Bonus 3.3.4

An 80 kg skier travels down a ski jump having a profile given by

$$y = y_0 - ax + bx^3 ;$$

$y_0 = 50\,m$, $a = 1$, $b = \frac{1}{12500}\,m^{-2}$

Goal : Plot normal force exerted by slope on skier as function of time from $x = 0$ to $x = 50\,m$

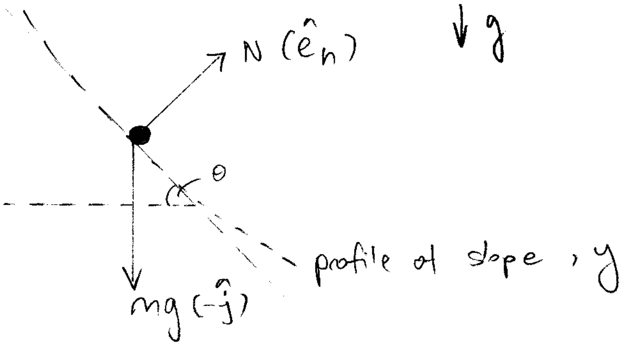
given : 1. $y = 50 - x + \frac{x^3}{12500}$

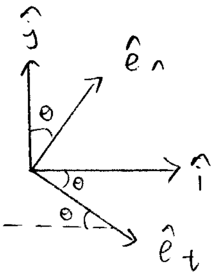
2. gravity , g

3. All units measured in SI units
m , kg , s

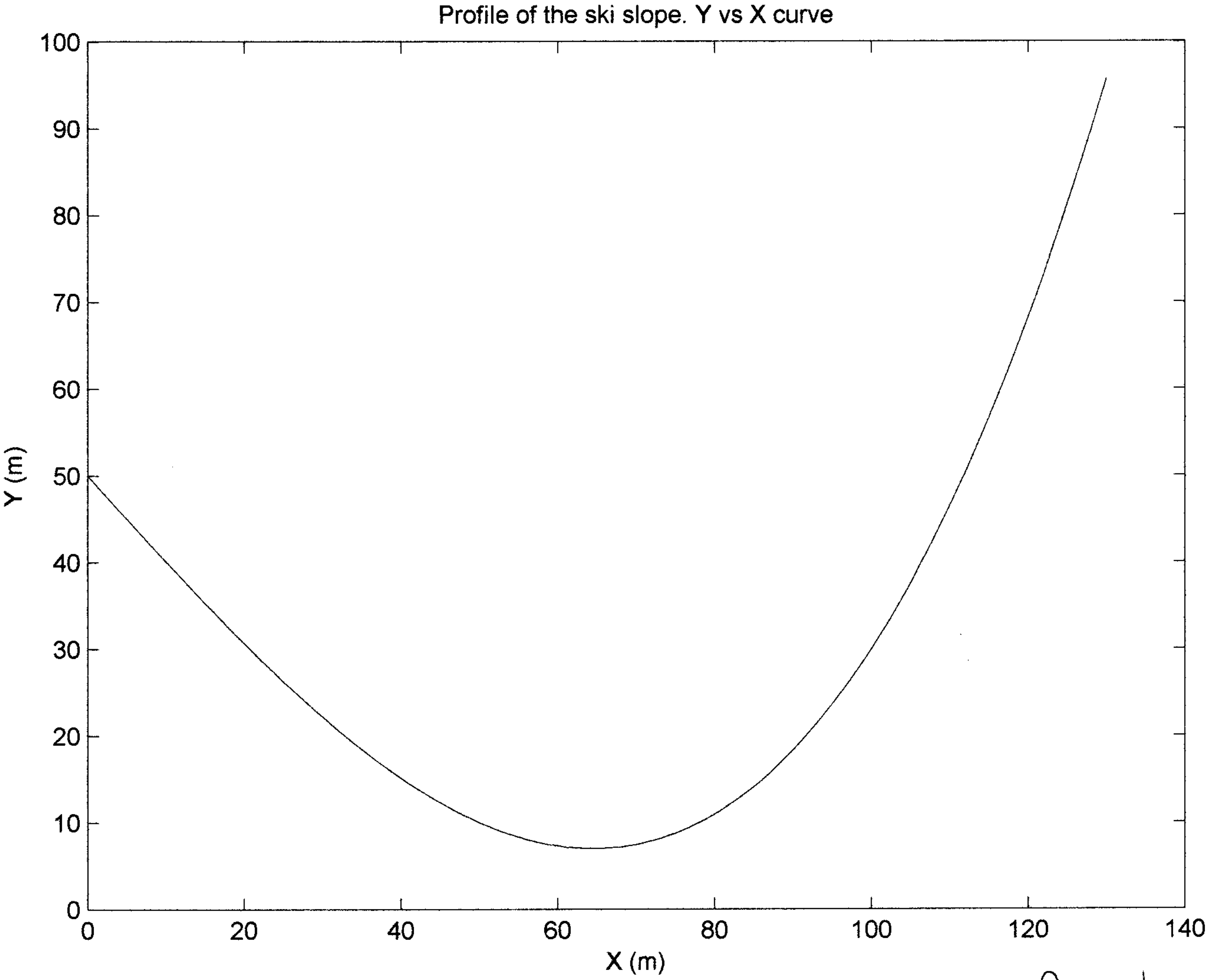
Solution :

1. FBD of skier idealized as particle





graph of $y = 50 - x + \frac{x^3}{12500}$



te: The question asks for N to be found across $0 \le x \le 50$

For purposes of checking if solution obeys the law of conservation of energy, the solution Should show that the maximum height the skier can achieve is 50 and not more. Here, I have extended the range of X

2. Apply Newton's Second law

$$m\,\underline{a}_p = \sum_i \underline{F}_i$$

$$m\,\underline{a}_p = N(\hat{e}_n) + mg(-\hat{j})$$

$\left\{ m\left(\dot{v}\hat{e}_t + \frac{v^2}{R_c}\hat{e}_n\right) = N(\hat{e}_n) + mg(-\hat{j}) \right\}$ —— $\circledast$

$\circledast \cdot \hat{i}$ (aim: solve for $\ddot{x}(t)$))

$$m\ddot{x}_i = N(\hat{e}_n \cdot \hat{i})$$

$$= N\sin\theta$$

$m\ddot{x}_i = N\sin\theta$ —— ①

$\circledast \cdot \hat{e}_n$ (aim: find an expression of N)

$$m\left(\frac{v^2}{R_c}\right) = N + mg(-\hat{j}\cdot\hat{e}_n)$$

$$= N - mg\cos\theta$$

$N = m\left(\frac{v^2}{R_c} + g\cos\theta\right)$ —— ②

3. Using Eq^n for the slope

$$y = 50 - x + \frac{1}{12500}x^3$$

$\frac{dy}{dx} = \frac{3x^2}{12500} - 1$ —— ③

$\tan\theta = \frac{dy}{dx}$ —— ④

$\frac{d^2y}{dx^2} = \frac{6x}{12500}$ —— ⑤

$$R_c = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$= \frac{\left[1+\tan^2\theta\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$R_c = \frac{\left[\sec^2\theta\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \text{ —— } ⑥$$

$$V^2 = \left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2$$

$$= \left(\frac{dy}{dx}\cdot\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2$$

$$= \left(\frac{dx}{dt}\right)^2\left(1+\left(\frac{dy}{dx}\right)^2\right)$$

$$V^2 = \dot{x}^2\left(1+\tan^2\theta\right)$$

$$V^2 = \dot{x}^2\sec^2\theta \text{ —— } ⑦$$

4. Making sense of eq^{ns} ①–⑦

from ①

$$m\ddot{x}_i = N\sin\theta$$

Substitute ②, ③, ④, ⑤, ⑥, ⑦ → ①

$$m\ddot{x}_i = m\sin\theta\left[\frac{\dot{x}^2\sec^2\theta\left|\frac{d^2y}{dx^2}\right|}{\sec^3\theta} + g\cos\theta\right]$$

$$\ddot{x}_i = \sin\theta\left[\dot{x}^2\left|\frac{d^2y}{dx^2}\right|\cos\theta + g\cos\theta\right]$$

$$\ddot{x}_i = \underline{\underline{\frac{1}{2}\sin\left(2\tan^{-1}\left(\frac{-3}{12500}x^2+1\right)\right)}}\left[\dot{x}^2\left(\frac{6x}{12500}\right)+g\right]$$

5. Use Matlab to solve for $\ddot{x}$; ($V_x$dot)

```
function skier

%t = time taken
%let time frame be from 0 to 5s
%let 'z' be a 2X1 matrix, where z(1) = x, 'x' is position of the skier in
%the 'i' direction
%w.r.t an arbitrary position. z(2) = xdot = v, 'v' is the velocity of the
%skier in the 'i' direction

%initial conditions
x = 0; xdot = 0;

%column vector to hold initial conditions
zNot = [0,0]';

%timespan integrate over 9 seconds
tSpan = [0,9];

%ODE
[t,z] = ode45(@f,tSpan,zNot);

%---------------------------------------------------------------------------
figure(1);

% X vs Y plots the profile of the ski slope
X=(0:130);
Y = 50 - X + (X.^3)/12500;

%plot the results
plot(X,Y,'b-');
xlabel('X (m)');
ylabel('Y (m)');
title('Profile of the ski slope. Y vs X curve');
%---------------------------------------------------------------------------
figure(2);

%unpack variables from z
velocityX = z(:,2)';
positionX = z(:,1)';

% units of Kg and m/s^2
m = 80;
g = 9.81;

%N was broken up into n1 and n2 to facilitate multplication of row vectors
n1 = m*(positionX.*velocityX.^2*6/12500+g);
n2 = (cos(atan(3/12500*positionX.^2-1)));
N = n1.*n2;

%plot Normal force vs ~~position~~ time
plot(t,N,'color','b');
xlabel('Time (S)');
```

```
ylabel('Normal Force (N)' );
title('Normal Force of Slope on Skier (N) vs Time (s)');
%-----------------------------------------------------------------------------
figure(3);

% S vs yP plots the position of the skier on the 'ski slope'
S=(z(:,1));
yP = 50 -S  + (S.^3)/12500;

% X vs Y plots the profile of the ski slope
X=(0:130);
Y = 50 - X + (X.^3)/12500;

%plot the results
plot(X,Y,'b-',S,yP,'ro');
xlabel('X (m)');
ylabel('Y(m)  curve(1)');

%text annotations to differentiate curve 1 and curve 2
text(80,80,['curve (2) '],...
    'VerticalAlignment','bottom',...
    'HorizontalAlignment','left',...
    'FontSize',10);
text(8,45,['curve (1) '],...
    'VerticalAlignment','bottom',...
    'HorizontalAlignment','left',...
    'FontSize',10);
text(5,3,['Red circles represents the Skier.This graph shows the skier oscillating be
y=50, x=0 and x ~= 112'],...
    'VerticalAlignment','bottom',...
    'HorizontalAlignment','left',...
    'FontSize',10);

% superimposing Normal Force vs Position curve on Y vs X curve
h1 = gca;
h2 =  axes('Position',get(h1,'Position'));

%unpack variables from z
velocityX = z(:,2)';
positionX = z(:,1)';

% units of Kg and m/s^2
m = 80;
g = 9.81;

%N was broken up into n1 and n2 to facilitate multplication of row vectors
n1 = m*(positionX.*velocityX.^2*6/12500+g);
n2 = (cos(atan(3/12500*positionX.^2-1)));
N = n1.*n2;

%plot Normal force vs position
plot(z(:,1),N,'color','k');
```

```
Axis ('Square');
ylabel('Normal Force (N). curve(2)' );
title('Y (m) vs X (m) graph superimposed with Normal Force (N) vs X(m) graph'
set(h2,'YAxisLocation','right','Color','none','XTickLabel',[]);
set(h2,'XLim',get(h1,'XLim'),'Layer','top');
set(gcf,'PaperPositionMode','auto');
%------------------------------------------------------------------------


function zdot =  f(t,z)

%desired output
x = z(1); v= z(2);

%input for ODE
xdot = v;
vxDot = 0.5*(sin(2*(atan(-3/12500*x^2 + 1))))*(xdot^2*(6/12500*x)+9.81);

%pack up input
z1dot = xdot;
z2dot = vxDot;

%this is what the function returns
zdot = [z1dot;z2dot];
```

6. Express Normal Force, N in terms of x & $\dot{x}$

from ②

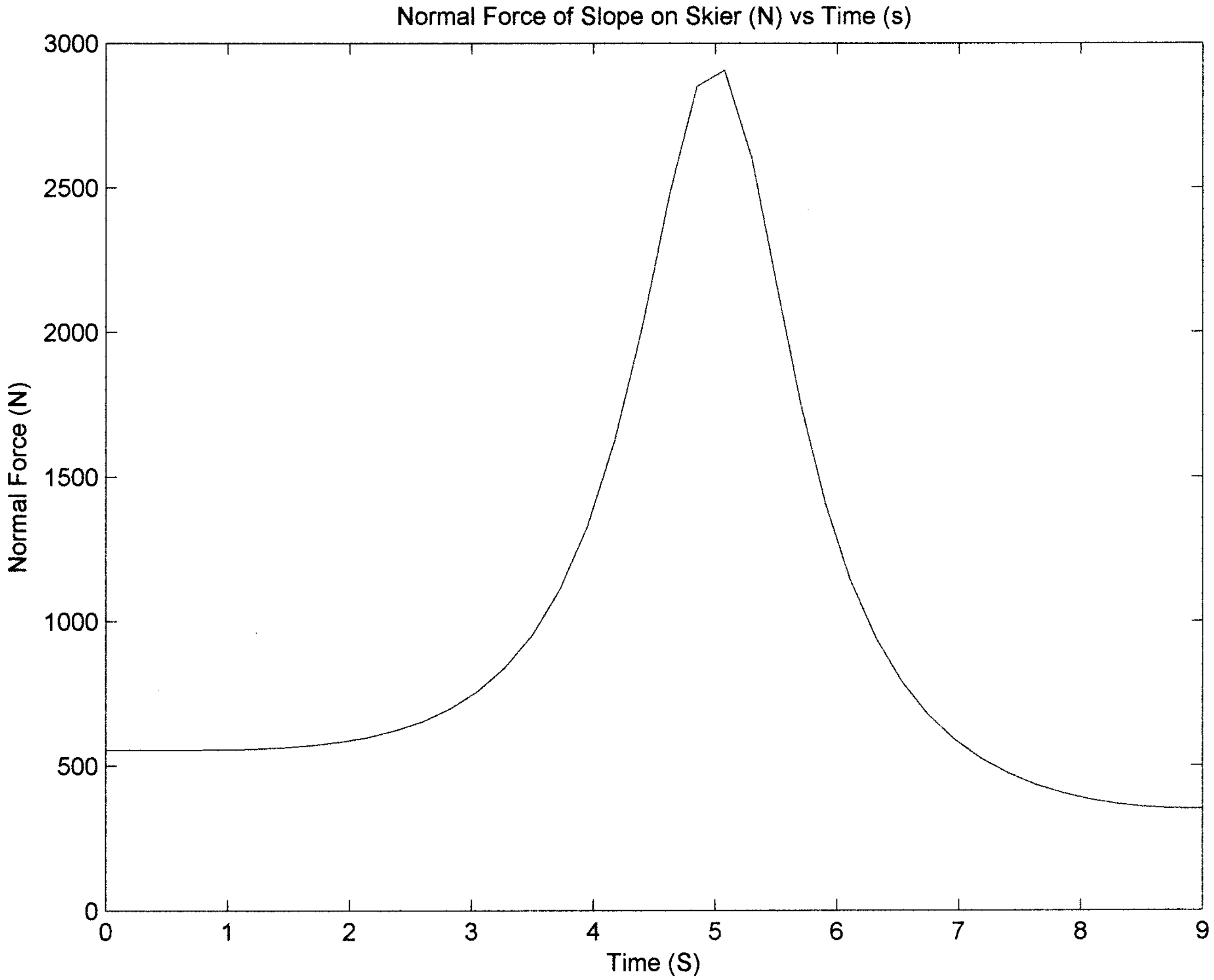
$$N = m\left(\frac{V^2}{R_C} + g\cos\theta\right)$$

Substitute ⑥ & ⑦

$$N = m\cos\left(\tan^{-1}\left(\frac{3}{12500}x^2 - 1\right)\right)\left[\dot{x}^2\left(\frac{6x}{12500}\right) +\right.$$

7. Since Matlab has solved for x & $\dot{x}$ I am able to plot N vs t

- look at Figure (2) of Matlab code.


Normal Force of Slope on Skier (N) vs Time (s)
Normal Force (N)
3000
2500
2000
1500
1000
500
0
0
1
2
3
4
5
6
7
8
9
Time (S)

8. Superimposing the Normal Force as a function of (X) on the curve of Y-X, it is observed that, N_{max} occurs at the base of the slope.

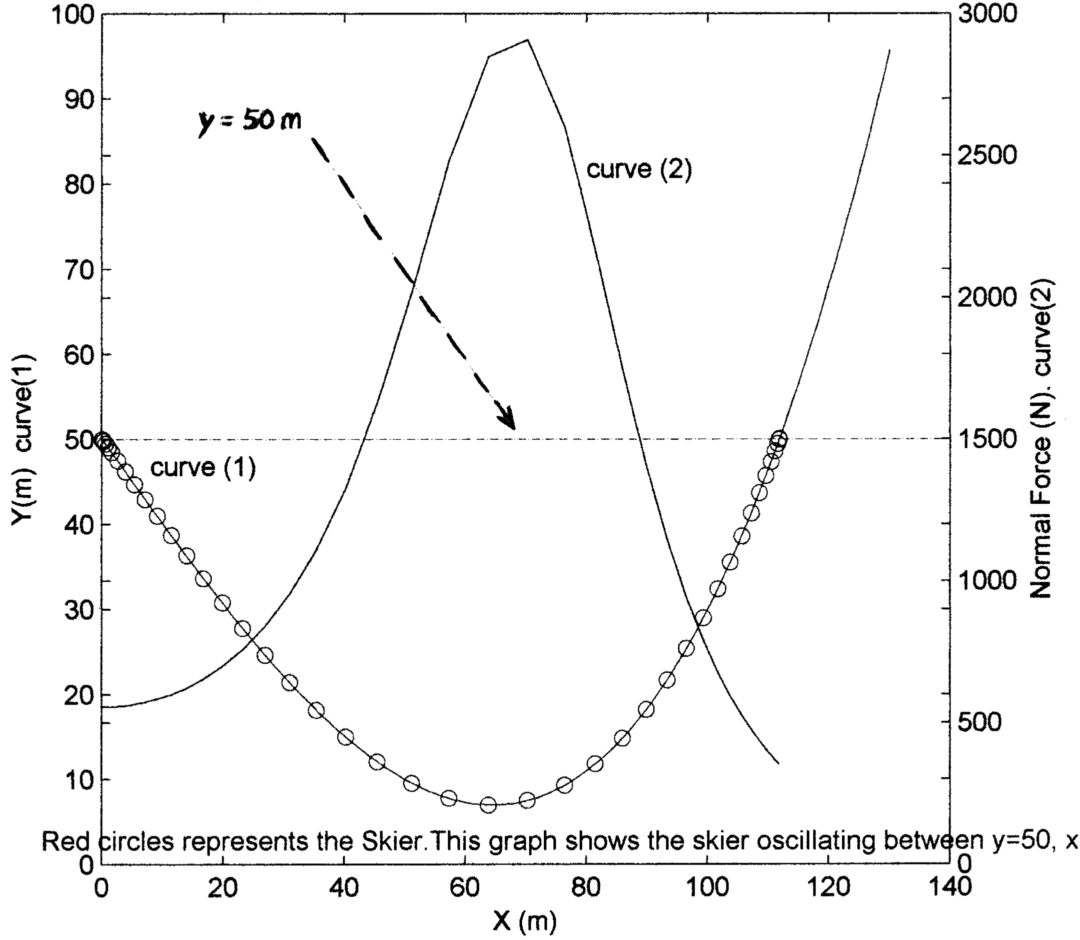
- At valley Velocity is greatest and R_c the lo

$\Rightarrow \frac{V^2}{R_c}$ is maximum

$\Rightarrow$ Curve (2) matches up with reality.

9. If matlab code is integrated for a longer time period, by increasing t span, it is observed that the skier oscillates in the manner shown in the following graph. (assuming no energy loss - closed System)

10. Indeed, the skier never goes pass $y=50$ m.
This verifies the validity of the solution.



Y (m) vs X (m) graph superimposed with Normal Force (N) vs X(m) graph

Red circles represents the Skier.This graph shows the skier oscillating between y=50, x=0 and