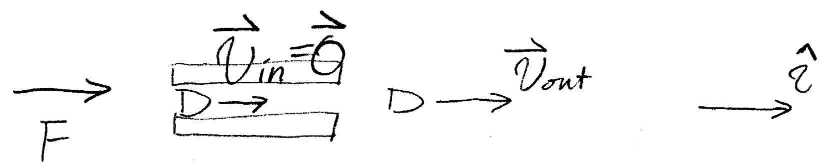
ENGRD/TAM 203 Spring 2006

HW 16 (Assigned on Mar. 16, due on Mar 30)

Solution by Dennis Yang

5.5.3

A gun shoots at 1500 spm (shots per minute). If the muzzle velocity is 3200 ft/s for the bullets which weight 0.22 lbm, What's the average force required in the horizontal direction to hold the gun if it fires in the horizontal direction?



Solution

$$\vec{F} = \dot{m}(\vec{V}_{out} - \vec{V}_{in})$$

$$= \dot{m}\vec{V}_{out}$$

($\vec{V}_{in} = \vec{0}$, assuming the bullets are initially at rest!)

$$\Longrightarrow \vec{F}\cdot\hat{\imath} = \dot{m}\vec{V}_{out}\cdot\hat{\imath}$$

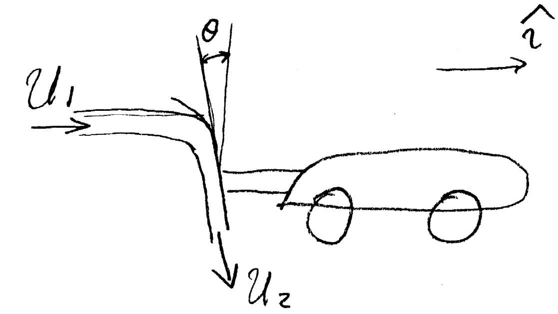
$$\Longrightarrow (F\hat{\imath})\cdot\hat{\imath} = \dot{m}(V_{out}\hat{\imath})\cdot\hat{\imath} = \dot{m}V_{out}$$

$$F = \left(1500\ \frac{\text{shot}}{\text{min}} \cdot \frac{1\ \text{min}}{60\ \text{s}} \cdot \frac{0.22\ \text{lbm/shot}}{32.2\ \text{lbm/slg}}\right) \cdot 3200\ \text{ft/s}$$

$$\boxed{F \approx 547\ \text{lbf}}$$

□

5.5.6

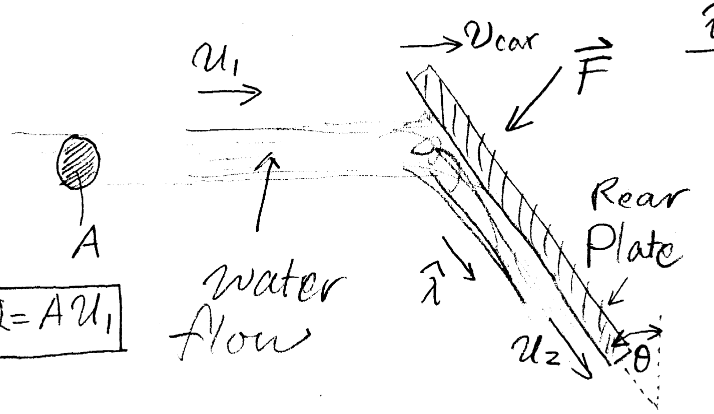


$\theta = 20^\circ$
$m_{car} = 3.2\ kg$
$Q = 0.06\ \ell/s$ (rate of the water jet)
$U_1 = 38\ m/s$

Neglect all frictions and gravitational effects.
$\vec{U}_2$ is the relative velocity of the water, w.r.t. the car.
Car is initially at rest!

Discussion

when consider this mass flow problem, we need to determine $\dot{m}$ and U_2.



$\vec{F}$ is the external force onto the water flow.

$$\vec{F} = \dot{m}\,(\vec{V}_{out} - \vec{V}_{in}) \quad (*)$$

In $(*)$, $\dot{m}$ is the rate of the mass which gets its velocity changed from $\vec{V}_{in}$ to $\vec{V}_{out}$.

In this case, $\dot{m}$ is equal to the mass rate of the water, at which it hits the rear plate of the car. Thus,

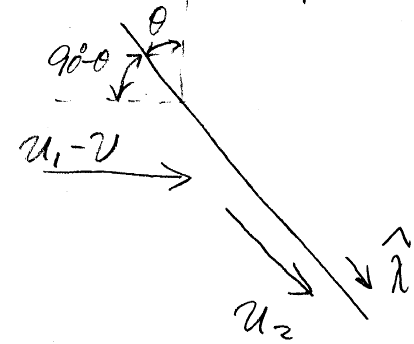
$$\dot{m} = \rho_{water} \cdot A \cdot (U_1 - V_{car})$$

$$\boxed{\dot{m} = \rho_{water} \cdot \frac{Q}{U_1} (U_1 - V_{car})}$$

Relative to the ground, $\vec{V}_{in} = u_1 \hat{\imath}$

and $\vec{V}_{out} = \vec{u}_2 + \vec{V}_{car}$

In this problem, the water <u>HITS</u> the rear plate of the car, and the moves along the plate (relative to the car). Thus, if assume same component tangent to the plate before and after, we have



$$(u_1 - v_{car}) \cos(90° - \theta) = u_2$$

$$\Rightarrow \quad u_2 = (u_1 - v_{car}) \sin\theta$$

and $\vec{u}_2 = u_2 \hat{\lambda} = u_2 (\sin\theta\, \hat{\imath} - \cos\theta\, \hat{\jmath})$

$$\Rightarrow \boxed{\vec{u}_2 = (u_1 - v_{car}) \sin^2\theta\, \hat{\imath} - (u_1 - v_{car}) \cos\theta \sin\theta\, \hat{\jmath}}$$

Thus, $\vec{F} = \dot{m}(\vec{V}_{out} - \vec{V}_{in})$

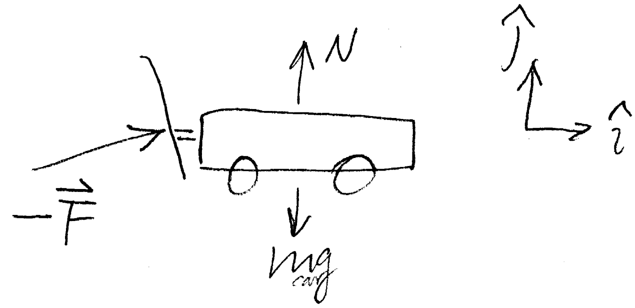
$$= \rho_{water} \frac{Q}{u_1} (u_1 - V_{car}) \Big((u_1 - V_{car}) \sin^2\theta\, \hat{\imath} - (u_1 - V_{car}) \cos\theta \sin\theta\, \hat{\jmath} + V_{car} \hat{\imath} - u_1 \hat{\imath} \Big)$$

$$\Rightarrow \vec{F} = \rho_{water} \frac{Q}{u_1} (u_1 - V_{car}) \Big[\big((u_1 - V_{car}) \sin^2\theta + V_{car} - u_1 \big) \hat{\imath} - (u_1 - V_{car}) \cos\theta \sin\theta\, \hat{\jmath} \Big]$$

$$\Rightarrow \boxed{\vec{F} = \rho_{water} \frac{Q}{u_1} (u_1 - V_{car}) \Big[-(u_1 - V_{car}) \cos^2\theta\, \hat{\imath} - (u_1 - V_{car}) \cos\theta \sin\theta\, \hat{\jmath} \Big]}$$

Now, for the car,

FBD



$$\sum_i \vec{F}_i = m\vec{a} \Longrightarrow -\vec{F} + m_{car} g(-\hat{j}) + N\hat{j} = m_{car}\dot{V}_{car}\hat{i} \quad (**)$$

$$(**)\cdot\hat{i} \Longrightarrow -\vec{F}\cdot\hat{i} = m_{car}\dot{V}_{car}\hat{i}\cdot\hat{i}$$

$$\Longrightarrow \rho_{water}\frac{Q}{U_1}(U_1 - V_{car})^2\cos^2\theta = m_{car}\dot{V}_{car}$$

$$\Longrightarrow \boxed{\dot{V}_{car} = \frac{\rho_{water}}{m_{car}}\frac{Q}{U_1}(U_1 - V_{car})^2\cos^2\theta} \quad (***)$$

$$(***) \Longrightarrow \int_{V_{car}\text{ at } t=0}^{V_{car}\text{ at } t=T} \frac{dV_{car}}{(U_1 - V_{car})^2} = \int_{t=0}^{t=T} \frac{\rho_{water}}{m_{car}}\frac{Q}{U_1}\cos^2\theta\, dt$$

$$\Longrightarrow \left.\frac{1}{U_1 - V_{car}}\right|_{\text{at } t=0}^{\text{at } t=T} = \frac{\rho_{water}}{m_{car}}\frac{Q}{U_1}\cos^2\theta\, T$$

$$\Longrightarrow \frac{1}{U_1 - V_{car}(T)} - \frac{1}{U_1} = \frac{\rho_{water}}{m_{car}}\frac{Q}{U_1}\cos^2\theta\, T$$

$\underline{V_{car}(0) = 0}$

$$\Longrightarrow \boxed{V_{car}(T) = U_1 - \frac{U_1}{\frac{\rho_{water}}{m_{car}}Q\cos^2\theta\, T + 1}} \quad (4*)$$

$$(4*) \Longrightarrow X(T)-X(0) = \int_0^T \left(u_1 - \frac{u_1}{\frac{\rho_{water}}{M_{car}} Q\cos^2\theta\, t+1}\right) dt$$

$$= u_1 \left(t - \frac{1}{\frac{\rho_{water}}{M_{car}} Q\cos^2\theta} \ln\left(\frac{\rho_{water}}{M_{car}} Q\cos^2\theta\, t + 1\right)\right)\Bigg|_0^T$$

$$\boxed{X(T)-X(0) = u_1\left(T - \frac{1}{\frac{\rho_{water}}{M_{car}} Q\cos^2\theta} \ln\left(\frac{\rho_{water}}{M_{car}} Q\cos^2\theta\, T+1\right)\right)}$$

$(5*)$

(a) Determine the acceleration of the car at the beginning.

$$(***) \Longrightarrow \dot{V}_{car}\Big|_{t=0} = \frac{\rho_{water}}{M_{car}} \frac{Q}{u_1} u_1^2 \cos^2\theta = \frac{\rho_{water} Q}{M_{car}} u_1 \cos^2\theta$$

"$V_{car}(0)=0$"

$$= \frac{1\,kg/l \cdot 0.06\, l/s}{3.2\,kg} 38\,m/s \cos^2 20^\circ$$

$$\approx 0.629\ m/s^2$$

$$\vec{a}_{car} \approx 0.629\ m/s^2\ \hat{i}$$

(b) Determine the acceleration of the car when $V_{car} = 1.0\,m/s$.

$$\dot{V}_{car}\Big|_{V_{car}=1.0\,m/s} = \frac{\rho_{water}}{M_{car}} \frac{Q}{u_1} (u_1 - V_{car})^2 \cos^2\theta = \frac{1\,kg/l \cdot 0.06\, l/s}{3.2\,kg} \frac{(38\,m/s - 1\,m/s)^2}{38\,m/s} \cos^2 20^\circ$$

$$\approx 0.596\ m/s^2$$

$$\vec{a}_{car} \approx 0.596\ m/s^2\ \hat{i}$$

(c) Determine the velocity of the car at $t = 3.9s$.

$$(4*) \Longrightarrow V_{car}(3.9s) = u_1 - \frac{u_1}{\frac{\rho_{water}}{m_{car}} Q \cos^2\theta \cdot 3.9s + 1}$$

$$= 38 m/s - \frac{38 m/s}{\frac{1 kg/l}{3.2kg} 0.06 l/s \cdot \cos^2 20° \cdot 3.9s + 1}$$

$$\approx 2.30 \ m/s$$

$$\boxed{\vec{v}_{car} = 2.30 \ m/s \ \hat{i}}$$

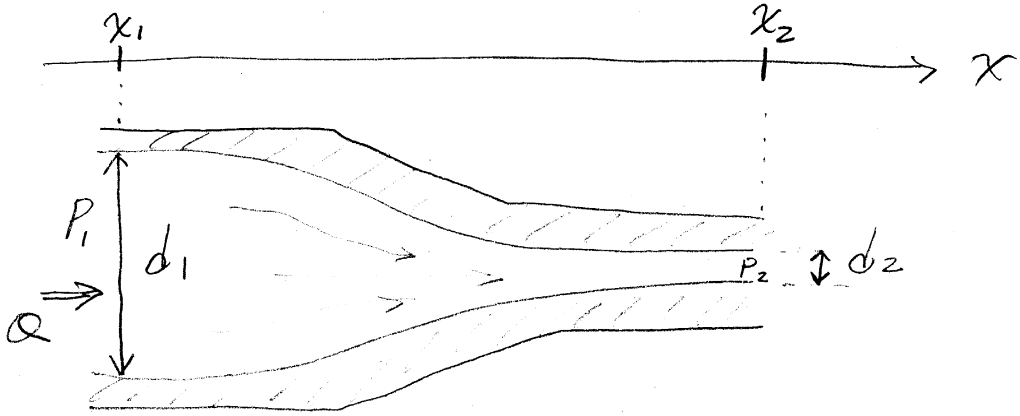
(d) Determine $x(3.9s) - x(0)$

$$(5*) \Rightarrow x(3.9s) - x(0) = 38 m/s \left(3.9s - \frac{1}{\frac{1 kg/l}{3.2kg} \cdot 0.06 l/s \cos^2 20°} \ln\left(\frac{1 kg/l}{3.2kg} 0.06 l/s \cos^2 20° \cdot 3.9s + 1\right)\right)$$

$$\approx 4.59 \ m$$

$\square$

5.5.9



$d_1 = 4\,cm$, $d_2 = 2\,cm$, $\Delta P = P_1 - P_2 = 50\,kPa$

$Q = 3\,l/s$ $\quad P_2 = 100\,kPa$

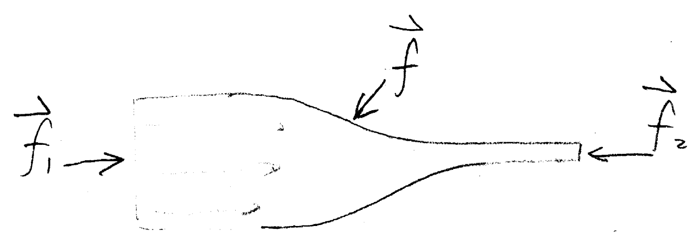
Determine the force exerted (x-component only!) on the area of pipe contraction by the water.

Solution

For the water flow :

$$\vec{F} = \dot{m}\,(\vec{v}_{out} - \vec{v}_{in})$$

where, $\vec{F}$ is the sum of the external forces onto the water flow btw x_1 and x_2.



$$\vec{F} = \vec{f}_1 + \vec{f} + \vec{f}_2 \text{ , where (next page)}$$

$$\vec{f}_1 = P_1 A_1 \hat{\imath}\ , \quad \vec{f}_2 = P_2 A_2 (-\hat{\imath})$$

$\vec{f}$ is the force exerted to the water by the pipe wall.

$$\vec{V}_{out} = \frac{Q}{A_2}\hat{\imath} = \frac{Q}{\frac{\pi}{4}d_2^2}\hat{\imath}$$

$$\vec{V}_{in} = \frac{Q}{A_1}\hat{\imath} = \frac{Q}{\frac{\pi}{4}d_1^2}\hat{\imath}$$

Thus,
$$P_1 A_1 \hat{\imath} + \vec{f} + P_2 A_2(-\hat{\imath}) = \rho Q\left(\frac{Q}{\frac{\pi}{4}d_2^2}\hat{\imath} - \frac{Q}{\frac{\pi}{4}d_1^2}\hat{\imath}\right) \quad (*)$$

$$(*)\cdot\hat{\imath} \Rightarrow P_1 A_1 + \vec{f}\cdot\hat{\imath} - P_2 A_2 = \rho Q\left(\frac{4Q}{\pi d_2^2} - \frac{4Q}{\pi d_1^2}\right)$$

$$\Rightarrow \vec{f}\cdot\hat{\imath} = P_2 A_2 + \frac{4\rho Q^2}{\pi}\left(\frac{1}{d_2^2} - \frac{1}{d_1^2}\right) - (P_2 + \Delta P)A_1$$

$$= 100\,kPa \cdot \frac{\pi}{4}(0.02\,m)^2 + \frac{4\cdot 10^3 kg/m^3 \left(3\,l/s \cdot \frac{m^3}{10^3 l}\right)^2}{\pi}\left(\frac{1}{(0.02m)^2} - \frac{1}{(0.04m)^2}\right)$$

$$- 150\,kPa \cdot \frac{\pi}{4}(0.04m)^2$$

$$\approx -135.6\,N$$

Thus, the x-component of the force exerted on the area of pipe contraction by the water is $\underline{(-\vec{f})\cdot\hat{\imath} = 135.6N}$ $\square$