ENGRD/TAM 203

HW 17 (Assigned on Mar. 28, due on April 4)

Solution by Dennis Yang

6.1.10 Assume an effective tire radius of 13.3 in. from the hub center to the ground. The two front chain rings have 39 and 52 teeth. The rear gear cluster has a rang of cogs from 12 teeth up to 28 teeth. When a person pedal at 75 rpm, what's the maximum and minimum rotational speed of the rear wheel ?

Solution

$$\dot{\theta}_{wheel} = \frac{N_{chain\text{-}ring}}{N_{rear\text{-}cog}} \cdot \dot{\theta}_{petal} \qquad (*)$$

$$(*) \Longrightarrow \dot{\theta}_{wheel\ max} = \frac{N_{chain\text{-}ring\ max}}{N_{rear\text{-}cog\ min}} \cdot \dot{\theta}_{petal} = \frac{52}{12} \cdot 75\,rpm = 325\,rpm$$

$$\boxed{\dot{\theta}_{wheel\ max} = 325\ rpm}$$

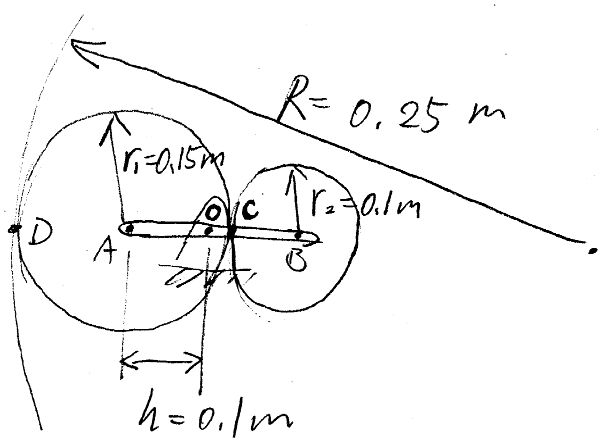
$$(*) \longrightarrow \dot{\theta}_{wheel\ min} = \frac{N_{chain\text{-}ring\ min}}{N_{rear\text{-}cog\ max}} \dot{\theta}_{petal} = \frac{39}{28} \cdot 75\,rpm \simeq 104.5\,rpm$$

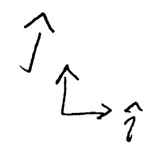
$$\boxed{\dot{\theta}_{wheel\ min} \simeq 104.5\ rpm}$$

$\square$

6.1.19 Body 1 is attached to the rigid link $\overline{AB}$ at A, and Body 2 is attached at B. The bodies roll without slip against each other. $\omega_{\overline{AB}} = -4\,\text{rad/s}\,\hat{k}$

Find $\vec{\omega}_1$, and $\vec{\omega}_2$ (Assume they are $\omega_1\hat{k}$ and $\omega_2\hat{k}$)





Solution

$$\vec{v}_A = \vec{v}_O + \vec{\omega}_{\overline{AB}} \times \vec{r}_{A/O}$$ (consider points A & O on link $\overline{AB}$)

$$\vec{v}_A = \vec{v}_D + \vec{\omega}_1 \times \vec{r}_{A/D}$$ (Consider points A & D on Body 1)

$$\Longrightarrow \cancel{\vec{v}_O}^{\,\vec{0}} + (-4\,\text{rad/s}\,\hat{k}) \times h(-\hat{\imath}) = \cancel{\vec{v}_D}^{\,\vec{0}} + \omega_1\hat{k} \times r_1\hat{\imath}$$

$$\Longrightarrow \quad 4\,\text{rad/s} \cdot h\,\hat{\jmath} = \omega_1 r_1\,\hat{\jmath} \qquad (*)$$

$$(*)\cdot\hat{\jmath} \Longrightarrow \quad 4\,\text{rad/s} \cdot h = \omega_1 r_1$$

$$\Longrightarrow \quad \omega_1 = 4\,\text{rad/s} \cdot \frac{h}{r_1} \approx 2.67\,\text{rad/s}$$

$$\Longrightarrow \boxed{\vec{\omega}_1 \approx 2.67\,\text{rad/s}\,\hat{k}}$$

$$\vec{v}_C = \vec{v}_A + \vec{\omega}_1 \times \vec{r}_{C/A} \quad \text{(points } C, A \text{ on body 1)}$$

$$\text{with } \vec{v}_A = \cancel{\vec{v}_O}^{\vec{0}} + \vec{\omega}_{\overline{AB}} \times \vec{r}_{A/O}$$

$$\Longrightarrow \vec{v}_C = \vec{\omega}_{\overline{AB}} \times \vec{r}_{A/O} + \vec{\omega}_1 \times \vec{r}_{C/A} \quad (**)$$

On the other hand,

$$\vec{v}_C = \vec{v}_B + \vec{\omega}_2 \times \vec{r}_{C/B} \quad \text{(points } C, B \text{ on body 2)}$$

$$\vec{v}_B = \cancel{\vec{v}_O}^{\vec{0}} + \vec{\omega}_{\overline{AB}} \times \vec{r}_{B/O} \quad \text{(points } B, O \text{ on link } \overline{AB})$$

$$\Longrightarrow \vec{v}_C = \vec{\omega}_{\overline{AB}} \times \vec{r}_{B/O} + \vec{\omega}_2 \times \vec{r}_{C/B} \quad (***)$$

$$(**), (***) \Longrightarrow \vec{\omega}_{\overline{AB}} \times \vec{r}_{A/O} + \vec{\omega}_1 \times \vec{r}_{C/A} = \vec{\omega}_{\overline{AB}} \times \vec{r}_{B/O} + \vec{\omega}_2 \times \vec{r}_{C/B}$$

$$\Longrightarrow (-4 \text{ rad/s } \hat{k}) \times h(-\hat{\imath}) + \omega_1 \hat{k} \times r_1 \hat{\imath} = (-4 \text{ rad/s } \hat{k}) \times (r_1 - h + r_2)\hat{\imath} + \omega_2 \hat{k} \times r_2(-\hat{\imath})$$

$$\Longrightarrow 4 \text{ rad/s} \cdot h \hat{\jmath} + \omega_1 r_1 \hat{\jmath} = -4 \text{ rad/s } (r_1 - h + r_2)\hat{\jmath} + \omega_2 r_2 (-\hat{\jmath}) \quad (4*)$$

$$(4*) \cdot \hat{\jmath} \Longrightarrow \cancel{4 \text{ rad/s} \cdot h} + \omega_1 r_1 = -4 \text{ rad/s } r_1 + \cancel{4 \text{ rad/s } h} - 4 \text{ rad/s } r_2 - \omega_2 r_2$$

$$\Longrightarrow \omega_2 = \frac{-4 \text{ rad/s } (r_1 + r_2) - \omega_1 r_1}{r_2}$$

$$\omega_2 = \frac{-4 \text{ rad/s } (0.15 \text{ m} + 0.1 \text{ m}) - 2.67 \text{ rad/s} \cdot 0.15 \text{ m}}{0.1 \text{ m}} \approx -14 \text{ rad/s} \qquad \boxed{\vec{\omega}_2 \approx -14 \text{ rad/s } \hat{k}}$$

$\boxminus$