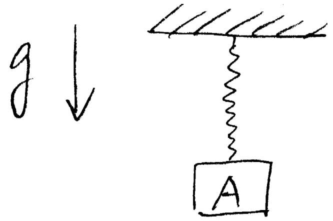
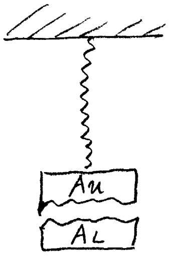
ENGRD/TAM 203 Spring 2006
HW14 (Assigned on Mar. 9, due on Mar. 16)
Solution by Dennis Yang

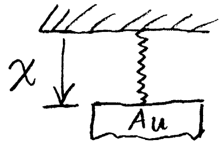
4.2.20 $m_A = 10\,kg$ and it is suspended in equilibrium by a linear spring with $k = 200\,N/m$. Assume an unstretched length of zero for the spring. At $t=0$ the lower half of A falls to the floor. At what speed will the upper half hit the ceiling?



initial state



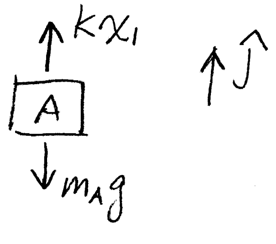
the lower half falls to the floor.



Solution

F.B.D.

initial state



initially at static equilibrium $\Longrightarrow \sum_i \vec{F}_i = \vec{0}$

$$\Longrightarrow kx_1\hat{j} + m_A g(-\hat{j}) = \vec{0} \quad (*)$$

$$(*)\cdot\hat{j} \Longrightarrow kx_1 - m_A g = 0 \Longrightarrow x_1 = \frac{m_A g}{k} \quad (*)$$

Right after the lower half falls off, the total mechanical energy for the upper half block and the spring is

$$\cancelto{0}{KE_1} + PE_1 = \frac{1}{2}kx_1^2 + \frac{m_A}{2}g(-x_1) \quad ①$$

the upper half
is initially at rest

Right before the upper half hits the ceiling, the total mechanical energy for the upper half block and the spring is

$$KE_2 + PE_2 = \frac{1}{2}\left(\frac{m_A}{2}\right)(\dot{x})^2 + \frac{1}{2}kx^2\Big|_{x=0} + \frac{m_A}{2}g(-x)\Big|_{x=0}$$

$$= \frac{1}{2}\frac{m_A}{2}\dot{x}^2 \quad ②$$

There is NO dissapitation, so the total mechanical energy is conserved!

$$\Longrightarrow \quad ② = ① \quad \Longrightarrow \quad \frac{1}{2}\frac{m_A}{2}\dot{x}^2 = \frac{1}{2}kx_1^2 - \frac{m_A}{2}gx_1$$

the substitution of (*) into above gives

$$\frac{1}{2}\frac{m_A}{2}\dot{x}^2 = \frac{1}{2}k\left(\frac{m_A g}{k}\right)^2 - \frac{m_A}{2}g\left(\frac{m_A g}{k}\right)$$

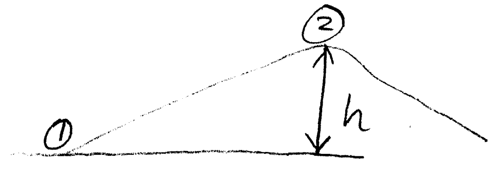
$$= 0$$

$$\Longrightarrow \boxed{\dot{x} = 0}$$

That is, the speed is zero when it reaches the ceiling!

$\square$

4.3.7 Someone takes 44 mins to get to the top of a mountain (elevation change of 3500ft). Assume a weight of 160 lb for bike and rider. What's the average power output in the ride? (in horsepower)



Solution

$$W = PE_{\textcircled{2}} - PE_{\textcircled{1}}$$

$$= mgh$$

(Assume the change of Kinetic energy is very small compared to the change of potential energy from ① to ②)

Average power: $\bar{P} = \frac{W}{\Delta t} = \frac{(mg)h}{\Delta t}$

$$= \frac{160\ \text{lb} \cdot 3500\ \text{ft}}{44\ \text{mins}}$$

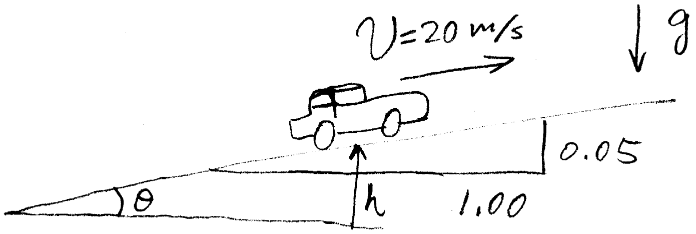
$$= 212.1\ \text{ft} \cdot \text{lb/s}$$

$$= 212.1\ \text{ft} \cdot \text{lb/s} \cdot \frac{1\ \text{hp}}{550\ \text{ft} \cdot \text{lb/s}}$$

$$\boxed{\bar{P} \approx 0.386\ \text{hp}}$$

□

4.3.11 What power is necessary to move a 1200 kg Car up a 5% grade at a Constant 20 m/s? Neglect air and road drag.



Solution

$$P = \frac{d}{dt} W = \frac{d}{dt}(mgh)$$

(no change in kinetic energy since $V \equiv 20$ m/s)

$$= mg\dot{h}$$

$$= mgV\sin\theta$$

$$= 1200\text{kg} \cdot 9.81\ \text{N/kg} \cdot 20\ \text{m/s} \cdot \frac{0.05}{\sqrt{1^2 + 0.05^2}}$$

$$\boxed{P \approx 11757\ \text{w}}$$

$\square$