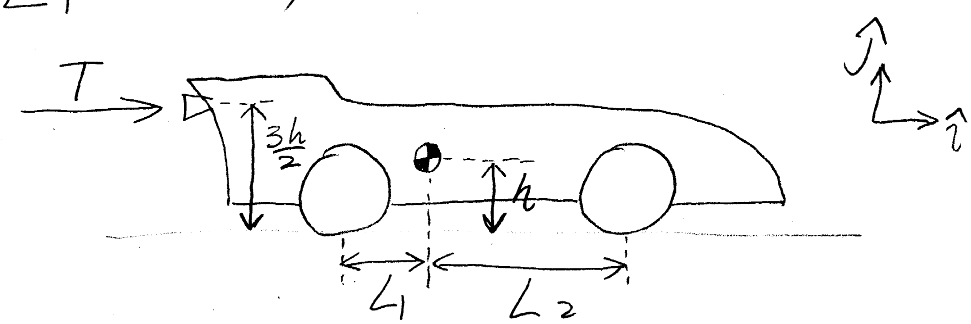
ENGRD/TAM 203
HW22 (Assigned on April 13, due on April 20)
Solution by Dennis Yang

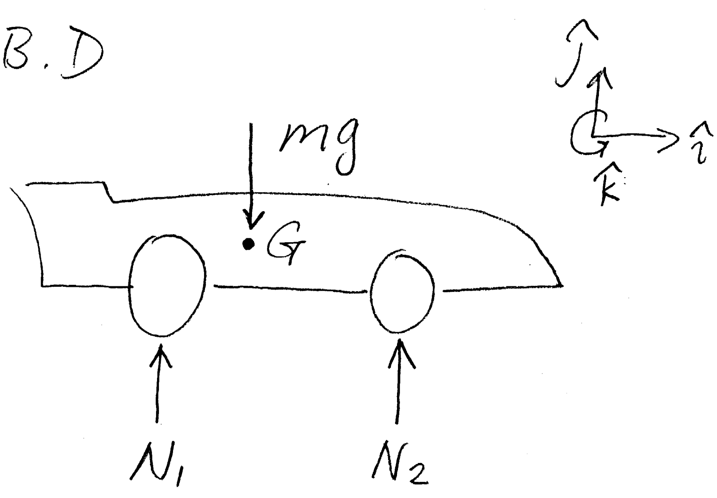
7.1.7 The Batmobile is propelled by a jet engine. Assume an acceleration of 0.9 g. How do the normal forces change from their static values? Neglect ground/tire forces in the $\hat{\imath}$ direction. $L_1 = 1\,m$, $L_2 = 1.8\,m$, and $h = 0.65\,m$.



Solution

When the car is at rest:

F.B.D



$$\sum_i \vec{F}_i = \vec{0} \implies N_1\hat{\jmath} + N_2\hat{\jmath} + mg(-\hat{\jmath}) = \vec{0} \quad (*)$$

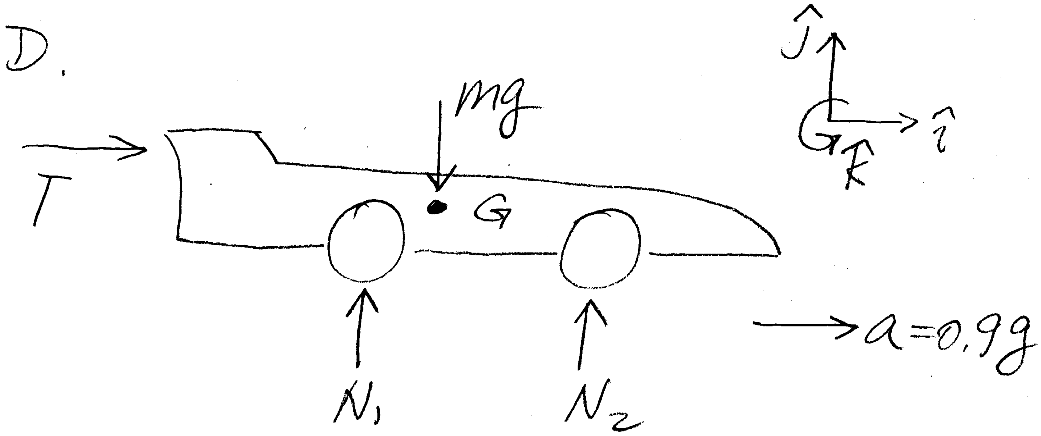
$$\sum_i \vec{M}_{i/G} = \vec{0} \implies N_2 L_2 \hat{k} + N_1 L_1(-\hat{k}) = \vec{0} \quad (**)$$

$$(*)\cdot\hat{j} \Longrightarrow N_1+N_2-mg=0$$
$$(**)\cdot\hat{k} \Longrightarrow N_2L_2-N_1L_1=0$$
$$\Longrightarrow \boxed{N_1=\frac{mgL_2}{L_1+L_2} \qquad N_2=\frac{mgL_1}{L_1+L_2}}$$

(a)

When the car is accelerating :

F. B. D.



$$\sum_i \vec{F}_i = m\vec{a} \Longrightarrow T\hat{i}+N_1\hat{j}+N_2\hat{j}+mg(-\hat{j})=ma\hat{i}$$

(3*)

$$\sum_i \vec{M}_{i/G}=\vec{0} \Longrightarrow N_2L_2\hat{k}+N_1L_1(-\hat{k})+T\cdot\frac{h}{2}(-\hat{k})=\vec{0}$$

(4*)

$$(3*)\cdot\hat{j} \Rightarrow N_1+N_2-mg=0$$
$$(4*)\cdot\hat{k} \Rightarrow N_2L_2-N_1L_1-T\frac{h}{2}=0$$
$$\Longrightarrow \boxed{N_1=\frac{mgL_2-\frac{1}{2}Th}{L_1+L_2} \qquad N_2=\frac{mgL_1+\frac{1}{2}Th}{L_2+L_1}}$$

(b)

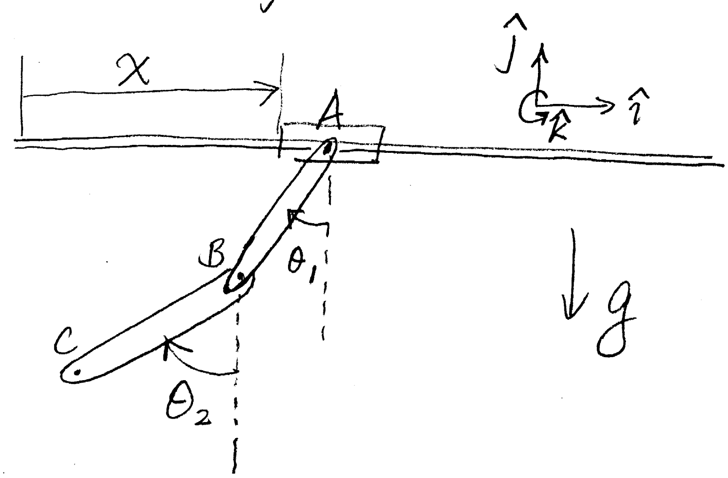
Where, by $(3*)\cdot\hat{i} \Rightarrow T=ma$, we have that

$$\underline{T=ma=m(0.9g)=0.9mg}$$

Compareing (a) and (b), we conclude that N_1 dropps by $\left(\frac{\frac{1}{2}Th}{L_1+L_2}\right)$ while N_2 increases by $\left(\frac{\frac{1}{2}Th}{L_1+L_2}\right)$, where $T = 0.9\,mg$.

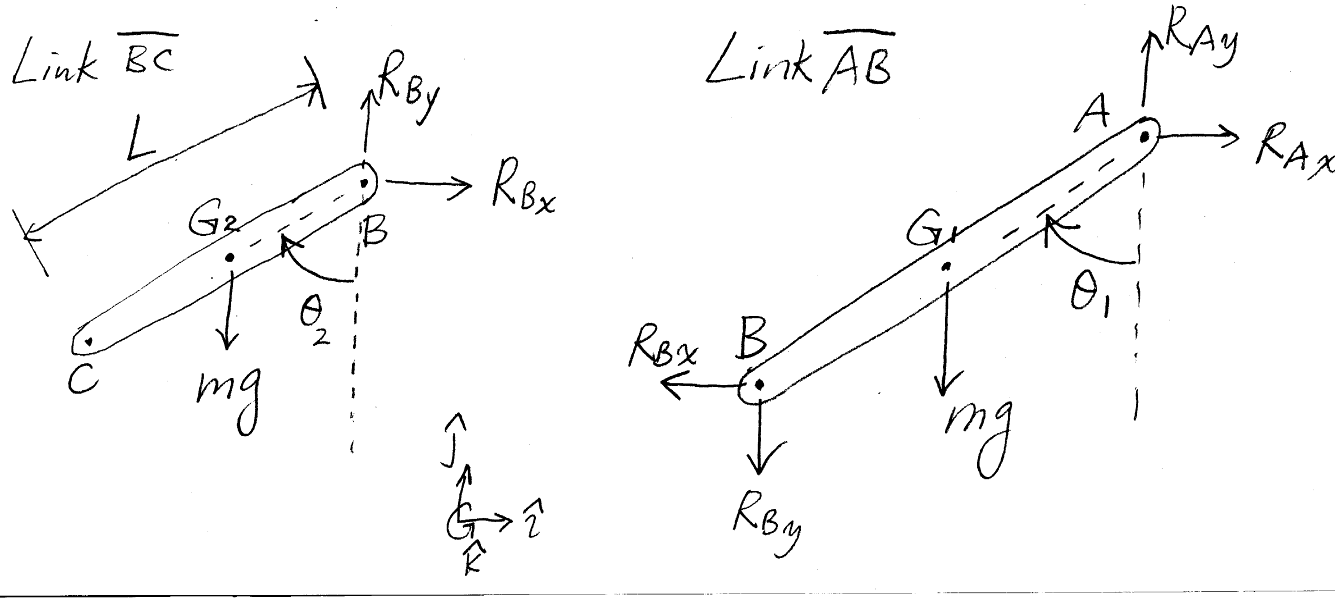
$\square$

7.1.10 Two pinned links are attached to a horizontally translating block. $\overline{AB} = \overline{BC} = 1.2\ m$. Each link has a mass of 5 kg. There exists a stable configuration for which both links simply translate to the right, each at a constant inclination angle, when $\ddot{x}$ is a constant. Find the angles when $\ddot{x} = g/2$.



Solution

F.B.D



For link $\overline{BC}$, its center of mass accelerates at $\vec{a}_{G2} = \ddot{x}\,\hat{\imath}$. Thus, about point B:

$$\sum_i \vec{M}_{i/B} = \dot{\vec{H}}_{/B} = \vec{r}_{G_2/B} \times m\vec{a}_{G_2} + \cancelto{\vec{0}}{I_{G_2}\vec{\alpha}}$$

$$\Longrightarrow \vec{r}_{G_2/B} \times mg(-\hat{\jmath}) = \vec{r}_{G_2/B} \times m\ddot{x}\,\hat{\imath}$$

$$\Longrightarrow \left[\frac{L}{2}\sin\theta_2(-\hat{\imath}) + \frac{L}{2}\cos\theta_2(-\hat{\jmath})\right] \times mg(-\hat{\jmath})$$

$$= \left[\frac{L}{2}\sin\theta_2(-\hat{\imath}) + \frac{L}{2}\cos\theta_2(-\hat{\jmath})\right] \times m\ddot{x}\,\hat{\imath}$$

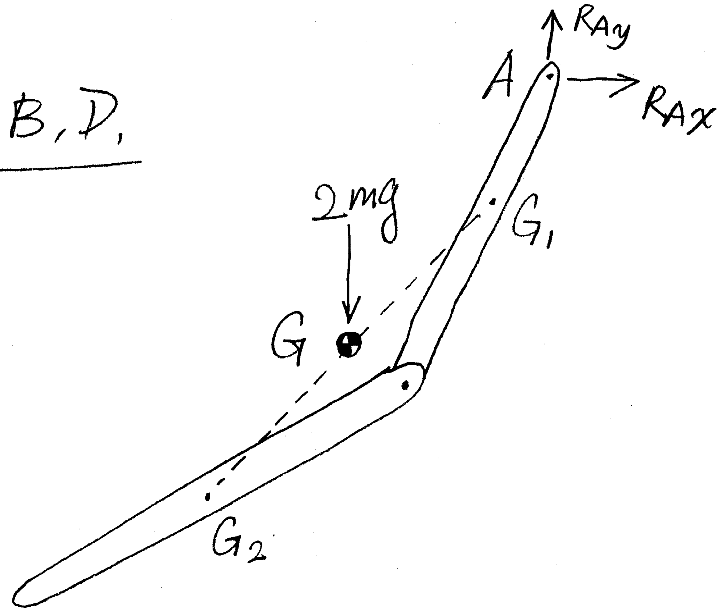
$$\Longrightarrow \frac{1}{2}mgL\sin\theta_2\,\hat{k} = \frac{1}{2}m\ddot{x}L\cos\theta_2\,\hat{k} \quad (*)$$

$$(*)\cdot\hat{k} \Longrightarrow \boxed{\frac{1}{2}mgL\sin\theta_2 = \frac{1}{2}m\ddot{x}L\cos\theta_2} \quad ①$$

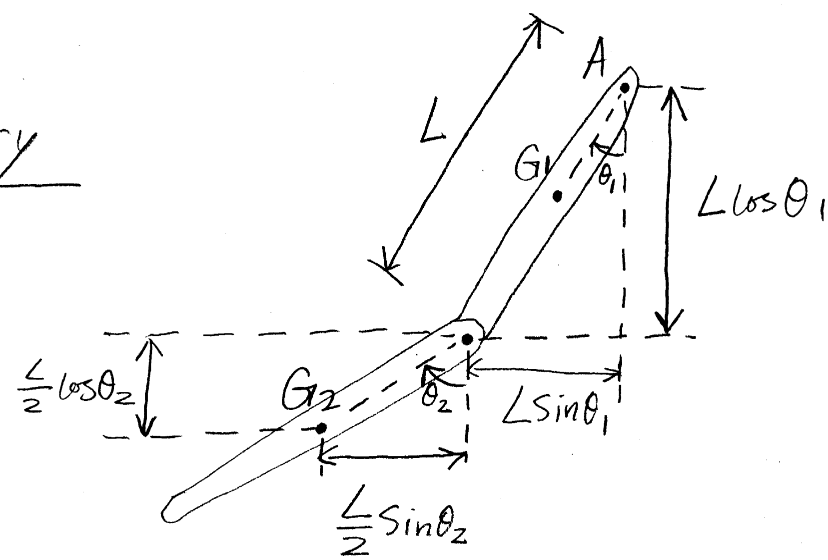
$$\Longrightarrow \boxed{\tan\theta_2 = \frac{\ddot{x}}{g}}$$

Now, for link $\overline{AB}$, to avoid the unnecessary calculation of R_{Bx}, R_{By}, etc., we can look at Links $\overline{AB}$ & $\overline{BC}$ together as if they are parts of a rigid body since the configuration does NOT change in our case anyway!

F.B.D.



Geometry



$$\vec{r}_{G/A} = \frac{m\vec{r}_{G_1/A} + m\vec{r}_{G_2/A}}{2m}$$

$$= \frac{m\left(\frac{L}{2}\sin\theta_1(-\hat{\imath}) + \frac{L}{2}\cos\theta_1(-\hat{\jmath})\right) + m\left(\left(L\sin\theta_1 + \frac{L}{2}\sin\theta_2\right)(-\hat{\imath}) + \left(L\cos\theta_1 + \frac{L}{2}\cos\theta_2\right)(-\hat{\jmath})\right)}{2m}$$

$$\boxed{\vec{r}_{G/A} = L\left(\tfrac{3}{4}\sin\theta_1 + \tfrac{1}{4}\sin\theta_2\right)(-\hat{\imath}) + L\left(\tfrac{3}{4}\cos\theta_1 + \tfrac{1}{4}\cos\theta_2\right)(-\hat{\jmath})}$$

2

Note: $\vec{a}_G = \ddot{x}\,\hat{\imath}$

About point A,

$$\sum_i \vec{M}_{i/A} = \dot{\vec{H}}_{/A} = \vec{r}_{G/A} \times 2m\vec{a}_G + I_G \cancel{\vec{\alpha}}^{\,\vec{0}}$$

$$\Rightarrow \vec{r}_{G/A} \times 2mg(-\hat{j}) = \vec{r}_{G/A} \times 2m\ddot{x}\,\hat{i}$$

$$\underset{②}{\Rightarrow} 2mgL\left(\tfrac{3}{4}\sin\theta_1 + \tfrac{1}{4}\sin\theta_2\right)\hat{k} = 2m\ddot{x}L\left(\tfrac{3}{4}\cos\theta_1 + \tfrac{1}{4}\cos\theta_2\right)\hat{k} \qquad (**)$$

$$(**)\cdot\hat{k} \Rightarrow 2mgL\left(\tfrac{3}{4}\sin\theta_1 + \tfrac{1}{4}\sin\theta_2\right) = 2m\ddot{x}L\left(\tfrac{3}{4}\cos\theta_1 + \tfrac{1}{4}\cos\theta_2\right)$$

$$\Rightarrow mgL\left(\tfrac{3}{2}\sin\theta_1\right) + \underline{\tfrac{1}{2}mgL\sin\theta_2}$$
$$= m\ddot{x}L\left(\tfrac{3}{2}\cos\theta_1\right) + \underline{\tfrac{1}{2}m\ddot{x}L\cos\theta_2}$$

$$\underset{\text{by } ①}{\Rightarrow} mgL\left(\tfrac{3}{2}\sin\theta_1\right) = m\ddot{x}L\left(\tfrac{3}{2}\cos\theta_1\right)$$

$$\Rightarrow \boxed{\tan\theta_1 = \frac{\ddot{x}}{g}}$$

Thus, $\tan\theta_1 = \tan\theta_2 = \dfrac{\ddot{x}}{g} = \dfrac{\frac{1}{2}g}{g} = \dfrac{1}{2}$

$$\Rightarrow \boxed{\theta_1 = \theta_2 \approx 26.6^\circ}$$

$\square$