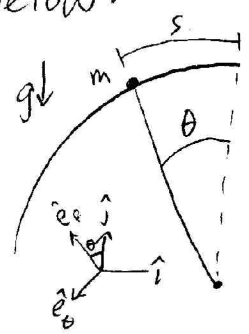
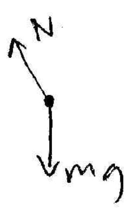
Example 4.3. a marble of mass 0.03 kg is released from rest at an angle $\theta = \frac{\pi}{24}$ radians (from vertical) on a surface which is a semicircle of radius $r = 0.8\,m$. Ignoring friction, how fast will it be going when it reaches $\theta = \frac{\pi}{12}$ rad?

<u>Bonus IV</u>: Why doesn't the "check" for example 4.3 on pages 196-197 agree exactly with the first method?

→ The first method that the book uses to solve this problem is a kinetic energy and work balance, summarized below:



FBD of m



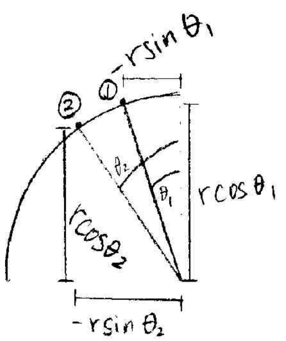
$$\sum \vec{F} = -mg\hat{j} + N\hat{e}_r$$
$$= (-mg\cos\theta + N)\hat{e}_r + mg\sin\theta\,\hat{e}_\theta.$$

$$v_2 = \left[\underbrace{\cancel{v_1^2}}_{0} + \frac{2}{m}\int_{s_1}^{s_2} F\,ds\right]^{\frac{1}{2}}$$

note: $ds = r\,d\theta$

$$\left[\frac{2}{\cancel{m}}\int_{\frac{\pi}{24}}^{\frac{\pi}{12}} \cancel{m}g\sin\theta\; r\,d\theta\right]^{\frac{1}{2}} = \left[-2gr\cos\theta\Big|_{\frac{\pi}{24}}^{\frac{\pi}{12}}\right]^{\frac{1}{2}} = \boxed{0.6328876\,\frac{m}{s}}$$

The book proceeds to "check" its answer by approximating the path of the particle as a straight line between its position at $\theta = \frac{\pi}{24}$ and $\theta = \frac{\pi}{12}$. However, their answer has an error of 12% from their original calculation of $0.633\,\frac{m}{s}$. Why is this?



$r = 0.8\,m$

$\theta_1 = \pi/24$

$\theta_2 = \pi/12$

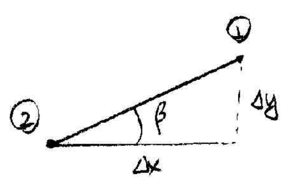
With the origin located at the center of the circle, as in the text:

$$(x_1, y_1) = (-r\sin\theta_1, r\cos\theta_1)$$
$$= (-0.1044, 0.7932)$$
$$(x_2, y_2) = (-r\sin\theta_2, r\cos\theta_2)$$
$$= (-0.2071, 0.7727)$$

The book's error is in calculating these coordinates

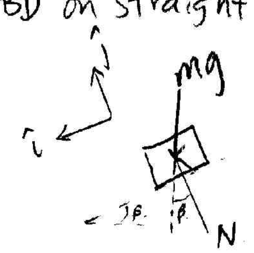
The distance d between 2 points, using the distance formula, is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-0.2071 - (-0.1044))^2 + (0.7727 - 0.7932)^2} = 0.1046\,m$$



$$\beta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{0.7727 - 0.7932}{-0.2071 - (-0.1044)}\right) = 0.1963$$

FBD on straight slope:



$$\{\Sigma \vec{F}\}\cdot\hat{\imath} = \cancel{m}a = \cancel{m}g\sin\beta \Rightarrow a = g\sin\beta$$

$$\int_{v_0}^{v_f} v\,dv = \int_{x_0}^{x_f} a\,ds$$

$$\left.\frac{v^2}{2}\right|_{v_0}^{v_f} = \left. ax \right|_{x_0}^{x_f}$$

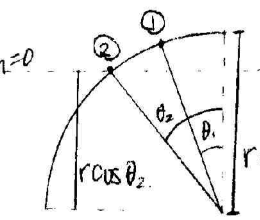
$$\frac{v_f^2}{2} - \cancel{\frac{v_0^2}{2}}^{\,0} = a(x_f - x_0)$$

$$v_f = \sqrt{2a\Delta x} = \sqrt{2(9.81\,\tfrac{m}{s^2})(\sin 0.1963)(0.1046\,m)}$$

$$= \boxed{0.63288876\ \tfrac{m}{s}}$$

↑ Exact same solution as the 1st method

NOTE: A much simpler method of solving the original problem (or checking the answer to the original problem) is to use conservation of energy methods discussed in later sections. Since the only force doing work on the particle is gravity (no friction), which is a conservative force, we see that:



$$E_K|_2 + \cancel{E_p}|_2 = \cancel{E_K}|_1 + E_p|_1 + \cancel{W_{nc}}|_{1-2}$$

0 $(h_2 = 0)$ $\quad$ 0 (released from rest) $\quad$ 0 (no nonconservative forces doing work)

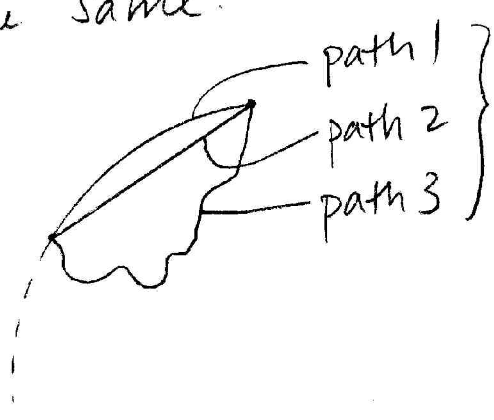
So: $\frac{1}{2}\cancel{m}v_2^2 = \cancel{m}gh_1$

$$v_2^2 = 2gh_1 = 2g\Delta y = 2gr(\cos\theta_1 - \cos\theta_2)$$

$$v_2 = \sqrt{2gr(\cos\theta_1 - \cos\theta_2)} = \sqrt{2(9.81\ \tfrac{m}{s^2})(0.8\ m)(0.991 - 0.966)}$$

$$= \boxed{0.6328876\ \tfrac{m}{s}}$$

The exact same answer with much less work. The conservation of energy analysis confirms that the final velocity of the particle is independent of its path, meaning that as long as the particle is only acted on by conservative forces, its velocity at point 2 will be the same.



Same answer for any path.