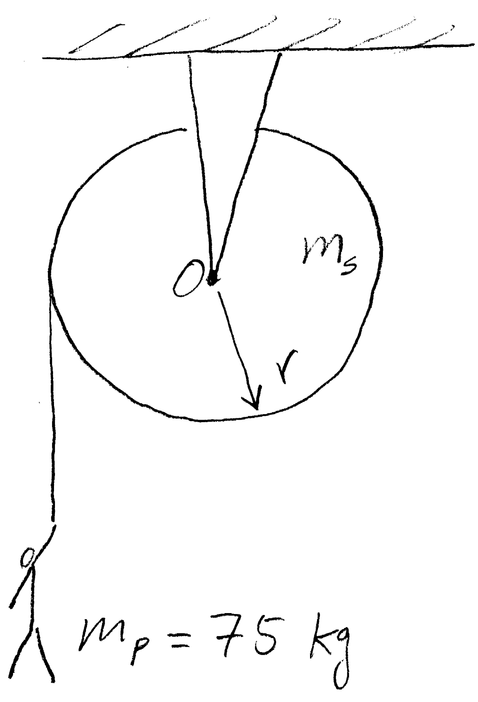
ENGRD / TAM 203

HW24 (Assigned on April 20, due on April 27)

Solution by Dennis Yang

7.2.33 The spool has a mass of 80 kg and a radius of 1.1 m. The axle/spool interface provides a friction so that 11 N.m of torque is needed to start the spool turning and remains at this level once the spool is rotating. The person's mass is 75 kg. What is his vertical acceleration? (neglect the mass of the cable)

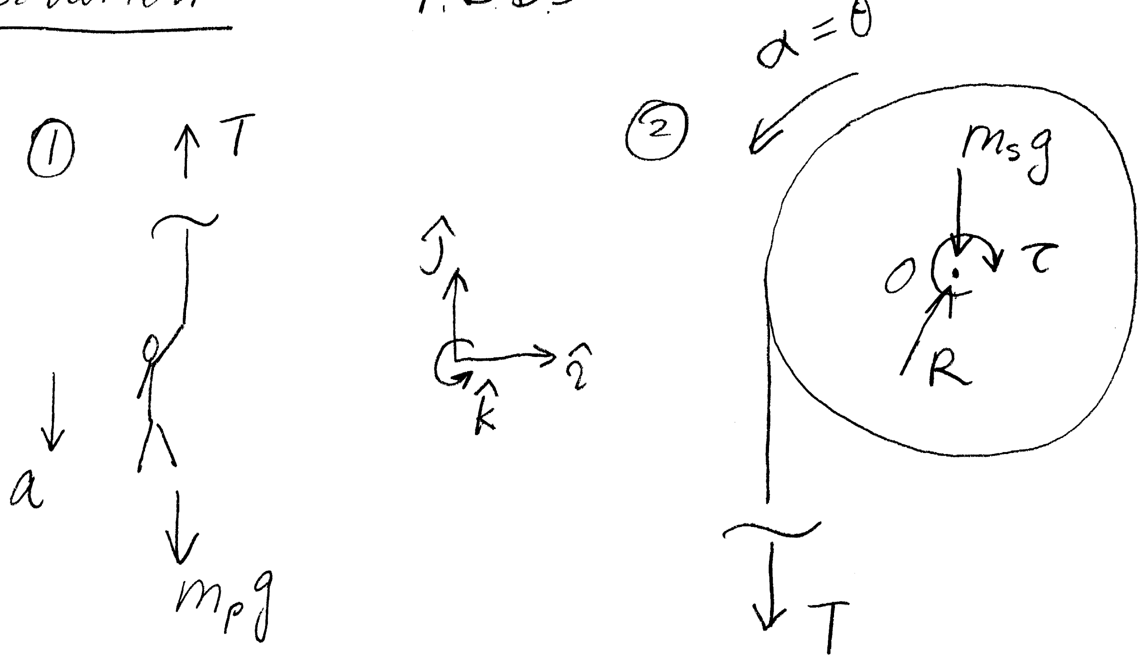


$m_s = 80$ kg

$r = 1.1$ m

Solution

F.B.Ds



Since we neglect the mass of the cable, the tension is the same along the vertical part of the cable.

(c)

Because the cable is inextensiable, $\underline{a = \alpha r = \ddot{\theta} r}$

For ① :

(a)

$$\sum \vec{F}_i = m\vec{a} \implies m_p g(-\hat{j}) + T(\hat{j}) = m_p a(-\hat{j})$$

For ② :

(b)

$$\sum \vec{M}_{i/o} = I_{/o}\vec{\alpha} \implies Tr\hat{k} + \tau(-\hat{k}) = I_{/o}\ddot{\theta}\hat{k}$$

$$(a)\cdot\hat{j} \Rightarrow -m_p g + T = -m_p a \quad (e)$$

$$(b)\cdot\hat{k} \Rightarrow Tr - \tau = I_{/O}\ddot{\theta} = \underbrace{\frac{m_s r^2}{2}}_{I_{/O} \text{ for circular plate}}\ddot{\theta} \quad (d)$$

$$(d) \xRightarrow[r\ddot{\theta}=a]{} Tr - \tau = \frac{m_s r}{2} a$$

$$\Rightarrow T - \frac{\tau}{r} = \frac{m_s}{2} a \quad (f)$$

$$(f)-(e) \Rightarrow m_p g - \frac{\tau}{r} = \left(\frac{m_s}{2} + m_p\right) a$$

$$\Rightarrow \boxed{a = \frac{m_p g - \tau/r}{m_s/2 + m_p}}$$

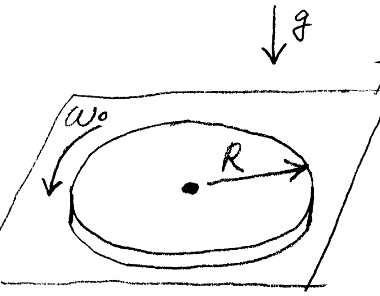
$$a = \frac{75\,kg \cdot 9.8\,m/s^2 - 11\,N\cdot m/1.1\,m}{80\,kg/2 + 75\,kg}$$

$$\boxed{a \approx 6.30\ m/s^2}$$

$$\boxed{\vec{a} \approx 6.30\ m/s^2\,(-\hat{j})}$$

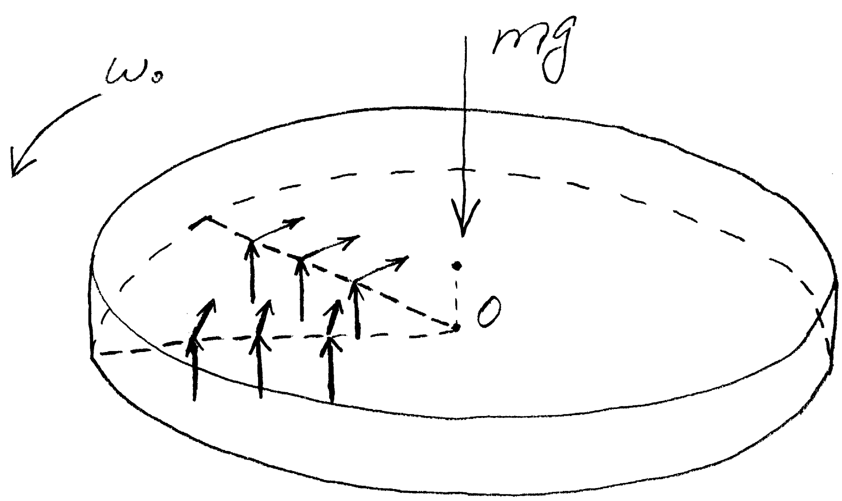
$\square$

7.2.45 A uniform disc of radius R is spun at an angular speed ω_0 and then carefully placed, flat side down, on a horizontal surface. How long will the disc be rotating on the surface if the coefficient of dynamic friction is μ? (Assume the pressure exerted by the disc on the surface is uniform.)



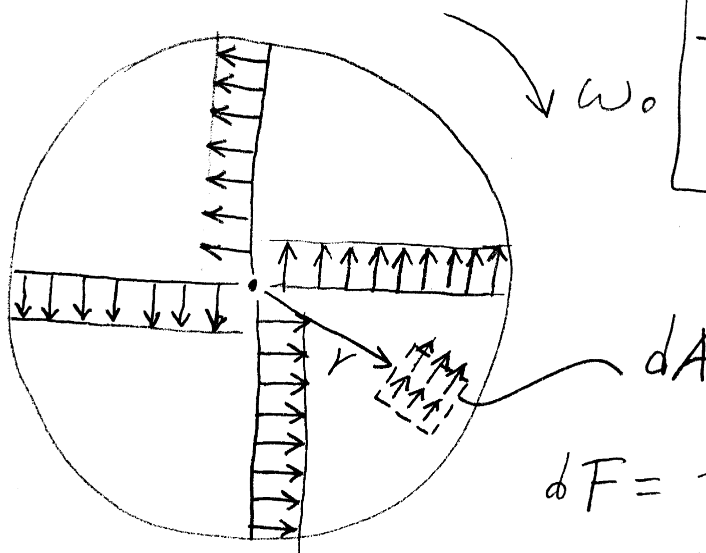
Solution

FBD



Note: the normal force and the friction are both uniformly distributed at the bottom of the disc, and the above drawing only gives a representive illustration.

Now, just look at tangential tractions (stresses) on the bottom



Note: this is a view from the bottom side.

Assume the total mass of the disc is "m".

$$dF = \tau \, dA = p\mu \, dA = \frac{mg}{A}\mu \, dA$$

$$\vec{M} = \iint_{\substack{\text{bottom}\\ \text{surface}}} \overbrace{r \cdot \underbrace{\tau \, dA}_{dF} \cdot (-\hat{k})}^{\vec{r} \times \vec{dF}}$$

$$= \int_0^{2\pi}\int_0^R r \cdot \underbrace{\frac{mg}{A}\mu}_{\tau} \cdot \underbrace{r\,dr\,d\theta}_{dA} \cdot (-\hat{k})$$

$$= \frac{mg}{A}\mu \left[\int_0^{2\pi} d\theta\right] \cdot \left[\int_0^R r^2 dr\right] (-\hat{k})$$

$$= \frac{mg}{\pi R^2}\mu \cdot 2\pi \cdot \frac{R^3}{3} (-\hat{k}) \qquad (A = \pi R^2)$$

$$\boxed{\vec{M} = \frac{2}{3} mg\mu R (-\hat{k})}$$

$$\vec{M} = I\vec{\alpha} \implies \frac{2}{3} mg\mu R(-\hat{k}) = \underbrace{\frac{mR^2}{2}}_{I}\alpha\hat{k} \quad (*)$$

$$(*)\cdot\hat{k} \implies -\frac{2}{3} mg\mu R = \frac{mR^2}{2}\alpha$$

$$\implies \alpha = -\frac{4}{3}\frac{g\mu}{R}$$

$$0 = \omega_0 + \alpha t^*$$

$$\implies t^* = -\frac{\omega_0}{\alpha}$$

$$= -\frac{\omega_0}{-\frac{4}{3}\frac{g\mu}{R}}$$

$$\boxed{t^* = \frac{3\omega_0 R}{4g\mu}}$$

$\square$