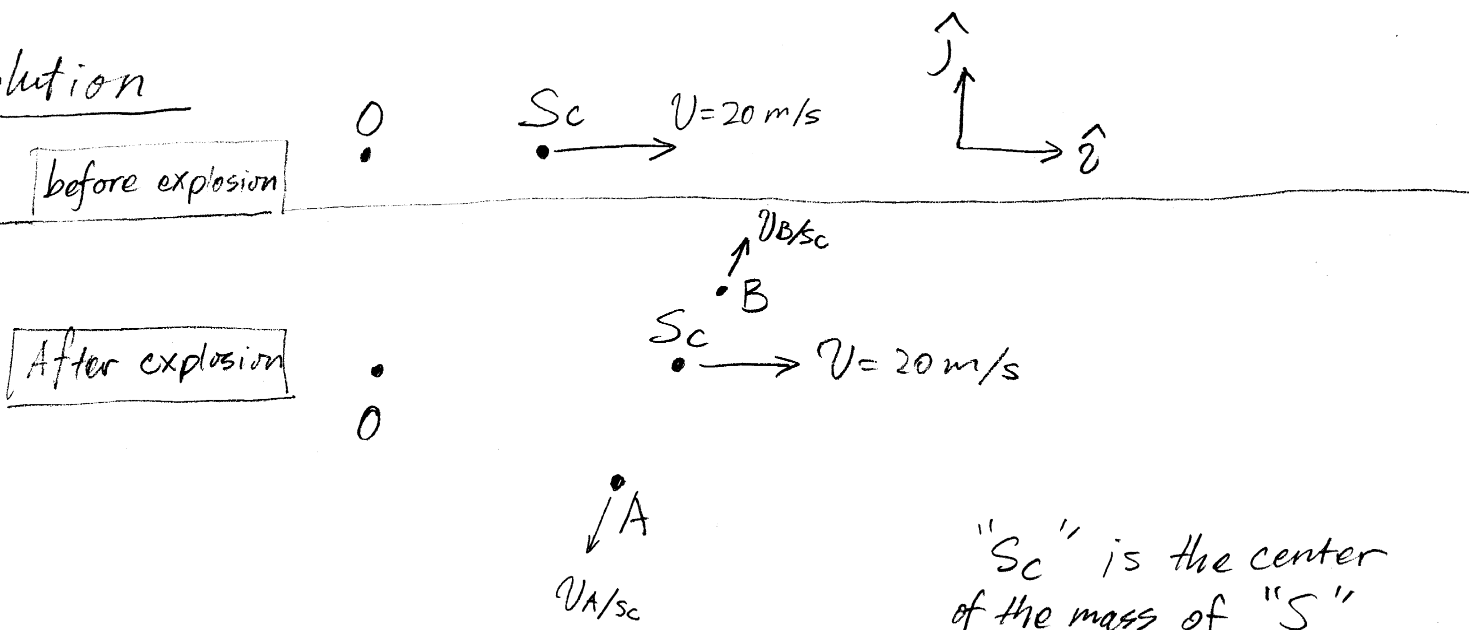
ENGRD/TAM203 Spring 2006
HW15 (Assigned on Mar 14, due on Mar 28)
Solution by Dennis Yang

5.1.13 At $t = 0s$, a satellite S is ejected at $\vec{v} = 20\ m/s\ \hat{\imath}$ from O. At $t = 3s$, S explodes into two pieces: "A" with $m_A = 100\ kg$ and "B" with $m_B = 500\ kg$. At $t = 5s$, B is at $\vec{r}_{B/O} = 100\ m\ \hat{\imath} + 10\ m\ \hat{\jmath}$. Where is A and how fast does it travel?

Solution



There is NO external force $\Longrightarrow \sum_i \vec{F}_i = \vec{0}$

$$\Longrightarrow (m_A + m_B)\dot{\vec{v}}_{Sc/O} = \vec{0}$$

$$\Longrightarrow \vec{v}_{Sc/O} \equiv 20\ m/s\ \hat{\imath}$$

At $t = 5\,s$, $\vec{r}_{SC/O} = \int_0^{5s} \vec{v}_{SC/O}\, dt$

$$= 20\, m/s\, \hat{i} \cdot 5\, s$$

$$= 100\, m\, \hat{i}$$

$$m_A \vec{r}_{A/O} + m_B \vec{r}_{B/O} = (m_A + m_B)\, \vec{r}_{SC/O}$$

$$\Longrightarrow m_A \vec{r}_{A/O} = (m_A + m_B)\, \vec{r}_{SC/O} - m_B \vec{r}_{B/O}$$

$$\Longrightarrow \vec{r}_{A/O} = \frac{m_A + m_B}{m_A} \vec{r}_{SC/O} - \frac{m_B}{m_A} \vec{r}_{B/O}$$

$$\Longrightarrow \vec{r}_{A/O} = \frac{100\,kg + 500\,kg}{100\,kg} \cdot 100\, m\, \hat{i} - \frac{500\,kg}{100\,kg} (100\, m\, \hat{i} + 10\, m\, \hat{j})$$

$$= 600\, m\, \hat{i} - (500\, m\, \hat{i} + 50\, m\, \hat{j})$$

$$\boxed{\vec{r}_{A/O} = 100\, m\, \hat{i} - 50\, m\, \hat{j}}$$

Right before explosion, i.e., $t = 3\,s$.

$$\vec{r}_{A/O} = \vec{r}_{B/O} = \vec{r}_{SC/O} = \int_0^{3s} \vec{v}_{SC/O}\, dt = 20\, m/s\, \hat{i} \cdot 3\, s$$

$$\Longrightarrow \vec{r}_{A/O}\Big|_{t=3s} = \vec{r}_{B/O}\Big|_{t=3s} = 60\, m\, \hat{i}$$

$$\vec{v}_{A/O} = \frac{\vec{r}_{A/O}\big|_{t=5s} - \vec{r}_{A/O}\big|_{t=3s}}{5s - 3s}$$

$$= \frac{(100\,m\,\hat{\imath} - 50\,m\,\hat{\jmath}) - 60\,m\,\hat{\imath}}{2\,s}$$

$$\boxed{\vec{v}_{A/O} = 20\,m/s\,\hat{\imath} - 25\,m/s\,\hat{\jmath}}$$

□

5.1.19

A boat, initially stationary in water, fires a shell in the $\hat{\imath}$ direction. The muzzle speed of the shell was 1000 m/s. The shell impacts a target at 990 m/s. The shell was in the gun for 0.1 seconds, and the force exerted on the shell decreased linearly from the initial force to zero over the firing interval. $m_s = 10$ kg. $m_{boat} = 990$ kg. Find the final velocity of the boat at the maximal force on the shell when it was in the gun.

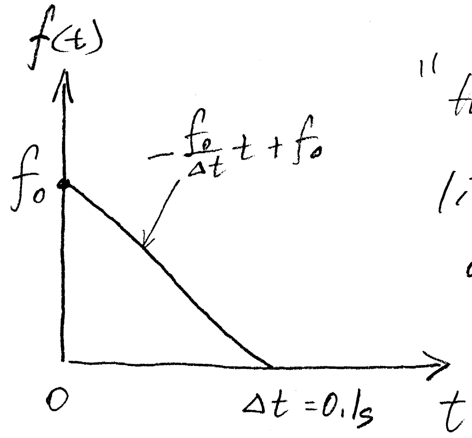
Solution

$$\vec{V}_{boat} + \vec{V}_{s/boat} = \vec{V}_s$$

$$\Longrightarrow \vec{V}_{boat} = \vec{V}_s - \vec{V}_{s/boat}$$

$$= 990 \text{ m/s } \hat{\imath} - 1000 \text{ m/s } \hat{\imath}$$

$$\Longrightarrow \boxed{\vec{V}_{boat} = -10 \text{ m/s } \hat{\imath}}$$



"the force exerted on the shell decreased linearly from the initial force to zero over the firing interval"

$$\underline{f_{max} = f_0}$$

F. B. D

$f(t) \longrightarrow$ shell $\qquad \longrightarrow \hat{\imath}$

$$m_s \cdot \vec{0} + \int_0^{\Delta t} \vec{f}(t)\, dt = m_s \vec{v}_s$$

$$\Longrightarrow \int_0^{\Delta t} \left(-\frac{f_0}{\Delta t} t + f_0\right) \hat{\imath}\, dt = m_s \vec{v}_s$$

$$\Longrightarrow \left(-\frac{f_0}{2\Delta t} t^2 \Big|_0^{\Delta t} + f_0 t \Big|_0^{\Delta t}\right) \hat{\imath} = m_s \vec{v}_s$$

$$\Longrightarrow \frac{f_0 \Delta t}{2} \hat{\imath} = m_s \cdot v_s \hat{\imath} \qquad (*)$$

$$(*) \cdot \hat{\imath} \Longrightarrow \frac{f_0 \Delta t}{2} = m_s v_s$$

$$\Longrightarrow f_0 = \frac{2 m_s v_s}{\Delta t} = \frac{2 \cdot 10\,kg \cdot 990\,m/s}{0.1\,s}$$

$$\Longrightarrow \boxed{f_{max} = f_0 = 1.98 \times 10^5\, N}$$

□

5.2.6

$m_A = 0.2\,kg, \quad m_B = 0.4\,kg, \quad m_C = 0.3\,kg$

$\vec{r}_{A/O} = 5cm\,\hat{\imath}, \quad \vec{r}_{B/O} = 10cm\,\hat{\imath}, \quad \vec{r}_{C/O} = 15cm\,\hat{\imath}$

$\vec{v}_A = -3\,cm/s\,\hat{\jmath}, \quad \vec{v}_B = 3\,cm/s\,\hat{k}, \quad \vec{v}_C = -3\,cm/s\,\hat{\jmath}$

$\vec{H}_O = ?$

Solution

$$\vec{H}_O = \vec{r}_{A/O} \times m_A \vec{v}_A + \vec{r}_{B/O} \times m_B \vec{v}_B + \vec{r}_{C/O} \times m_C \vec{v}_C$$

$$= (5\,cm\,\hat{\imath}) \times (0.2\,kg)(-3\,cm/s\,\hat{\jmath})$$
$$+ (10\,cm\,\hat{\imath}) \times (0.4\,kg)(3\,cm/s\,\hat{k})$$
$$+ (15\,cm\,\hat{\imath}) \times (0.3\,kg)(-3\,cm/s\,\hat{\jmath})$$

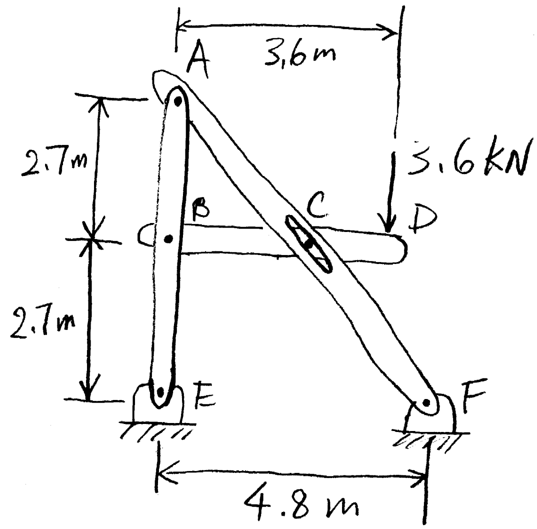
$$= -3\,kg \cdot cm^2/s\,\hat{k} + 12\,kg \cdot cm^2/s\,(-\hat{\jmath})$$
$$-13.5\,kg\,cm^2/s\,\hat{k}$$

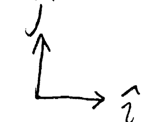
$$\boxed{\vec{H}_O = -12\,kg\,cm^2/s\,\hat{\jmath} - 16.5\,kg\,cm^2/s\,\hat{k}}$$

$\square$

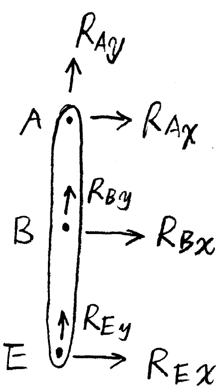
BJ 6,98

Determine the components of all forces on member ABE

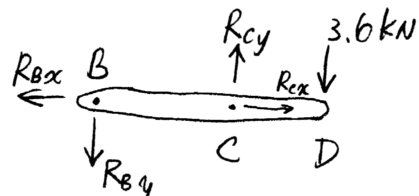




F.B.D

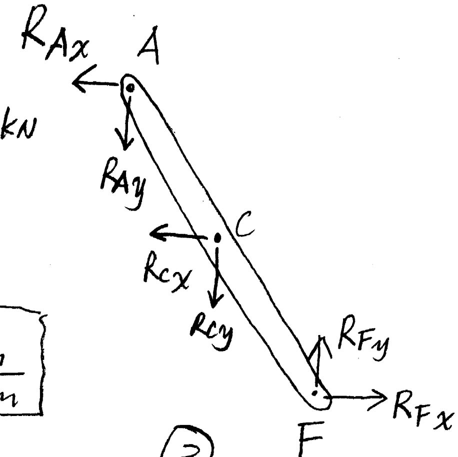


①

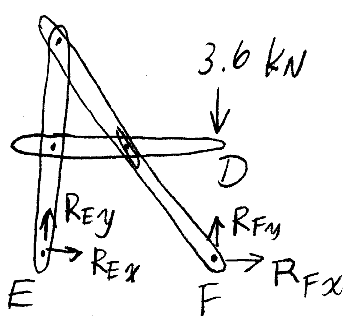


$$\boxed{\frac{R_{cy}}{R_{cx}} = \frac{4.8m}{5.4m}}$$

②



③



④

FBD④ : $\sum_i \vec{M}_{i/F} = \vec{0}$

$$\Rightarrow 4.8\,m(-\hat{\imath}) \times (R_{Ex}\hat{\imath} + R_{Ey}\hat{\jmath}) + (1.2\,m(-\hat{\imath}) + 2.7\,m\hat{\jmath}) \times 3.6\,kN(-\hat{\jmath}) = \vec{0}$$

$$\Rightarrow -4.8\,m \cdot R_{Ey}\hat{k} + 1.2\,m \cdot 3.6\,kN\,\hat{k} = \vec{0} \quad (*)$$

$$(*) \cdot \hat{k} \Rightarrow -4.8\,m\,R_{Ey} + 1.2\,m \cdot 3.6\,kN = 0$$

$$\Rightarrow \underline{R_{Ey} = 0.9\,kN}$$

$$\frac{|\vec{r}_{B/C}|}{2.7\,m} = \frac{4.8\,m}{2.7\,m + 2.7\,m}$$

$$\Rightarrow |\vec{r}_{B/C}| = 2.4\,m$$

FBD ② : $\sum_i \vec{M}_{i/C} = \vec{0}$

$$\Rightarrow 2.4\,m(-\hat{\imath}) \times (R_{Bx}(-\hat{\imath}) + R_{By}(-\hat{\jmath})) + (3.6\,m - 2.4\,m)\hat{\imath} \times 3.6\,kN(-\hat{\jmath}) = \vec{0}$$

$$\Rightarrow 2.4\,m \cdot R_{By}\hat{k} + 1.2\,m \cdot 3.6\,kN(-\hat{k}) = \vec{0} \quad (**)$$

$$(**) \cdot \hat{k} \Rightarrow 2.4\,m\,R_{By} - 1.2\,m \cdot 3.6\,kN = 0$$

$$\Rightarrow \underline{R_{By} = 1.8\,kN}$$

$$\sum_i \vec{F}_i = \vec{0}$$

$$\Rightarrow R_{Bx}(-\hat{\imath}) + R_{By}(-\hat{\jmath}) + R_{Cy}\hat{\jmath} + R_{Cx}\hat{\imath} + 3.6\,kN(-\hat{\jmath}) = \vec{0} \quad (***)$$

$$(***) \cdot \hat{\imath} \Rightarrow -R_{Bx} + R_{Cx} = 0 \quad ①$$

$$(***) \cdot \hat{\jmath} \Rightarrow -R_{By} + R_{Cy} - 3.6\,kN = 0 \quad ②$$

②, $R_{By} = 1.8\,kN \implies R_{Cy} = 3.6\,kN + 1.8\,kN$

$$\implies \underline{R_{Cy} = 5.4\,kN}$$

$$\frac{R_{Cy}}{R_{Cx}} = \frac{4.8\,m}{5.4\,m} \implies \underline{R_{Cx} = 6.075\,kN}$$

$$① \implies \underline{R_{Bx} = R_{Cx} = 6.075\,kN}$$

<u>FBD ①</u> : $\sum_i \vec{M}_{i/A} = \vec{0}$

$$\implies 2.7\,m(-\hat{j}) \times (R_{Bx}\hat{\imath} + R_{By}\hat{j}) + 5.4\,m(-\hat{j}) \times (R_{Ey}\hat{j} + R_{Ex}\hat{\imath}) = \vec{0}$$

$$\implies 2.7\,m \cdot R_{Bx}\hat{k} + 5.4\,m \cdot R_{Ex}\hat{k} = \vec{0} \quad ****$$

$$**** \cdot \hat{k} \implies 2.7\,m\, R_{Bx} + 5.4\,m\, R_{Ex} = 0$$

$$\implies \underline{R_{Ex} = -3.0375\,kN}$$

$$\sum_i \vec{F}_i = \vec{0}$$

$$\implies (R_{Ax}\hat{\imath} + R_{Ay}\hat{j}) + (R_{Bx}\hat{\imath} + R_{By}\hat{j}) + (R_{Ex}\hat{\imath} + R_{Ey}\hat{j}) = \vec{0} \quad 5*$$

$$5* \cdot \hat{\imath} \implies R_{Ax} + R_{Bx} + R_{Ex} = 0 \quad ③$$

$$5* \cdot \hat{j} \implies R_{Ay} + R_{By} + R_{Ey} = 0 \quad ④$$

③, $R_{Bx} = 6.075\,kN$, $R_{Ex} = -3.0375\,kN$

$\Longrightarrow$ $\underline{R_{Ax} = -3.0375\,kN}$

④, $R_{By} = 1.8\,kN$, $R_{Ey} = 0.9\,kN$

$\Longrightarrow$ $\underline{R_{Ay} = -2.7\,kN}$

Therefore,

$$\boxed{\begin{aligned} \vec{R_A} &= R_{Ax}\hat{\imath} + R_{Ay}\hat{\jmath} = -3.0375\,kN\,\hat{\imath} - 2.7\,kN\,\hat{\jmath} \\ \vec{R_B} &= R_{Bx}\hat{\imath} + R_{By}\hat{\jmath} = 6.075\,kN\,\hat{\imath} + 1.8\,kN\,\hat{\jmath} \\ \vec{R_E} &= R_{Ex}\hat{\imath} + R_{Ey}\hat{\jmath} = -3.0375\,kN\,\hat{\imath} + 0.9\,kN\,\hat{\jmath} \end{aligned}}$$

$\square$