ENGRD/TAM 203

HW18 (Assigned on Mar. 30, due on April 6)

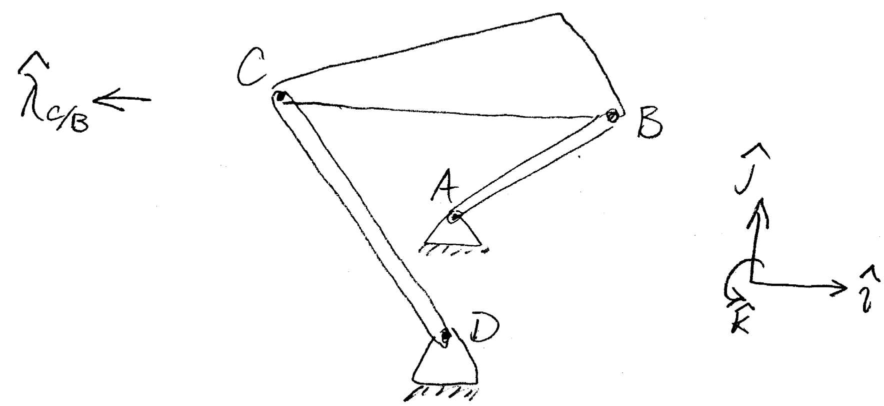
Solution by Dennis Yang

6.1.23

$\vec{r}_{C/B} = -0.4m\hat{i}$, $\vec{r}_{A/B} = -0.2m\hat{i} - 0.1m\hat{j}$

$\vec{r}_{D/C} = 0.2m\hat{i} - 0.2m\hat{j}$. $\vec{\omega}_{\overline{AB}} = -2\hat{k}$ rad/s

Find $\vec{v}_C$ and $\vec{\omega}_{\overline{CD}}$.



Solution

Let $\vec{\omega}_{\overline{CD}} = \omega_{\overline{CD}}\hat{k}$

take $\hat{\lambda}_{C/B} = \dfrac{\vec{r}_{C/B}}{\|\vec{r}_{C/B}\|} = \dfrac{-0.4m\hat{i}}{0.4m} = -\hat{i}$

Consider link $\overline{CD}$: $\vec{v}_C = \vec{v}_D^{\,\nearrow \vec{0}} + \vec{\omega}_{\overline{CD}} \times \vec{r}_{C/D} = \vec{\omega}_{\overline{CD}} \times (-\vec{r}_{D/C})$

$= \omega_{\overline{CD}}\hat{k} \times (-0.2m\hat{i} + 0.2m\hat{j})$

$\Longrightarrow \vec{v}_C = -0.2m \cdot \omega_{\overline{CD}}\hat{j} + 0.2m \cdot \omega_{\overline{CD}}(-\hat{i})$

(a)

consider link $\overline{AB}$: $\vec{v}_B = \vec{v}_A^{\,\vec{0}} + \vec{\omega}_{\overline{AB}} \times \vec{r}_{B/A}$

$$= \vec{\omega}_{\overline{AB}} \times (-\vec{r}_{A/B})$$

$$= -2\,\text{rad/s}\,\hat{k} \times (0.2\text{m}\,\hat{i} + 0.1\text{m}\,\hat{j})$$

$$\vec{v}_B = -2\,\text{rad/s} \cdot 0.2\text{m}\,\hat{j} + 2\,\text{rad/s} \cdot 0.1\text{m}\,\hat{i}$$

(b)

consider link $\overline{CB}$:

Both C and B are on the rigid link $\overline{CB}$.

thus $(\vec{v}_C - \vec{v}_B) \cdot \hat{\lambda}_{C/B} = 0$

(i.e., point C can NOT move (w.r.t. point B) along the direction of $\hat{\lambda}_{C/B}$, so the body has NO elongation or contraction along $\hat{\lambda}_{C/B}$)

by (a), (b) $\Rightarrow (\vec{v}_C - \vec{v}_B) \cdot \hat{\lambda}_{C/B} = (\vec{v}_C - \vec{v}_B) \cdot (-\hat{i}) = 0$

$$\Longrightarrow 0.2\text{m} \cdot \omega_{\overline{CD}} + 2\,\text{rad/s} \cdot 0.1\text{m} = 0$$

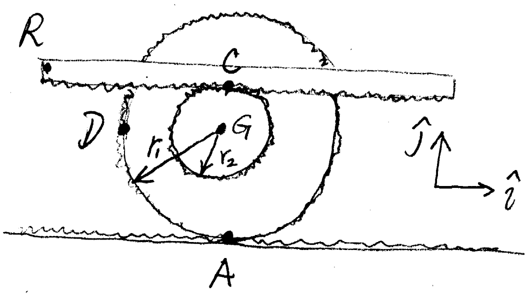
$$\Longrightarrow \omega_{\overline{CD}} = -1\,\text{rad/s} \qquad \boxed{\vec{\omega}_{\overline{CD}} = -1\,\text{rad/s}\,\hat{k}}$$

by (a) $\Rightarrow$ $\boxed{\vec{v}_C = 0.2\,\text{m/s}\,\hat{i} + 0.2\,\text{m/s}\,\hat{j}}$

□

6.2.2 $\vec{v}_G = 1.2\ m/s\ \hat{\imath}$ $r_1 = 0.15\ m$, $r_2 = 0.1\ m$

Find $\vec{\omega}$ and $\vec{v}_R, \vec{v}_D$ (using the instantaneous center of rotation)



Solution — Point A is the instantaneous center of rotation

Let $\vec{\omega} = \omega \hat{k}$.

$$\vec{v}_G = \vec{\omega} \times \vec{r}_{G/A} = \omega \hat{k} \times r_1 \hat{\jmath} = -\omega r_1 \hat{\imath} \quad (1)$$

$$\vec{v}_R = \vec{v}_C = \vec{\omega} \times \vec{r}_{C/A} = \omega \hat{k} \times (r_1 + r_2)\hat{\jmath} = -\omega(r_1 + r_2)\hat{\imath} \quad (2)$$

$$\vec{v}_D = \vec{\omega} \times \vec{r}_{D/A} = \omega \hat{k} \times (-r_1 \hat{\imath} + r_1 \hat{\jmath}) = -\omega r_1 \hat{\jmath} - \omega r_1 \hat{\imath} \quad (3)$$

$\vec{v}_G = 1.2\ m/s\ \hat{\imath}$ & (1) $\Rightarrow -\omega r_1 = 1.2\ m/s$

$$\Rightarrow \omega = \frac{1.2\ m/s}{-0.15\ m} = -8\ rad/s$$

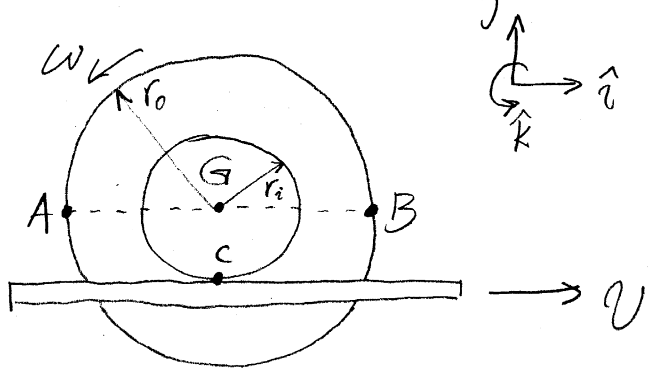
$$\boxed{\vec{\omega} = -8\ rad/s\ \hat{k}}$$

$$\vec{v}_R = -(-8\ rad/s)(0.15\ m + 0.1\ m)\hat{\imath} \Rightarrow \boxed{v_R = 2\ m/s\ \hat{\imath}}$$

$$\vec{v}_D = -(-8\ rad/s)\ 0.15\ m/s\ \hat{\jmath} - (-8\ rad/s)\ 0.15\ m/s\ \hat{\imath} \Rightarrow \boxed{\vec{v}_D = 1.2\ m/s\ \hat{\imath} + 1.2\ m/s\ \hat{\jmath}}$$

□

6.2.9



find $\vec{v}_A$, $\vec{v}_B$.

Solution

$$\vec{v}_A = \vec{v}_c + \vec{\omega} \times \vec{r}_{A/c}$$

$$= v\hat{i} + \omega\hat{k} \times (-r_o\hat{i} + r_i\hat{j})$$

$$= v\hat{i} + (-\omega r_o\hat{j} - \omega r_i\hat{i})$$

$$\boxed{\vec{v}_A = (v - \omega r_i)\hat{i} - \omega r_o\hat{j}}$$

$$\vec{v}_B = \vec{v}_c + \vec{\omega} \times \vec{r}_{B/c}$$

$$= v\hat{i} + \omega\hat{k} \times (r_o\hat{i} + r_i\hat{j})$$

$$= v\hat{i} + \omega r_o\hat{j} - \omega r_i\hat{i}$$

$$\boxed{\vec{v}_B = (v - \omega r_i)\hat{i} + \omega r_o\hat{j}}$$

□