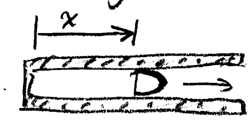
ENGRD/TAM 203 Spring 2006

HW 10 (Assigned on Feb. 23, due on Mar. 2)

Solution by Dennis Yang

3.5.4 The force of the gas on the 0.02 kg bullet is given by $f = f_0 - f_1 t$. For the bullet, $V_i = 0$ at $x = 0$ and $t = 0$ and $V_f = 1000$ kph at the exit of the gun.

$f_0 = 7000$ N and $f_1 = 2.9 \times 10^6$ N/s



a) What is the total linear impulse acting on the bullet?

The motion is 1-D. We have

$$I = \int_0^{t_f} f(t)\,dt = mV_f - mV_i$$

$$= m(V_f - \cancel{V_i}^{\,0})$$

$$= mV_f = 0.02\,\text{kg} \cdot 1000\,\text{kph} = 0.02\,\text{kg} \cdot \frac{1000{,}000\,\text{m}}{3600\,\text{s}}$$

$$\Longrightarrow \boxed{\begin{array}{l} I \approx 5.56\ \text{kg}\cdot\text{m/s} \\ \vec{I} \approx 5.56\ \text{kg}\cdot\text{m/s}\ \hat{\imath} \end{array}}$$

b) When does the bullet exit the gun?

$$I = \int_0^{t_f} f(t)\,dt = m v_f - m\cancel{v_i}^{\,0}$$

$$\Longrightarrow \int_0^{t_f} (f_0 - f_1 t)\,dt = m v_f$$

$$\Longrightarrow f_0 t_f - \tfrac{1}{2} f_1 t_f^2 = m v_f$$

$$\Longrightarrow \tfrac{1}{2} f_1 t_f^2 - f_0 t_f + m v_f = 0$$

$$\Longrightarrow t_f = \frac{f_0 + \sqrt{f_0^2 - 2 f_1 m v_f}}{f_1} \quad \text{or} \quad \frac{f_0 - \sqrt{f_0^2 - 2 f_1 m v_f}}{f_1}$$

$f(t_f) = f_0 - f_1 t_f > 0$ (the expanding gas pushes the bullet out of the gun)

$$\underset{f_0>0,\ f_1>0}{\Longrightarrow} \boxed{t_f < \frac{f_0}{f_1}}$$

Thus, we only take $t_f = \dfrac{f_0 - \sqrt{f_0^2 - 2 f_1 m v_f}}{f_1} \quad \left(< \dfrac{f_0}{f_1}\right)$

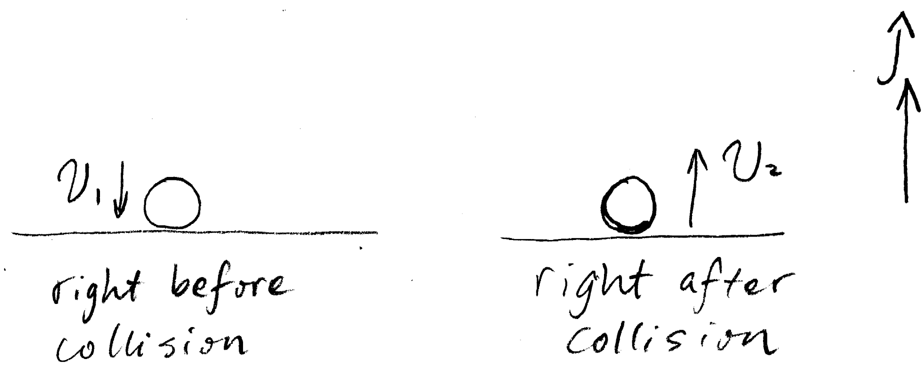
$$= \frac{7000\ \text{N} - \sqrt{(7000\ \text{N})^2 - 2 \times 2.9 \times 10^6\ \text{N/s} \times 5.56\ \text{kg·m/s}}}{2.9 \times 10^6\ \text{N/s}}$$

$$\Longrightarrow \boxed{t_f \approx 1.00 \times 10^{-3}\ \text{s}}$$

□

3.5.8 A 0.2 kg bouncing ball is dropped from a height h and rebounds to a height of 1.6 m. What is the linear impulse applied to the ball while it is in contact with the floor?

Solution



$$\vec{I} = m\vec{v}_2 - m\vec{v}_1$$
$$= m v_2 \hat{j} - m v_1(-\hat{j})$$
$$= m(v_2 + v_1)\hat{j}$$

In addition: $v_1^2 - 0^2 = 2(-g)(-h)$

$0^2 - v_2^2 = 2(-g) \cdot 1.6\,\text{m}$

$\Longrightarrow$ $v_1 = \sqrt{2gh}$, $v_2 = \sqrt{2 \times 9.81\,\text{m/s}^2 \times 1.6\,\text{m}} \approx 5.6\,\text{m/s}$

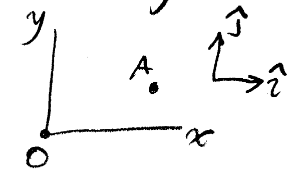
Thus, $\boxed{\vec{I} \approx 0.2\,\text{kg}\,(5.6\,\text{m/s} + \sqrt{2gh})\,\hat{j}}$

$\square$

3.5.37 Find the angular momentum of A about O

$m_A = 6\,kg$, $\vec{V}_A = 5.2\,m/s\,\hat{\imath} - 3.4\,m/s\,\hat{\jmath}$

$\vec{r}_{A/O} = 5.6\,m\,\hat{\imath} + 2.5\,m\,\hat{\jmath}$



Solution

$$\vec{H}_{A/O} = \vec{r}_{A/O} \times m_A \vec{V}_A$$

$$= 6\,kg\cdot(5.6\,m\,\hat{\imath} + 2.5\,m\,\hat{\jmath}) \times (5.2\,m/s\,\hat{\imath} - 3.4\,m/s\,\hat{\jmath})$$

$$= 6\,kg\,(2.5\,m\cdot 5.2\,m/s\,(-\hat{k}) + 5.6\,m\,(-3.4\,m/s)\,\hat{k})$$

$$= 6\,kg\cdot(-2.5\,m * 5.2\,m/s - 5.6\,m * 3.4\,m/s)\,\hat{k}$$

$$\boxed{\vec{H}_{A/O} = -192.24\ kg\cdot m^2/s\ \hat{k}}$$

□