real thing, from an old aircraft autopilot) for still more.

Procedure

1)   Read this handout all of the way through.

2)   Turn on the air source.

3)   Place the black balance weight on the rod so that if the sphere is released with no spin the rod does not tend to fall down or pop upright from a horizontal position. (This may be a bit tricky; if so, just get it close.)

    a)   Where is the center of mass of the system (sphere, rod, and disk) after the gyroscope is balanced?
    b)   What effect does gravity have on the motion of the balanced gyroscope?

4)   Without spinning the ball, point the rod in some particular direction (up, or towards the door, for example). Carefully release the rod, and watch it for several seconds. Does it keep pointing in the same direction? Touch the rod lightly with a small strip of paper. How much force is required to change the orientation of the rod? In which direction does the rod move? Rotate the table underneath the air bearing. Does the rod move?

5)   Get the ball spinning and repeat step (4). One good way to do this is to roll the rod between your hands; stop any wobbling motion by holding the tip lightly and briefly. Avoid touching the ball itself. Caution: do not allow the rod to touch the base, and do not jar the ball while it is spinning.

    a)   What is the effect of spin on the gyroscope motion?
    b)   Why are navigation gyroscopes set spinning?

6)   While the ball is spinning, apply forces to the end of the rod using one of the pieces of Teflon on a string. The ball should continue to rotate freely as you apply the force, because of the low friction of the Teflon. Gently move the end of the rod. (Keep the rod from touching the bearing cup, or the rod may spin wildly). What is the relationship between the force you are applying and the velocity of the tip of the rod (estimated magnitude and direction)? Remember that tension is always in the direction of the string.

7)   For a more quantitative look at the motion of a gyroscope:

   a)   Add another weight to the rod so that the gyroscope is no longer balanced. Record its
        mass and position on the rod, for use in calculations later (See Figure IV.1). Weigh it if
        necessary.

   b)   Get the ball spinning, but not wobbling, and point the rod towards one of the three
        support screws on the air bearing platform. With the rod horizontal, simultaneously
        release the rod and start the handheld digital stopwatch. The spinning ball and rod will
        begin to move (precess) in a horizontal plane. Depending upon the precession rate, you
        may want to stop the timer after one full revolution, or after only one-third or two-thirds
        of a revolution.

   c)   Halfway through the timing interval, use the optical tachometer to measure the spin rate
        of the ball (this gives an average). The light beam from the tachometer should be aimed
        at the reflective tape on the black balance weight. The tachometer measures the rate of
        the pulses of light returning from the tape, and displays the result in r.p.m. Hold the it at a
        distance of 10 cm or so. For higher accuracy, try to follow the precession of the rod with
        the tachometer. This may require practice and patience. If you find it more convenient,
        measure the spin rate at the start of the precession period and again at the end, and then
        find the average.

   d)   Repeat the procedure for at least two additional spin rates. Try to use a wide range of spin
        rates; e.g., 200, 400, and 600 r.p.m.

8)   Remove the weight and repeat step (7) with at least two more weights, for a total of at least three
          different weights and three different spin rates per weight. The spin rates need not be the same as
          the ones you used before, but they should cover a similarly wide range of r.p.m.

9)   Hold the bicycle wheel while someone else gets it spinning. Twist it different ways. Hold your
          hands level and turn your body in a circle. How do the forces you apply depend on the direction
          you twist the axle and on the rotation speed and sense?

10)   Play with the electric gyroscope. Can you determine the direction of rotation of the aircraft gyro
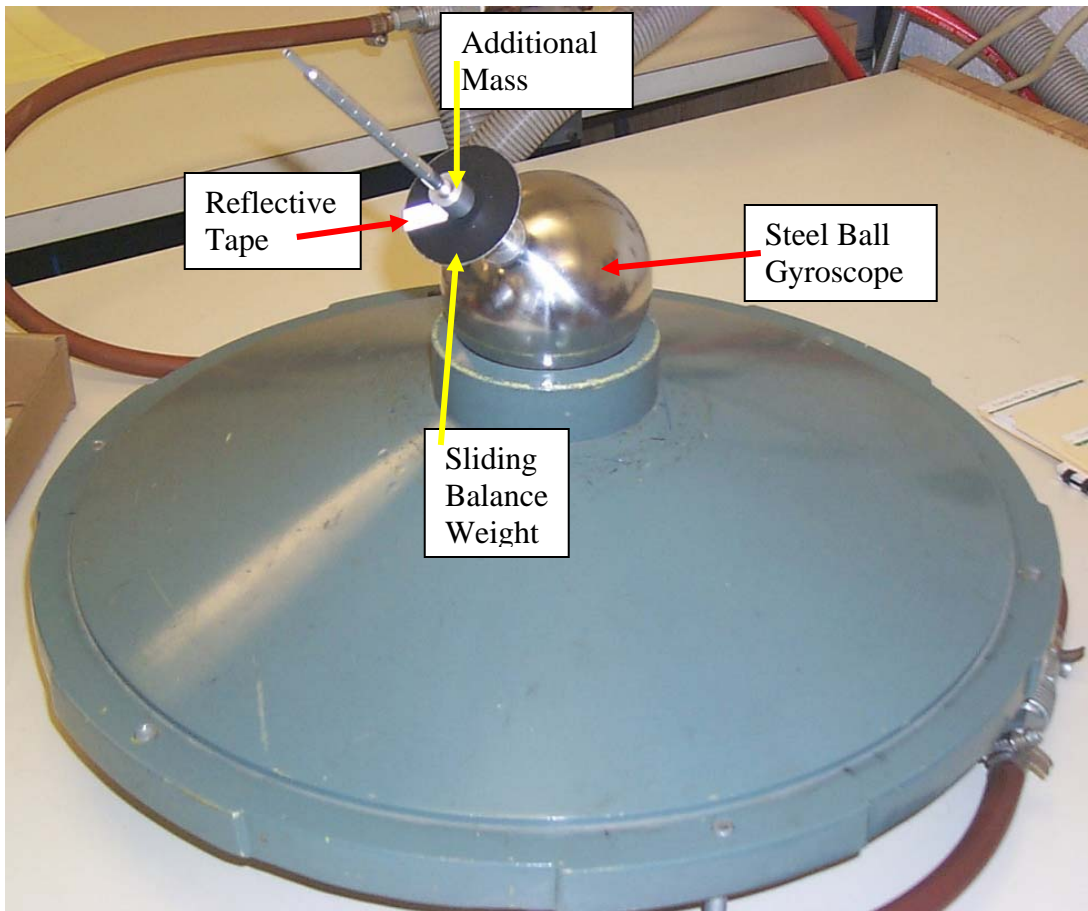   by applying torques and observing the precession?

QUESTIONS [Note: Since vectors are involved, it is not only clearer, but imperative, to draw clear,
unambiguous diagrams showing the spin, torque, and precession relations. Also remember to follow the
format given in the Introduction.]

1)   Answer all of the questions in the procedure above. (Please indicate clearly which question you
          are answering by using full self-contained sentences.)

2)   Suppose that the rod on one spinning air gyroscope is pointed North, at an angle of 42.5 degrees
          from the horizontal (i.e., along the earth's axis of rotation). A second air gyroscope is pointed east,
          with its rod horizontal. Assume that the ball is perfectly balanced, and that air friction is
          negligible. How does the orientation of the spinning gyroscopes change, over a period of several
          hours?

3)   Use your data from parts 7 and 8 to do the following:

   a)   Plot the precessional period vs. the spin rate, using units of seconds and radians/ second.
             Use a different color for each different applied torque. (The gravitational torque comes
             from gravity acting at the added weights. For a horizontal rod the torque is the product of
             the force times the distance of the weight from the center of the sphere.)

   b)   What is the relationship between the precessional period and the spin rate? (You should
             be able to see a simple formula from looking at your plot.)

   c)   For a fixed torque show that the product of the precessional rate and the spin rate is a
             constant. (The precessional rate is defined to be inversely proportional to the precessional
             period.) Use units of radians per second.

   d)   The torque should be proportional to the product of the spin rate and the precession rate.
             Find the constant of proportionality and plot the relationship between torque and
             (spinrate × precessionrate)

   e)   You have now found a simple formula relating torque, spin rate and precession rate. In
             this formula there is a numerical constant. What determines the value of this constant?
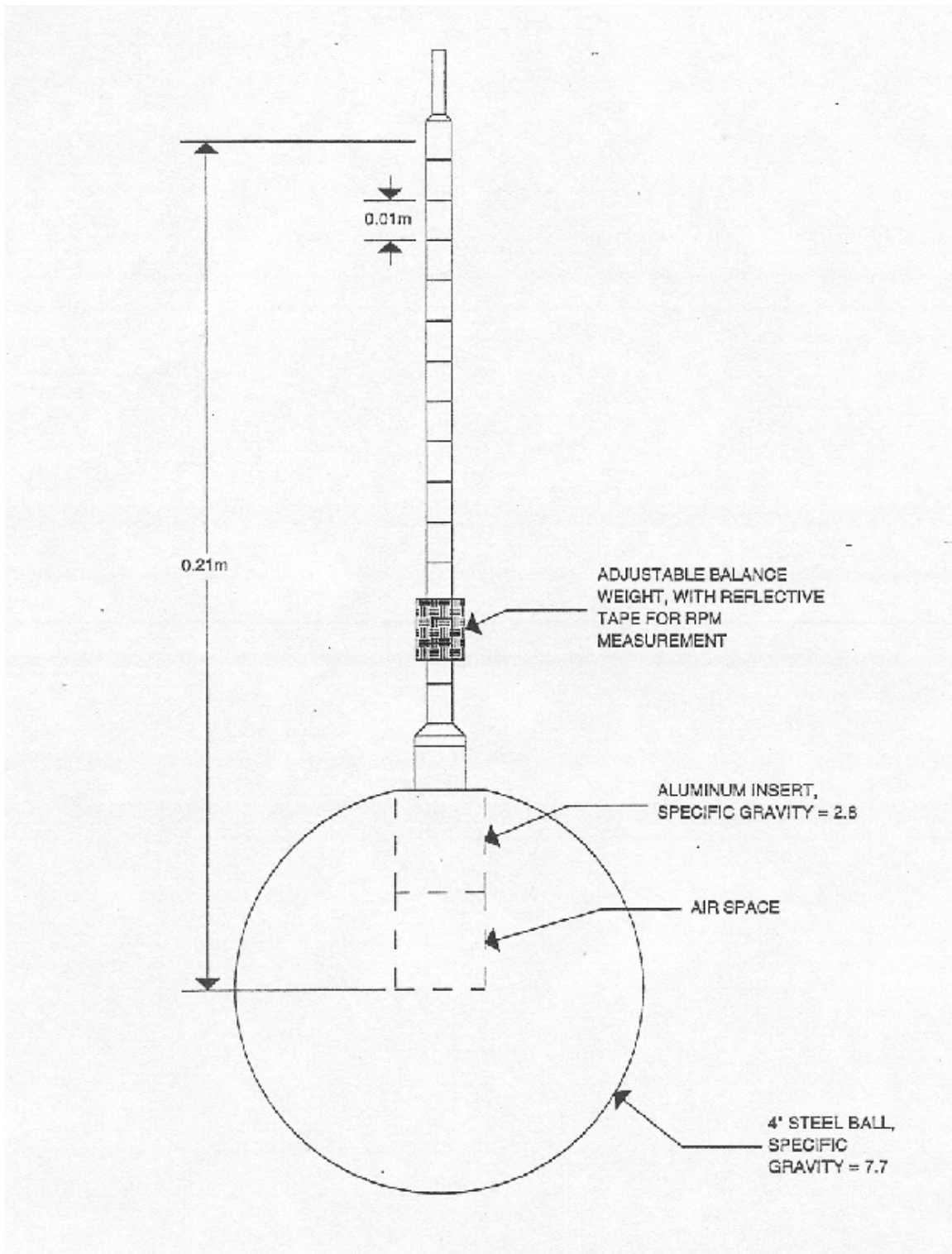
0.01m

0.21m

ADJUSTABLE BALANCE
WEIGHT, WITH REFLECTIVE
TAPE FOR RPM
MEASUREMENT

ALUMINUM INSERT,
SPECIFIC GRAVITY = 2.8

AIR SPACE

4' STEEL BALL,
SPECIFIC
GRAVITY = 7.7

FIGURE IV.1