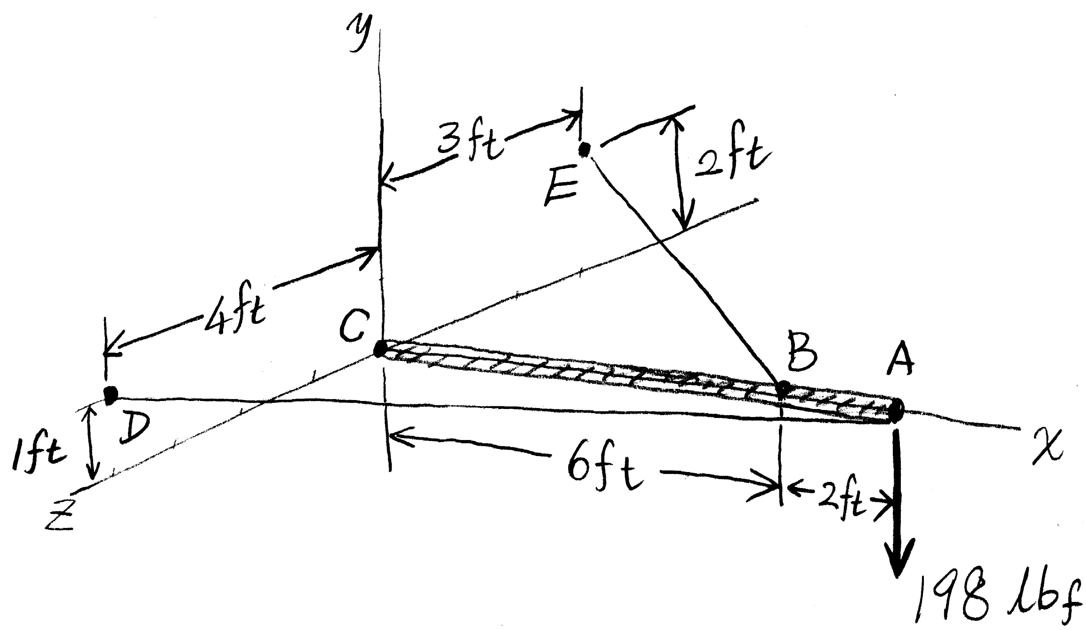
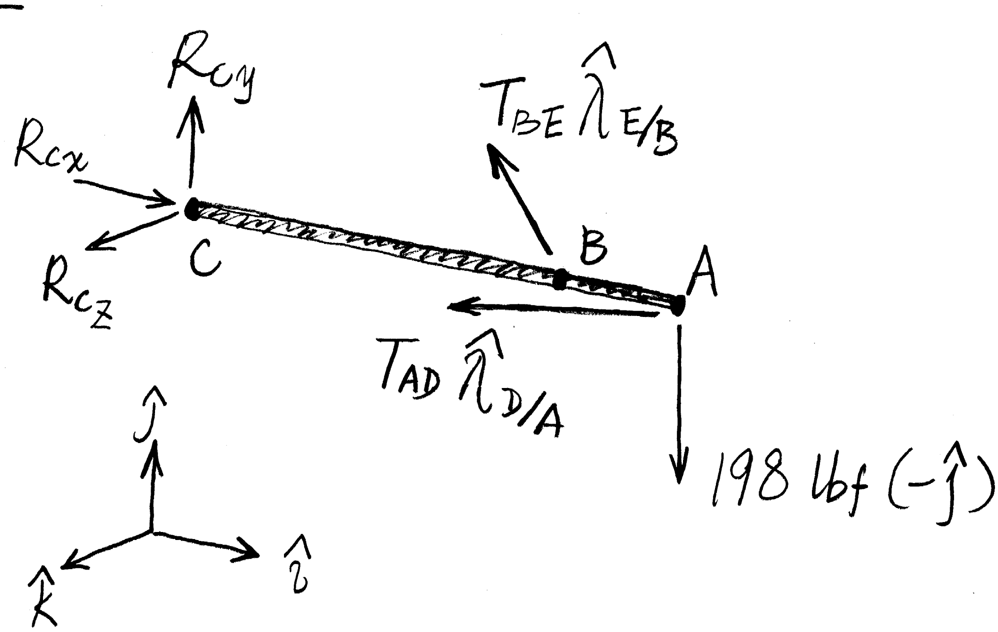
Solution by Dennis Yang

BJ 4.114 An 8-ft-long boom is held by a ball-and-socket joint at C two cables AD and BE. Find the tension in cable BE.



FBD



Solution: Our goal is to find tension in cable BE only. Thus, we should observe that the reaction force at C (i.e., $R_{Cx}\hat{i} + R_{Cy}\hat{j} + R_{Cz}\hat{k}$) and the force on the boom from cable AD (i.e., $T_{AD}\hat{\lambda}_{D/A}$) both pass through line DC. Thus, the moment produced by them about axis $\frac{\vec{r}_{D/C}}{\|\vec{r}_{D/C}\|}$ is ZERO!

Since the boom is at rest, we have

$$\sum_i \vec{M}_{i/C} = \vec{0} \Longrightarrow \underbrace{\left\{\sum_i \vec{M}_{i/C}\right\} \cdot \frac{\vec{r}_{D/C}}{\|\vec{r}_{D/C}\|}}_{\text{total moments about axis } \frac{\vec{r}_{D/C}}{\|\vec{r}_{D/C}\|}} = 0$$

$$\Longrightarrow \left\{\vec{r}_{B/C} \times T_{BE}\,\hat{\lambda}_{E/B} + \vec{r}_{A/C} \times 198\,\text{lbf}\,(-\hat{j})\right\} \cdot \frac{\vec{r}_{D/C}}{\|\vec{r}_{D/C}\|} = 0 \qquad (**)$$

$$\vec{r}_{B/C} = 6\text{ ft}\,\hat{i}, \qquad \vec{r}_{A/C} = 8\text{ ft}\,\hat{i}$$

$$\hat{\lambda}_{E/B} = \frac{-6\text{ ft}\,\hat{i} + 2\text{ ft}\,\hat{j} - 3\text{ ft}\,\hat{k}}{\sqrt{(-6\text{ ft})^2 + (2\text{ ft})^2 + (-3\text{ ft})^2}} = -\frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}$$

$$\frac{\vec{r}_{D/C}}{\|\vec{r}_{D/C}\|} = \frac{1\text{ ft}\,\hat{j} + 4\text{ ft}\,\hat{k}}{\sqrt{(1\text{ ft})^2 + (4\text{ ft})^2}} = \frac{1}{\sqrt{17}}(\hat{j} + 4\hat{k})$$

Then, (**) becomes

$$\left[6\,\text{ft}\,\hat{\imath}\times T_{BE}\left(-\frac{6}{7}\hat{\imath}+\frac{2}{7}\hat{\jmath}-\frac{3}{7}\hat{k}\right)\right]\cdot\frac{\hat{\jmath}+4\hat{k}}{\sqrt{17}}+\left[8\,\text{ft}\,\hat{\imath}\times 198\,\text{lbf}\,(-\hat{\jmath})\right]\cdot\frac{\hat{\jmath}+4\hat{k}}{\sqrt{17}}=0$$

$$\Rightarrow\left[6\,\text{ft}\,\hat{\imath}\times T_{BE}\left(-\frac{6}{7}\hat{\imath}+\frac{2}{7}\hat{\jmath}-\frac{3}{7}\hat{k}\right)\right]\cdot(\hat{\jmath}+4\hat{k})+1584\,\text{ft}\cdot\text{lbf}\,(-\hat{k})\cdot(\hat{\jmath}+4\hat{k})=0$$

$$\Rightarrow\left(\frac{12}{7}\,\text{ft}\cdot T_{BE}\,\hat{k}+\frac{18}{7}\,\text{ft}\cdot T_{BE}\,\hat{\jmath}\right)\cdot(\hat{\jmath}+4\hat{k})-6336\,\text{ft}\cdot\text{lbf}=0$$

$$\Rightarrow\quad\frac{18}{7}\,\text{ft}\cdot T_{BE}+\frac{48}{7}\,\text{ft}\cdot T_{BE}-6336\,\text{ft}\cdot\text{lbf}=0$$

$$\Rightarrow\quad T_{BE}=\frac{6336\,\text{ft}\cdot\text{lbf}}{\frac{66}{7}\,\text{ft}}$$

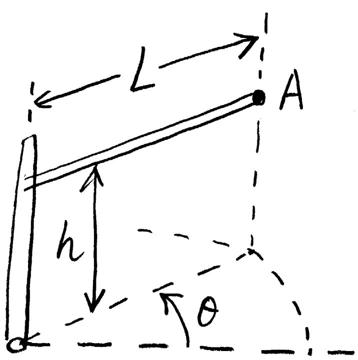
$$\Rightarrow\quad\boxed{T_{BE}=672\,\text{lbf}}$$

$\square$

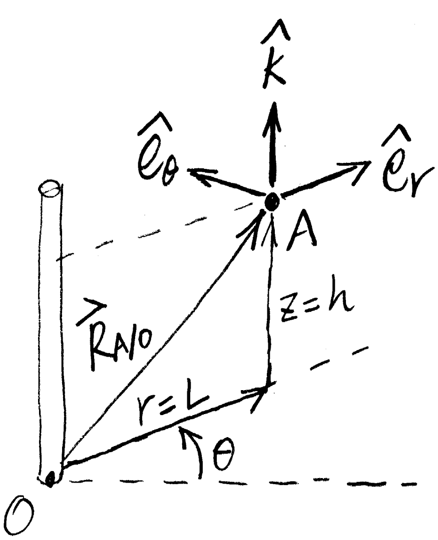
2.3.28 Let the arm length L be constant and $h(t)$, $\theta(t)$ be given as

$$h(t) = b[1 - \cos(\omega_2 t)]$$
$$\theta(t) = a[1 - \cos(\omega_1 t)]$$

What is the total acceleration at the tip of the probe (point A)?



Solution



Using the cylindrical coordinates:

$$\vec{a}_A = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta + \ddot{z}\hat{k}$$

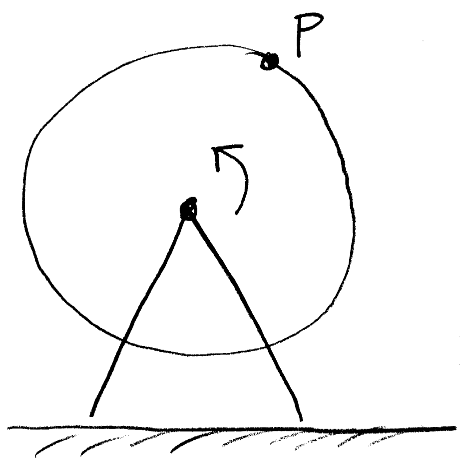
$$r \equiv L \Rightarrow \dot{r} = 0,\ \ddot{r} = 0$$

$$\theta(t) = a(1 - \cos(\omega_1 t))$$
$$\Rightarrow \dot{\theta} = a\omega_1 \sin(\omega_1 t),\ \ddot{\theta} = a\omega_1^2 \cos(\omega_1 t)$$

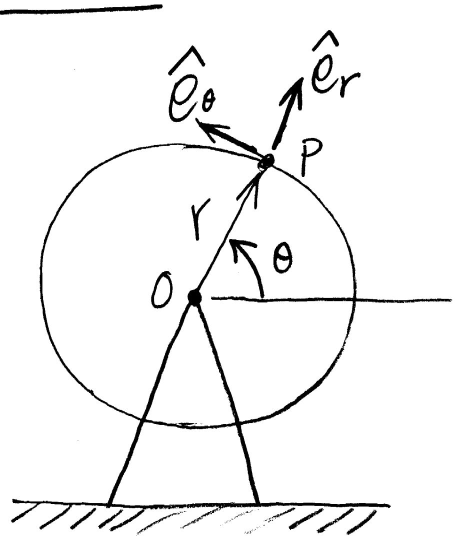
$$h(t) = b(1 - \cos(\omega_2 t))$$
$$\Rightarrow \dot{h} = b\omega_2 \sin(\omega_2 t),\ \ddot{h} = b\omega_2^2 \cos(\omega_2 t)$$

Thus,
$$\vec{a}_A = -L(a\omega_1 \sin(\omega_1 t))^2 \hat{e}_r + L(a\omega_1^2 \cos(\omega_1 t))\hat{e}_\theta + b\omega_2^2 \cos(\omega_2 t)\hat{k}$$
$$= -La^2\omega_1^2 \sin^2(\omega_1 t)\hat{e}_r + La\omega_1^2 \cos(\omega_1 t)\hat{e}_\theta + b\omega_2^2 \cos(\omega_2 t)\hat{k}$$

2.4.1 A person P on a Ferris wheel travels in a circle with a 30-foot radius and experiences an acceleration of magnitude 0.33 ft/s². The person's speed is constant during the ride. What is the Ferris wheel's rotational speed?



Solution



$$\vec{a}_P = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$$

$$\vec{v}_P = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$r \equiv 30\text{ ft} \implies \dot{r} = \ddot{r} = 0$$

Thus $\vec{v}_P = r\dot{\theta}\hat{e}_\theta$

$\implies \|\vec{v}_P\| = |r\dot{\theta}|$, which is constant

$\implies \dot{\theta}$ is constant

$\implies \ddot{\theta} = 0$

Thus, $\vec{a}_P = -r\dot{\theta}^2 \hat{e}_r$

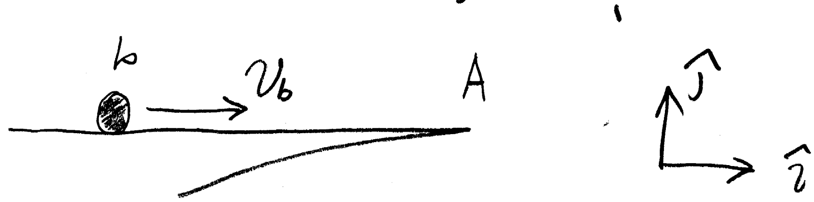
$$\Longrightarrow \|\vec{a}_P\| = r\dot{\theta}^2 = 0.33 \text{ ft/s}^2$$

$$\Longrightarrow 30 \text{ ft} \cdot \dot{\theta}^2 = 0.33 \text{ ft/s}^2$$

$$\Longrightarrow \boxed{\dot{\theta} \approx 0.105 \text{ rad/s}}$$

$\square$

2.4.8 The small particle b slides to the right at a constant speed v_b toward A, the end of the table. When it passes A, it begins to accelerate down-ward at 32.2 ft/s². Thus there is a very marked difference in its acceleration just before and just after reaching A. Is the same true for the velocity? What are the direction and magnitude of the velocity vector the instant before reaching A as well as the instant after?



Solution

let t_A is the instant when b is exactly at A (i.e., has NOT passed A yet!) Then, $\vec{v}(t_A) = v_b \hat{i}$

For. $\Delta t > 0$, $\vec{v}(t_A+\Delta t) = \vec{v}(t_A) + \int_{t_A}^{t_A+\Delta t} \vec{a}\, dt$

If $\Delta t \to 0^+$ (the instant after),

$$\lim_{\Delta t \to 0^+} \vec{v}(t_A+\Delta t) = \vec{v}(t_A) + \lim_{\Delta t \to 0^+} \int_{t_A}^{t_A+\Delta t} \vec{a}\, dt$$

In this problem,

there is NO force on the particle with infinitely large magnitude

$\Longrightarrow$ the particle has NO acceleration with infinitely large magnitude

$$\Longrightarrow \lim_{\Delta t \to 0^+} \int_{t_A}^{t_A+\Delta t} \vec{a}\, dt = \vec{0}$$

$$\Longrightarrow \lim_{\Delta t \to 0^+} \vec{v}(t_A + \Delta t) = v_b \hat{i}$$

$\square$

(That is to say, we have the continuity of velocity !)