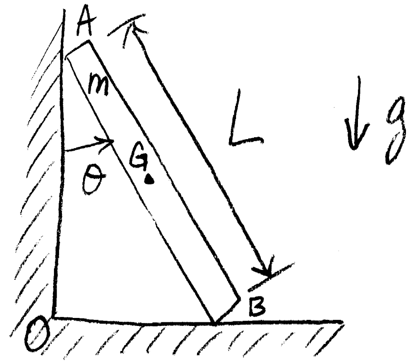
ENGRD/TAM 203
HW28 (Assigned on May 4)
Solution by Dennis Yang

7.5.9



No friction!

Initially $\theta = 15°$ and the board AB is at rest.
Use an energy formulation to determine $\dot{\theta}|_{\theta=45°}$ $(m = 10\,kg,\ L = 2m)$

Note: this problem can be included into Problem 7.3.22 with "$\mu = 0$". Thus, the differential equation for θ is obtained by taking the result of 7.3.22 and evaluating at $\mu = 0$.

Solution Let the total mechanical energy be $\mathcal{E}$. Then,

$$\boxed{\mathcal{E} = \frac{1}{2} I_G \dot{\theta}^2 + \frac{1}{2} m v_G^2 + mg\frac{L}{2}\cos\theta}$$

Specifically, $\vec{v}_G = \frac{d}{dt}(\vec{r}_{G/O}) = \frac{d}{dt}\left(\frac{L}{2}\sin\theta\,\hat{\imath} + \frac{L}{2}\cos\theta\,\hat{\jmath}\right)$

$$= \frac{L}{2}\cos\theta\,\dot{\theta}\,\hat{\imath} - \frac{L}{2}\sin\theta\,\dot{\theta}\,\hat{\jmath}$$

$$v_G^2 = \vec{v}_G \cdot \vec{v}_G = \frac{L^2}{4}\cos^2\theta\,\dot{\theta}^2 + \frac{L^2}{4}\sin^2\theta\,\dot{\theta}^2$$

$$= \frac{L^2}{4}\dot{\theta}^2$$

In addition, $I_G = \frac{mL^2}{12}$

Thus, $\mathcal{E} = \frac{1}{2}\left(\frac{mL^2}{12}\right)\dot{\theta}^2 + \frac{1}{2}m\left(\frac{L^2}{4}\dot{\theta}^2\right) + mg\frac{L}{2}\cos\theta$

$$\boxed{\mathcal{E} = \frac{mL^2}{6}\dot{\theta}^2 + mg\frac{L}{2}\cos\theta = \underline{\text{const.}}}$$

Since No friction, the total energy is conserved.

Therefore,

(initiall at rest)

$$\frac{mL^2}{6}\left(\dot{\theta}\Big|_{\theta=45°}\right)^2 + mg\frac{L}{2}\cos 45° = \frac{mL^2}{6}\Big(\underbrace{\dot{\theta}\Big|_{\theta=15°}}_{0}\Big)^2 + mg\frac{L}{2}\cos 15°$$

$$\Longrightarrow \quad \boxed{\dot{\theta}\Big|_{\theta=45°} = \left[\frac{3g}{L}\left(\cos 15° - \cos 45°\right)\right]^{1/2}}$$

$$\underline{\dot{\theta}\Big|_{\theta=45°} \simeq 1.95 \text{ rad/s}}$$

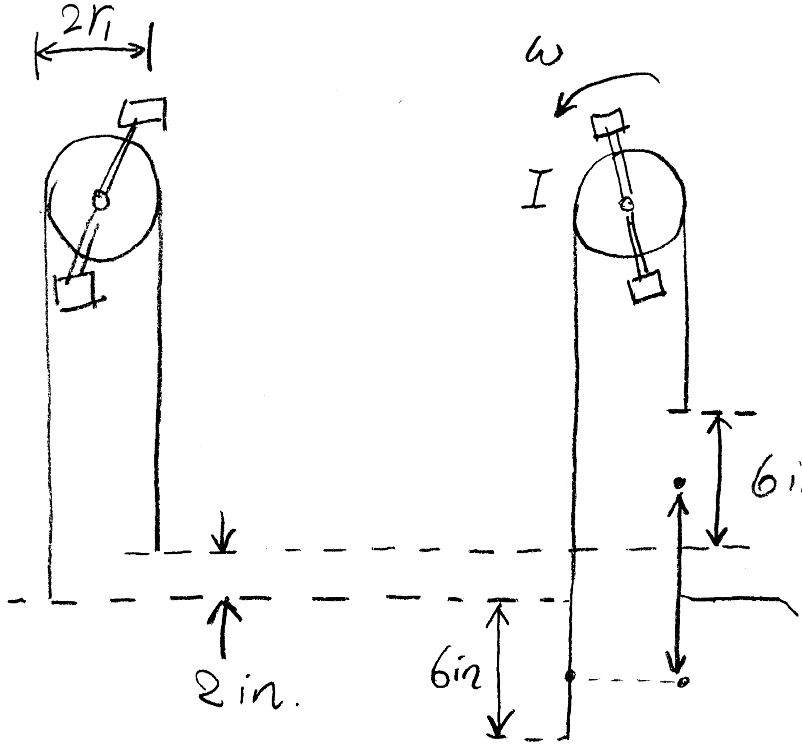
□

7.5.16

$W_{chain} = 0.73\,lb_m$

$L_{chain} = 56\,in$

$r_1 = 4\,in$



Initial

everything is at rest.

a moment later

determine <u>how fast does the crank rotate at this moment</u>.

<u>Solution</u>

The whole chain moves at speed of $|\omega r_1|$. Thus,

$$KE_{chain} = \frac{1}{2} m_{chain} (\omega r_1)^2$$

$$KE_{crank} = \frac{1}{2} I \omega^2$$

Then $KE_{chain} + KE_{crank} - \underset{\substack{\uparrow \\ \text{zero kinetic energy at the initial state ("at rest")}}}{0}$

$$= - \underbrace{\Delta PE}_{\text{Change of the total potantial energy.}}$$

$$\Delta PE = \Delta m g \Delta h$$

$$= \left(\frac{6\,in}{56\,in} m_{chain}\right) g \left(\frac{6\,in}{2} + 2\,in + \frac{6\,in}{2}\right)$$

$$= \frac{6\,in}{7} m_{chain}\, g$$

$$\Longrightarrow \frac{1}{2} m_{chain} (\omega r_1)^2 + \frac{1}{2} I \omega^2 = \frac{6\,in}{7} m_{chain}\, g$$

$$\Longrightarrow \left(\frac{1}{2} m_{chain} r_1^2 + \frac{1}{2} I\right) \omega^2 = \frac{6\,in}{7} m_{chain}\, g$$

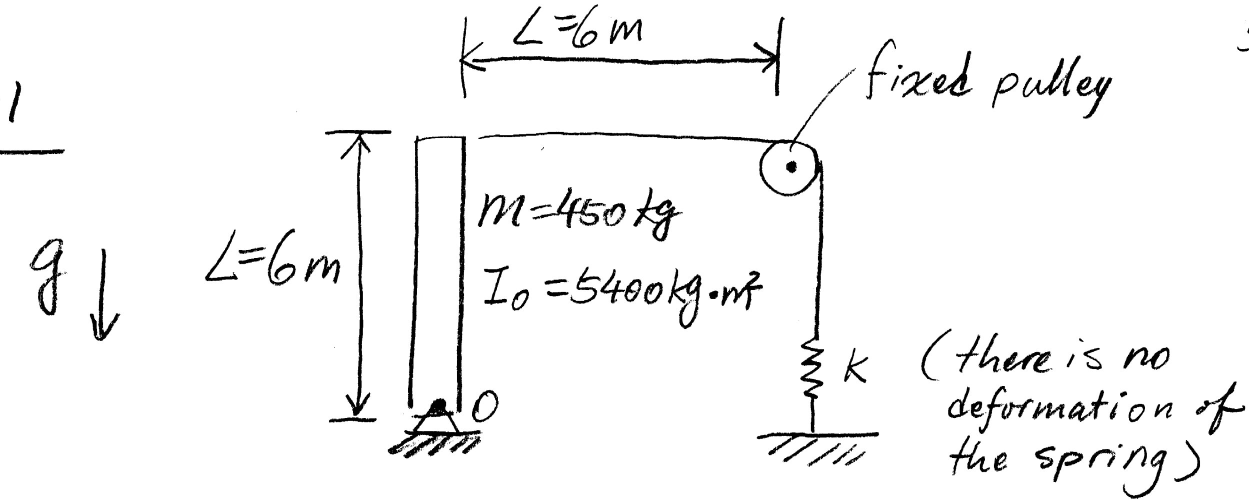
$$\Longrightarrow \boxed{\omega = \left[\frac{\frac{6\,in}{7} m_{chain}\, g}{\frac{1}{2} m_{chain} r_1^2 + \frac{1}{2} I}\right]^{1/2}}$$

Note:
$W_{chain} = m_{chain}\, g$
$1\,in = \frac{1}{12}\,ft.$

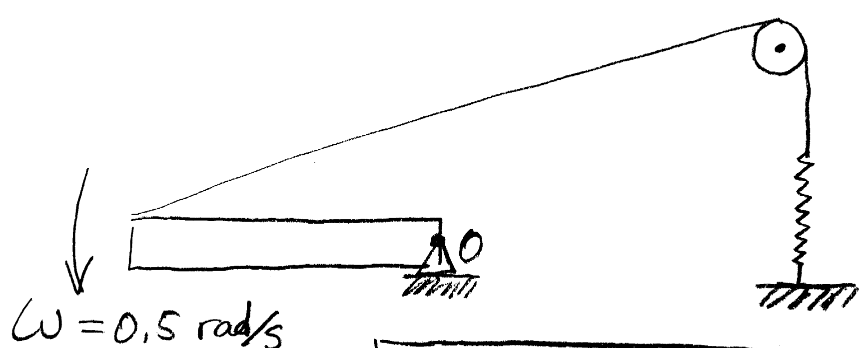
$$\underline{\omega \simeq 2.76 \text{ rad/s}}$$

$\square$

7.5.21



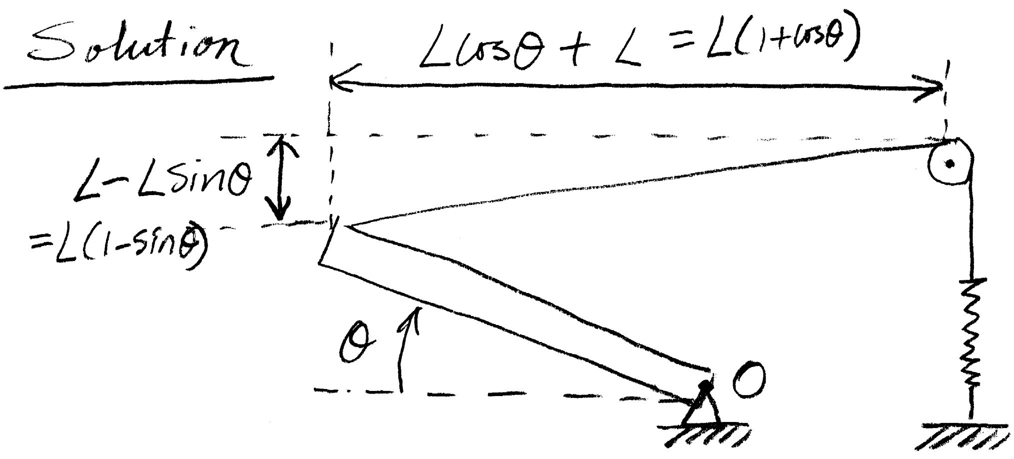
Initial position (at rest)



final position

(a) Determine a value of "k" so that $\omega = 0.5$ rad/s

Solution



Total kinetic energy:

$$\underline{KE = \frac{1}{2} I_o \dot{\theta}^2}$$

Total potential energy:

$$\underline{PE = mg\frac{L}{2}\sin\theta + \frac{1}{2}k\Big[\underbrace{\left(L^2(1+\cos\theta)^2 + L^2(1-\sin\theta)^2\right)^{1/2} - L}_{\text{the elongation of the spring}}\Big]^2}$$

$KE + PE = \text{Const.}$ (total energy is conserved)

$$\Longrightarrow \frac{1}{2} I_o \dot{\theta}^2 + mg\frac{L}{2}\sin\theta + \frac{1}{2}k\left[\left(L^2(1+\cos\theta)^2 + L^2(1-\sin\theta)^2\right)^{1/2} - L\right]^2 = \text{Const.}$$

$$\Longrightarrow \underbrace{\frac{1}{2} I_o \cancelto{0}{\dot{\theta}}^2 + mg\frac{L}{2}\sin 90^\circ + \frac{1}{2}k\underbrace{\left[\left(L^2(1+\cos 90^\circ)^2 + L^2(1-\sin 90^\circ)^2\right)^{1/2} - L\right]^2}_{=0}}_{\text{initial position, at rest, } \theta = 90^\circ}$$

$$= \underbrace{\frac{1}{2} I_o \dot{\theta}^2 + mg\frac{L}{2}\sin 0^\circ + \frac{1}{2}k\left[\left(L^2(1+\cos 0^\circ)^2 + L^2(1-\sin 0^\circ)^2\right)^{1/2} - L\right]^2}_{\text{final position, } \dot{\theta} = -\omega,\ \theta = 0^\circ.}$$

$$\Rightarrow mg\frac{L}{2} = \frac{1}{2}I_o\omega^2 + kL^2(3-\sqrt{5})$$

$$\Rightarrow \boxed{k = \frac{mg\frac{L}{2} - \frac{1}{2}I_o\omega^2}{L^2(3-\sqrt{5})}}$$

$$\underline{k \simeq 457 \ N/m}$$

(b) For the value of "k" determined in (a), will the bridge ~~fall~~ continue down?

<u>Solution</u> In (a), we've found that

$$PE = mg\frac{L}{2}\sin\theta + \frac{1}{2}kL^2\left[\left((1+\cos\theta)^2 + (1-\sin\theta)^2\right)^{1/2} - 1\right]^2$$

$$= mg\frac{L}{2}\sin\theta + \frac{1}{2}kL^2\left[\left(3 + 2\cos\theta - 2\sin\theta\right)^{1/2} - 1\right)^2$$

$$PE\Big|_{\theta=90^\circ} = mg\frac{L}{2} = 450\,kg \cdot 9.81\,N/kg \cdot 3\,m = 13243.5 \ N\cdot m$$

$$PE\Big|_{\theta=89^\circ} = mg\frac{L}{2}\cdot 0.99985 + \frac{1}{2}kL^2 \cdot 0.000305$$

$$= 13241.5 \ N\cdot m + 2.509 \ N\cdot m$$

$$= 13244.0 \ N\cdot m > PE\Big|_{\theta=90^\circ}$$

"$$PE\big|_{\theta=89^\circ} > PE\big|_{\theta=90^\circ}$$"

indicates that the <u>potential energy at the initial vertical position</u> is actually smaller than that of $\theta = 89^\circ$. Thus, if the bridge starts at very very small kinetic energy, then its total energy may still be smaller than $PE\big|_{\theta=89^\circ}$.

In this case, the bridge will NOT fall to the ground.

$\square$