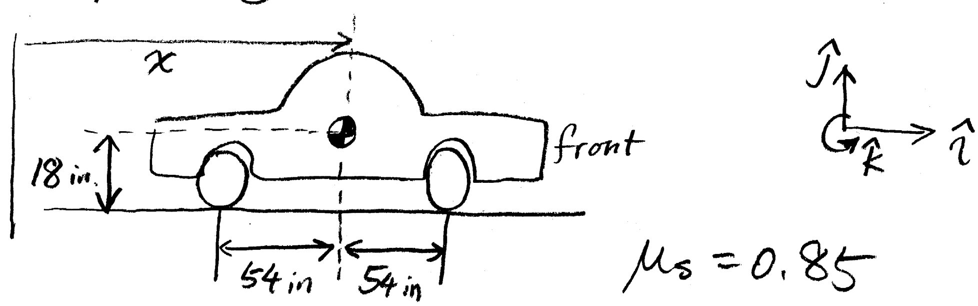
ENGRD/TAM 203

HW 21 (Assigned on April 11, due on April 18)

Solution by Dennis Yang

7.1.1. Assume back wheel drive, and the car engine can deliver as much power/torque as demanded, find the maximum attainable acceleration $\ddot{x}$ and the corresponding normal forces on the tires.

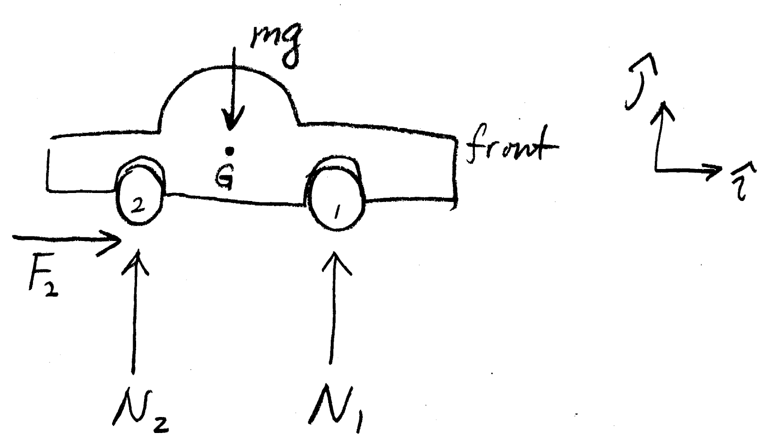


$\mu_s = 0.85$

Solution

(Approach A)

F.B.D.



$$\sum_i \vec{F}_i = m\vec{a} \Longrightarrow mg(-\hat{j}) + N_1\hat{j} + N_2\hat{j} + F_2\hat{i} = m\ddot{x}\hat{i} \qquad (*)$$

$$\sum_i \vec{M}_{i/G} = \dot{\vec{H}}_{/G} = \vec{0} \Rightarrow N_1\cdot(54\text{in})\hat{k} + N_2\cdot(54\text{in})(-\hat{k}) + F_2(18\text{ in})\hat{k} = \vec{0} \qquad (**)$$

$$(**)\cdot\hat{k} \Longrightarrow N_1\cdot 54\text{in} - N_2\cdot 54\text{in} + F_2\cdot 18\text{in} = 0 \quad ①$$

$$(*)\cdot\hat{j} \Longrightarrow -mg + N_1 + N_2 = 0 \quad ②$$

$$① \Longrightarrow 3N_1 - 3N_2 + F_2 = 0 \quad ③$$

$$②, ③ \Longrightarrow \boxed{N_1 = \frac{3mg - F_2}{6} \; ④, \quad N_2 = \frac{3mg + F_2}{6} \; ⑤}$$

Constraints : $\boxed{\underset{(a)}{N_1 \geq 0} \;\&\; \underset{(b)}{F_2 \leq N_2\mu_s}}$

$$④, (a) \Longrightarrow \frac{3mg - F_2}{6} \geq 0 \Longrightarrow \boxed{F_2 \leq 3mg} \quad (c)$$

$$⑤, (b) \Longrightarrow F_2 \leq \frac{3mg + F_2}{6}\cdot\mu_s$$

$$\Longrightarrow 6F_2 - \mu_s F_2 \leq 3\mu_s mg \quad (\mu_s = 0.85)$$

$$\Longrightarrow F_2 \leq \frac{3mg\mu_s}{6 - \mu_s} \quad (\mu_s = 0.85)$$

$$\Longrightarrow \boxed{F_2 \leq 0.495\, mg} \quad (d)$$

$$\text{(c) and (d)} \Longrightarrow \boxed{\boxed{F_2 \leq 0.495\,mg}}$$

$$\text{Thus, } \boxed{F_{2\,max} = 0.495\,mg}$$

$$\text{and } \boxed{\ddot{x}_{max} = \frac{F_{2\,max}}{m} = 0.495\,g} \approx \underline{16.00\,\text{ft}/\text{s}^2}$$

$$N_1 = \frac{3mg - F_{2max}}{6} = \frac{3mg - 0.495\,mg}{6}$$

$$= 0.4175\,mg = 0.4175 \cdot 4100\ \text{lbf}$$

$$\underline{= 1711.75\ \text{lbf}}$$

$$N_2 = \frac{3mg + F_{2\,max}}{6} = \frac{3mg + 0.495\,mg}{6}$$

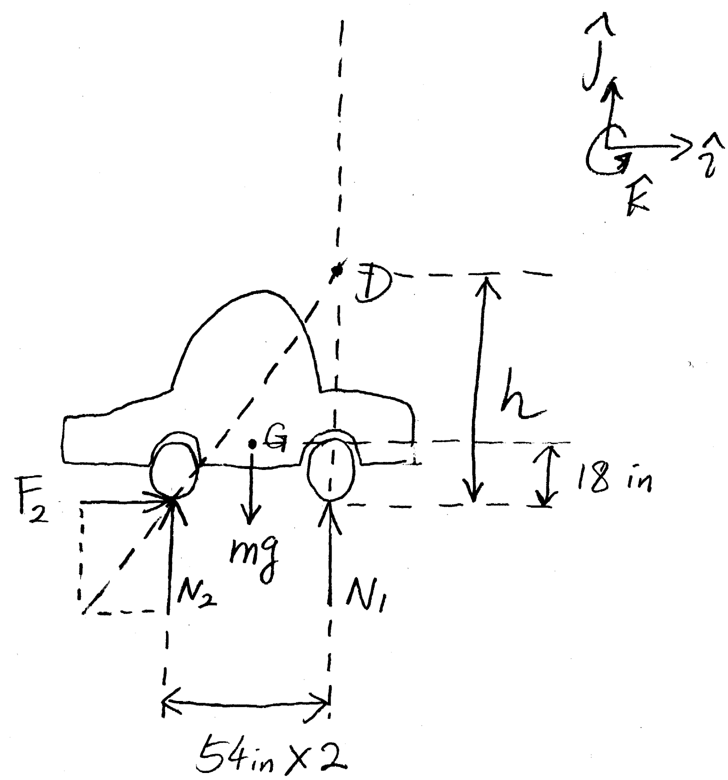
$$= 0.5825\,mg = 0.5825 \cdot 4100\ \text{lbf}$$

$$\underline{= 2388.25\ \text{lbf}}$$

$\square$

Solution
(Approach B)

F.B.D



$$\frac{h}{54\text{in} \times 2} = \frac{N_2}{F_2} \Longrightarrow h = \frac{N_2}{F_2} \cdot 108\text{ in}$$

$$\boxed{\sum \vec{M}_{/D} = \dot{\vec{H}}_{/D} = \vec{r}_{G/D} \times m\vec{a} + I_G \cancel{\vec{\alpha}}^{\,\vec{0}}} \quad (*)$$

$$\Longrightarrow N_2 \cdot 108\text{in}\,(-\hat{k}) + F_2 h \hat{k} + mg \cdot 54\text{in}\,\hat{k} = m\ddot{x}(h - 18\text{in})\hat{k}$$

$$(*)\cdot\hat{k} \Longrightarrow -N_2 \cdot 108\text{in} + F_2 \cdot \left(\frac{N_2}{F_2} 108\text{in}\right) + mg \cdot 54\text{in} = m\ddot{x}\left(\frac{N_2}{F_2} 108\text{in} - 18\text{in}\right)$$

$$\Longrightarrow 3mg = m\ddot{x}\left(\frac{N_2}{F_2} \cdot 6 - 1\right)$$

$$\Longrightarrow \boxed{\ddot{x} = \frac{3g}{6\frac{N_2}{F_2} - 1}}$$

Since $F_2 \leq N_2 \mu_s \Longrightarrow \frac{N_2}{F_2} \geq \frac{1}{\mu_s}$

$$\Longrightarrow 6\frac{N_2}{F_2} - 1 \geq \frac{6}{\mu_s} - 1$$

$$\Longrightarrow \ddot{x} \leq \frac{3g}{\frac{6}{\mu_s} - 1}$$ (equality holds when $N_2 \mu_s = F_2$)

we have $\ddot{x}_{max} = \frac{3g}{\frac{6}{\mu_s} - 1} = \frac{3g\mu_s}{6 - \mu_s}$

$$= 0.495\, g$$

Then, $F_2 = m\ddot{x}_{max}$

$N_2 = F_2/\mu_s$

$N_1 = mg - N_2$

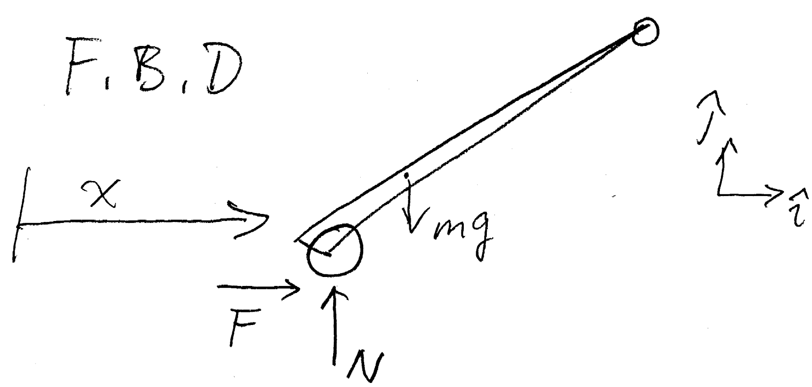
$\square$

7.1.6 In case A the rear-wheel-drive dragster travels 200 ft, all the while doing a wheelie. In case B the dragster keeps all tires on the road and supports 85% of its weight on the rear tires. In Both cases, the tires roll without slip. $\mu_s = 0.82$. How long does it take to finish in both cases. Assume they're operating at the limit of slip/non-slip conditions.

Solution

F.B.D

Case A



at the limit of slip/non-slip condition $\Rightarrow F = N\mu_s$

$$\sum_i \vec{F}_i = m\vec{a} \Rightarrow N\hat{j} + mg(-\hat{j}) + N\mu_s\hat{\imath} = m\ddot{x}\hat{\imath} \quad (*)$$

$$(*)\cdot\hat{j} \Rightarrow N - mg = 0 \Rightarrow N = mg$$

$$(*)\cdot\hat{\imath} \Rightarrow N\mu_s = m\ddot{x}$$

$$\Rightarrow \ddot{x} = \frac{N\mu_s}{m} = \frac{mg\mu_s}{m}$$

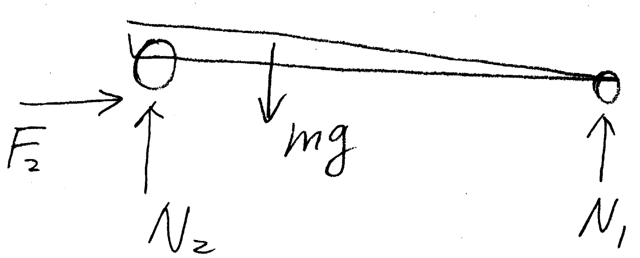
$$\Rightarrow \boxed{\ddot{x} = \mu_s g}$$

$$\frac{1}{2}\ddot{x}\,\Delta t_A^2 = \Delta x \quad (\text{since } \ddot{x} = \mu_s g \text{ is constant})$$

$$\Rightarrow \Delta t_A = \left(\frac{2\Delta x}{\ddot{x}}\right)^{1/2} = \left(\frac{2 \times 200\,ft}{0.82 \times 32.2\,ft/s^2}\right)^{1/2} \approx \underline{\underline{3.89\,s}}$$

Case B

F.B.D



at the limit of slip/non-slip condition $\Rightarrow F_2 = N_2 \mu_s$

$$\sum_i \vec{F}_i = m\vec{a} \Rightarrow F_2\hat{i} + N_2\hat{j} + N_1\hat{j} + mg(-\hat{j}) = m\ddot{x}\,\hat{i} \quad (**)$$

$$(**)\cdot\hat{i} \Rightarrow F_2 = m\ddot{x}$$

$$\Rightarrow \ddot{x} = \frac{F_2}{m} = \frac{N_2\mu_s}{m}, \quad \text{where } N_2 = mg \cdot 0.85$$

$$\Rightarrow \ddot{x} = 0.85\,\mu_s g.$$

$$\Rightarrow \Delta t_B = \left(\frac{2\Delta x}{\ddot{x}}\right)^{1/2} = \left(\frac{2 \times 200\,ft}{0.85 \times 0.82 \times 32.2\,ft/s^2}\right)^{1/2} \approx \underline{\underline{4.22\,s}}$$

$\square$