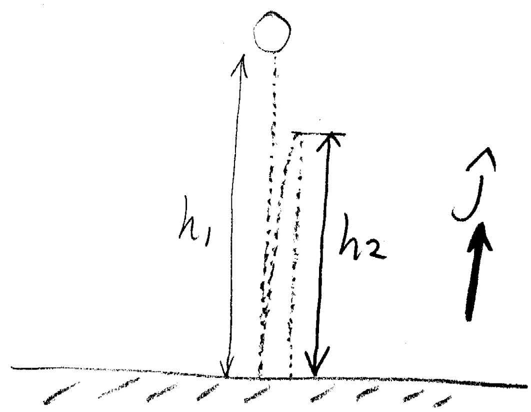
ENGRD/TAM 203 Spring 2006

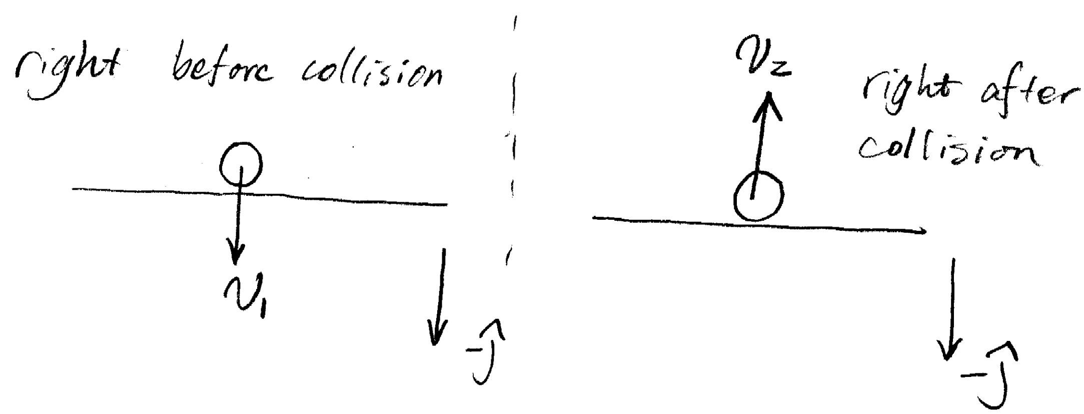
HW 11 (Assigned on Feb 28, due on Mar 7)

Solution by Dennis Yang

3.8.2 If a tennis ball is dropped from a height of 100 in and it rebounces more than 53 in. and less than 58 in., what is the range of e?

Solution





Note: The tennis ball collides onto the ground in $(-\hat{j})$ direction. Before the collision, the velocity of the ball is $v_1(-\hat{j})$, and after the collision, the velocity of the ball is $v_2\hat{j}$. The velocity of the ground is $\vec{0}$ before and after the collision.

Thus, in $(-\hat{j})$ direction,

$$e = \frac{\vec{0}\cdot(-\hat{j}) - (v_2\hat{j})\cdot(-\hat{j})}{[v_1(-\hat{j})]\cdot(-\hat{j}) - \vec{0}\cdot(-\hat{j})} = \frac{v_2}{v_1}.$$

In addition,

$$v_1^2 - 0^2 = 2(-g)(-h_1)$$

$$\Longrightarrow \quad v_1 = \sqrt{2gh_1}$$

$$0^2 - v_2^2 = 2(-g)\cdot h_2$$

$$\Longrightarrow \quad v_2 = \sqrt{2gh_2}$$

Thus, $e = \dfrac{\sqrt{2gh_2}}{\sqrt{2gh_1}} = \sqrt{\dfrac{h_2}{h_1}}$

$$\Longrightarrow \quad \sqrt{\frac{53\text{ in.}}{100\text{ in.}}} \leq e \leq \sqrt{\frac{58\text{ in.}}{100\text{ in.}}}$$

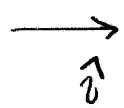
That is, $\boxed{0.728 \leq e \leq 0.762}$

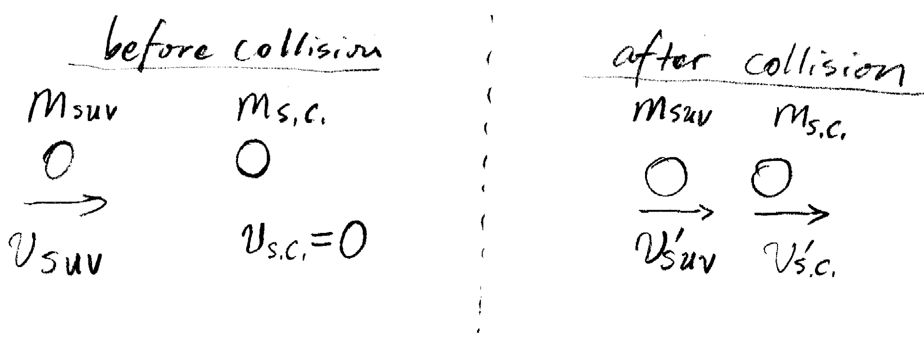
$\square$

3.8.3 A 5000 lbm SUV plows into the rear of a 2200 lbm sportscar. $e = 0$. Just before the collision the SUV was moving at 20 mph and the sportscar was stationary. The entire collision takes 0.3 s.

a) What are the two vehicles' speeds immediately after the collision?

Solution





$$e = \frac{v'_{s.c.} - v'_{suv}}{v_{suv} - \cancel{v_{s.c.}}^{\,0}} = \frac{v'_{s.c.} - v'_{suv}}{v_{suv}} = 0$$

$$\Longrightarrow \underline{v'_{s.c.} = v'_{suv}}$$

In addition, the total momentum of the system is conserved, i.e.,

$$m_{suv}\vec{v}_{suv} + m_{s.c.}\cancel{\vec{v}_{s.c.}}^{\,\vec{0}} = m_{suv}\vec{v}'_{suv} + m_{s.c.}\vec{v}_{s.c.}$$

$$\Longrightarrow m_{suv} v_{suv}\hat{\imath} = m_{suv} v'_{suv}\hat{\imath} + m_{s.c.} v_{s.c.}\hat{\imath} \quad (*)$$

$$(*)\cdot\hat{\imath} \Rightarrow m_{suv}V_{suv} = m_{suv}V'_{suv} + m_{s.c.}V'_{s.c.}$$

$$\Longrightarrow m_{suv}V_{suv} = m_{suv}V'_{suv} + m_{s.c.}V'_{suv}$$

$V'_{s.c.} = V'_{suv}$

$$\Longrightarrow V'_{suv} = \frac{m_{suv}}{m_{suv} + m_{s.c.}} V_{suv}$$

$$= \frac{5000\ lbm/(32.2\,ft/s^2)}{(5000\ lbm + 2200\ lbm)/(32.2\,ft/s^2)} \cdot 20\ mph$$

$$= \frac{5000}{7200} \cdot 20\ mph$$

$$\approx 13.89\ mph = 20.37\ ft/s$$

Thus, $\boxed{V'_{s.c.} = V'_{suv} = 20.37\ ft/s}$

b) What time-average accelerative loads do the two vehicles experience?

Solution

$$\bar{a}_{suv} = \frac{V'_{suv} - V_{suv}}{\Delta t} = \frac{20.37\ ft/s - 20\ mph \times 1.467\ \frac{ft/s}{mph}}{0.3\ s} \approx -29.9\ ft/s^2$$

$$\bar{a}_{s.c.} = \frac{V'_{s.c.} - V_{s.c.}}{\Delta t} = \frac{20.37\ ft/s - 0\ ft/s}{0.3\ s} \approx 67.9\ ft/s^2$$

□