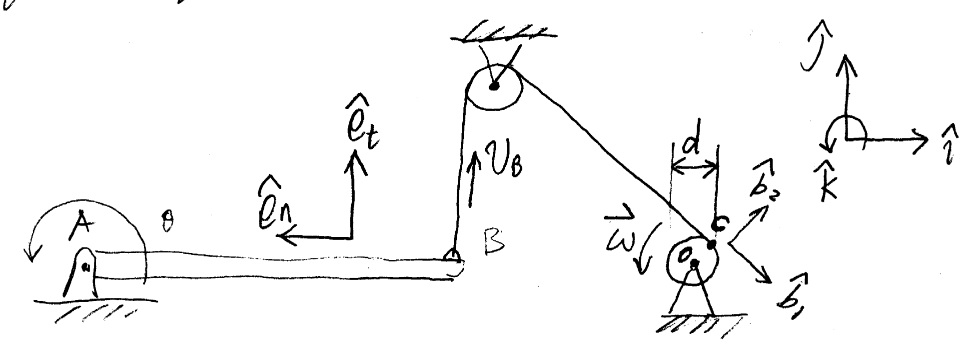
ENGRD/TAM 203
HW19 (Assigned on April 4. Due on 4/11)
Solution by Dennis Yang

6.3.9 The length of bar $\overline{AB}$ is 14 ft. $d = 8$ in. The angular velocity of C is -10 rad/s $\hat{k}$ and the angular acceleration is -0.5 rad/s² $\hat{k}$. Determine the acceleration of B for the illustrated instant.



Solution At the illustrated instant, $\hat{e}_t = \hat{\jmath}$, $\hat{e}_n = -\hat{\imath}$ and $\theta = 0$ rad, $r = \|\vec{r}_{B/A}\| = 14$ ft.

$$\vec{a}_B = \dot{v}_B \hat{e}_t + \frac{v_B^2}{\|\vec{r}_{B/A}\|}\hat{e}_n$$

$$= \dot{v}_B \hat{\jmath} - \frac{v_B^2}{\|\vec{r}_{B/A}\|}\hat{\imath}$$

$$v_B = \vec{v}_C \cdot \hat{b}_1 = (\vec{\omega} \times \vec{r}_{C/O}) \cdot \hat{b}_1 = \left(\left(-10 \text{ rad/s } \hat{k}\right) \times \frac{d}{2}\hat{b}_2\right) \cdot \hat{b}_1$$

$$= -10 \text{ rad/s} \cdot \frac{d}{2}(-\hat{b}_1) \cdot \hat{b}_1$$

$$= 10 \text{ rad/s} \cdot \frac{d}{2} = 10 \text{ rad/s} \cdot 4 \text{ in} = \underline{\frac{10}{3} \text{ ft/s}}$$

$$\dot{v}_B = (\vec{\alpha} \times \vec{r}_{C/O}) \cdot \hat{b}_1 = (-0.5\ \text{rad/s}^2\ \hat{k} \times \tfrac{d}{2}\ \hat{b}_2) \cdot \hat{b}_1$$

$$= -0.5\ \text{rad/s}^2 \cdot \tfrac{d}{2}\ (-\hat{b}_1) \cdot \hat{b}_1$$

$$= 0.5\ \text{rad/s}^2 \cdot \tfrac{d}{2} = 0.5\ \text{rad/s}^2\ 4\ \text{in} = \underline{\tfrac{1}{6}\ \text{ft/s}^2}$$

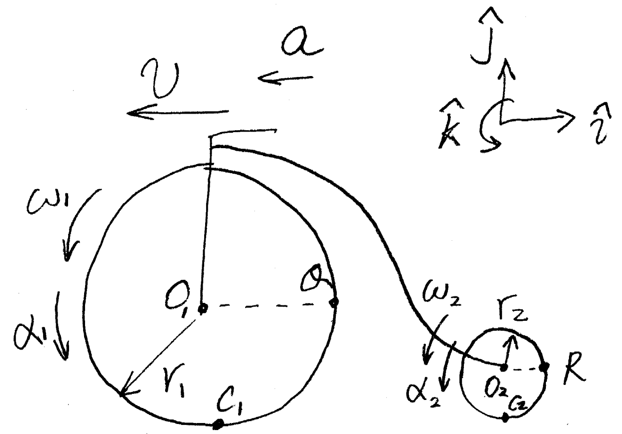
$$\vec{a}_B = \dot{v}_B\ \hat{\jmath} - \frac{v_B^2}{\|\vec{r}_{B/A}\|}\ \hat{\imath}$$

$$= \tfrac{1}{6}\ \text{ft/s}^2\ \hat{\jmath} - \frac{(10/3\ \text{ft/s})^2}{14\ \text{ft}}\ \hat{\imath}$$

$$\boxed{\vec{a}_B \approx -0.794\ \text{ft/s}^2\ \hat{\imath} + 0.167\ \text{ft/s}^2\ \hat{\jmath}}$$

$\square$

6.3.12



$\vec{v} = 10\,\text{mph}\,(-\hat{\imath})$

$\vec{a} = 4\,\text{ft/s}^2\,(-\hat{\imath})$

$r_1 = 25\,\text{in}$

$r_2 = 9\,\text{in}$

Find $\dfrac{\|\vec{a}_Q\|}{\|\vec{a}_R\|}$

Solution

for $i = 1, 2$,

$\vec{\omega}_1 = \omega_1\hat{k}$, $\vec{\alpha}_1 = \alpha_1\hat{k}$

$\vec{\omega}_2 = \omega_2\hat{k}$, $\vec{\alpha}_2 = \alpha_2\hat{k}$

$\vec{v}_{C_1} = \vec{v}_{C_2} = \vec{0}$

$v = 10\,\text{mph} = 14.67\,\text{ft/s}$

$$\begin{aligned}\vec{v}_{O_i} &= \vec{v}_{C_i} + \vec{\omega}_i \times \vec{r}_{O_i/C_i}\\ &= \omega_i\hat{k} \times r_i\hat{\jmath}\\ &= \omega_i r_i(-\hat{\imath}) = v(-\hat{\imath}) \quad (*)\end{aligned}$$

(a)

$$(*)\cdot\hat{\imath} \implies -\omega_i r_i = -v \implies \boxed{\omega_i = \frac{v}{r_i} \quad i = 1, 2}$$

$$\vec{a}_{O_i} = \frac{d\vec{v}_{O_i}}{dt} = \frac{d}{dt}\left(\omega_i r_i(-\hat{\imath})\right) = r_i\frac{d\omega_i}{dt}(-\hat{\imath}) = r_i\alpha_i(-\hat{\imath})$$

also, $\vec{a}_{O_i} = a(-\hat{\imath}) \implies a(-\hat{\imath}) = r_i\alpha_i(-\hat{\imath})$ $(**)$

$$(**)\cdot\hat{\imath} \implies -a = -r_i\alpha_i$$

(b)

$$\implies \boxed{\alpha_i = \frac{a}{r_i}} \quad i = 1, 2.$$

$$\vec{a}_Q = \vec{a}_{O_1} + \vec{\alpha}_1 \times \vec{r}_{Q/O_1} + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}_{Q/O_1})$$

$$= a(-\hat{\imath}) + \alpha_1 \hat{k} \times r_1 \hat{\imath} + \omega_1 \hat{k} \times (\omega_1 \hat{k} \times r_1 \hat{\imath})$$

$$= -a\hat{\imath} + \alpha_1 r_1 \hat{\jmath} + \omega_1^2 r_1 (-\hat{\imath})$$

$$= \underline{-(a + \omega_1^2 r_1)\hat{\imath} + \alpha_1 r_1 \hat{\jmath}}$$

$$\vec{a}_R = \vec{a}_{O_2} + \vec{\alpha}_2 \times \vec{r}_{R/O_2} + \vec{\omega}_2 \times (\vec{\omega}_2 \times \vec{r}_{R/O_2})$$

$$= a(-\hat{\imath}) + \alpha_2 \hat{k} \times r_2 \hat{\imath} + \omega_2 \hat{k} \times (\omega_2 \hat{k} \times r_2 \hat{\imath})$$

$$= -a\hat{\imath} + \alpha_2 r_2 \hat{\jmath} + \omega_2^2 r_2 (-\hat{\imath})$$

$$= \underline{-(a + \omega_2^2 r_2)\hat{\imath} + \alpha_2 r_2 \hat{\jmath}}$$

$$\frac{\|\vec{a}_Q\|}{\|\vec{a}_R\|} = \frac{\sqrt{(a+\omega_1^2 r_1)^2 + (\alpha_1 r_1)^2}}{\sqrt{(a+\omega_2^2 r_2)^2 + (\alpha_2 r_2)^2}} = \frac{\sqrt{\left(a + \left(\frac{v}{r_1}\right)^2 r_1\right)^2 + \left(\frac{a}{r_1} \cdot r_1\right)^2}}{\sqrt{\left(a + \left(\frac{v}{r_2}\right)^2 r_2\right)^2 + \left(\frac{a}{r_2} r_2\right)^2}}$$

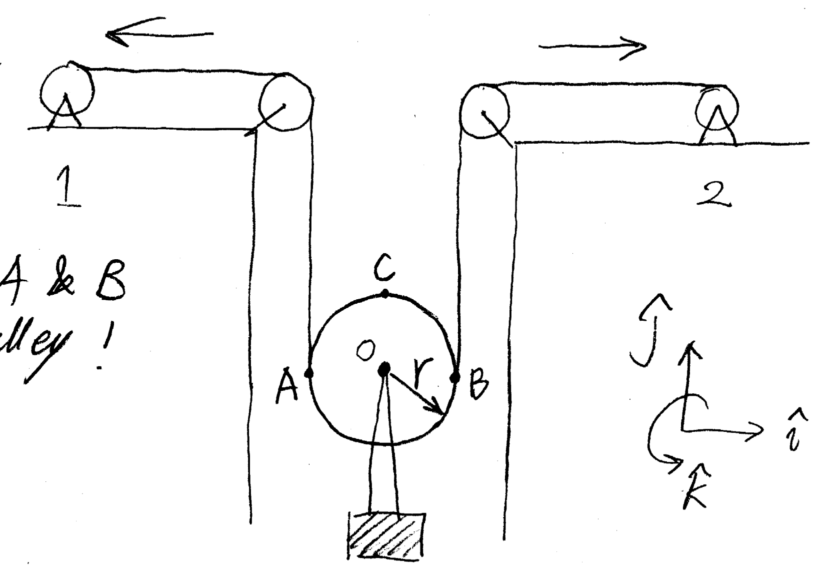
(by (a),(b))

$$= \frac{\sqrt{\left(a + \frac{v^2}{r_1}\right)^2 + a^2}}{\sqrt{\left(a + \frac{v^2}{r_2}\right)^2 + a^2}}$$

$$= \frac{\sqrt{\left(4\,\mathrm{ft/s^2} + \frac{(14.67\,\mathrm{ft/s})^2}{25\,\mathrm{in}}\right)^2 + (4\,\mathrm{ft/s})^2}}{\sqrt{\left(4\,\mathrm{ft/s^2} + \frac{(14.67\,\mathrm{ft/s})^2}{9\,\mathrm{in}}\right)^2 + (4\,\mathrm{ft/s})^2}} \approx \underline{0.369}$$

$\square$

6.3.25 Rope is being drawn into two motorized reels. At the illustrated moment Reel 1 and 2 are taking rope in at rates v_1 ft/s and v_2 ft/s respectively. The rate is constant for Reel 1 but is accelerating (at a_2 ft/s²) for Reel 2. What is the acceleration of point C located at the top of the pulley?



Note: Points A & B are on the pulley!

Solution

Assume the velocity of O : $v_o\hat{j}$

the acceleration of O: $a_o\hat{j}$

the angular velocity of the pulley: $\omega\hat{k}$

the angular acceleration of the pulley: $\alpha\hat{k}$

We know, by the rope being inextensable,

$$\vec{v}_A\cdot\hat{j} = v_1 \quad (1), \quad \vec{v}_B\cdot\hat{j} = v_2 \quad (2), \quad \vec{a}_A\cdot\hat{j} = 0 \quad (3), \quad \vec{a}_B\cdot\hat{j} = a_2 \quad (4)$$

Now consider the pulley

$$\vec{v}_A = \vec{v}_0 + \vec{\omega} \times \vec{r}_{A/0}$$

$$= v_0 \hat{j} + \omega \hat{k} \times (r(-\hat{i})) = v_0 \hat{j} + \omega r(-\hat{j})$$

$$= (v_0 - \omega r)\hat{j}$$

Thus, by ①, $(v_0 - \omega r)\hat{j} \cdot \hat{j} = v_0 - \omega r = v_1$

$$\Longrightarrow \boxed{v_1 = v_0 - \omega r} \quad (a)$$

$$\vec{v}_B = \vec{v}_0 + \vec{\omega} \times \vec{r}_{B/0}$$

$$= v_0 \hat{j} + \omega \hat{k} \times r \hat{i} = v_0 \hat{j} + \omega r \hat{j}$$

$$= (v_0 + \omega r)\hat{j}$$

Thus, by ②, $(v_0 + \omega r)\hat{j} \cdot \hat{j} = v_0 + \omega r = v_2$

$$\Longrightarrow \boxed{v_2 = v_0 + \omega r} \quad (b)$$

$$(a) + (b) \Longrightarrow \boxed{v_0 = \frac{v_1 + v_2}{2}}$$

$$(b) - (a) \Longrightarrow \boxed{\omega = \frac{v_2 - v_1}{2r}}$$

Next, we are going to consider the accelerations of points A & B, as they are points on the pulley.

$$\vec{a}_A = \vec{a}_0 + \vec{\alpha}\times\vec{r}_{A/0} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{A/0})$$
$$= a_0\hat{j} + \alpha\hat{k}\times r(-\hat{i}) + \frac{v_2-v_1}{2r}\hat{k}\times\left(\frac{v_2-v_1}{2r}\hat{k}\times r(-\hat{i})\right)$$
$$= a_0\hat{j} + \alpha r(-\hat{j}) + \frac{(v_2-v_1)^2}{4r}\hat{i}$$
$$= (a_0-\alpha r)\hat{j} + \frac{(v_2-v_1)^2}{4r}\hat{i} \quad (*)$$

Thus, by ③, $(*)\cdot\hat{j} = a_0 - \alpha r = 0$

$$\Longrightarrow \boxed{a_0 = \alpha r} \quad (c)$$

$$\vec{a}_B = \vec{a}_0 + \vec{\alpha}\times\vec{r}_{B/0} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{B/0})$$
$$= a_0\hat{j} + \alpha\hat{k}\times r\hat{i} + \frac{v_2-v_1}{2r}\hat{k}\times\left(\frac{v_2-v_1}{2r}\hat{k}\times r\hat{i}\right)$$
$$= a_0\hat{j} + \alpha r\hat{j} + \frac{(v_2-v_1)^2}{4r}(-\hat{i})$$
$$= (a_0+\alpha r)\hat{j} + \frac{(v_2-v_1)^2}{4r}(-\hat{i}) \quad (**)$$

Thus, by ④, $(**)\cdot\hat{j} = a_0 + \alpha r = a_2$

$$\Longrightarrow \boxed{a_0 + \alpha r = a_2} \quad (d)$$

$$(c),(d) \Longrightarrow \boxed{a_0 = \frac{a_2}{2}, \quad \alpha = \frac{a_2}{2r}}$$

Finally, we will consider point C by its motion relative to point O:

$$\vec{a}_c = \vec{a}_o + \vec{\alpha}\times\vec{r}_{c/o} + \vec{\omega}\times(\vec{\omega}\times\vec{r}_{c/o})$$

$$= a_o\hat{j} + \alpha\hat{k}\times r\hat{j} + \omega\hat{k}\times(\omega\hat{k}\times r\hat{j})$$

$$= \frac{a_2}{2}\hat{j} + \frac{a_2}{2r}\hat{k}\times r\hat{j} + \frac{v_2-v_1}{2r}\hat{k}\times\left(\frac{v_2-v_1}{2r}\hat{k}\times r\hat{j}\right)$$

$$= \frac{a_2}{2}\hat{j} + \frac{a_2}{2}(-\hat{i}) + \frac{(v_2-v_1)^2}{4r}(-\hat{j})$$

$$\boxed{\vec{a}_c = -\frac{a_2}{2}\hat{i} + \left(\frac{a_2}{2} - \frac{(v_2-v_1)^2}{4r}\right)\hat{j}}$$

$\square$