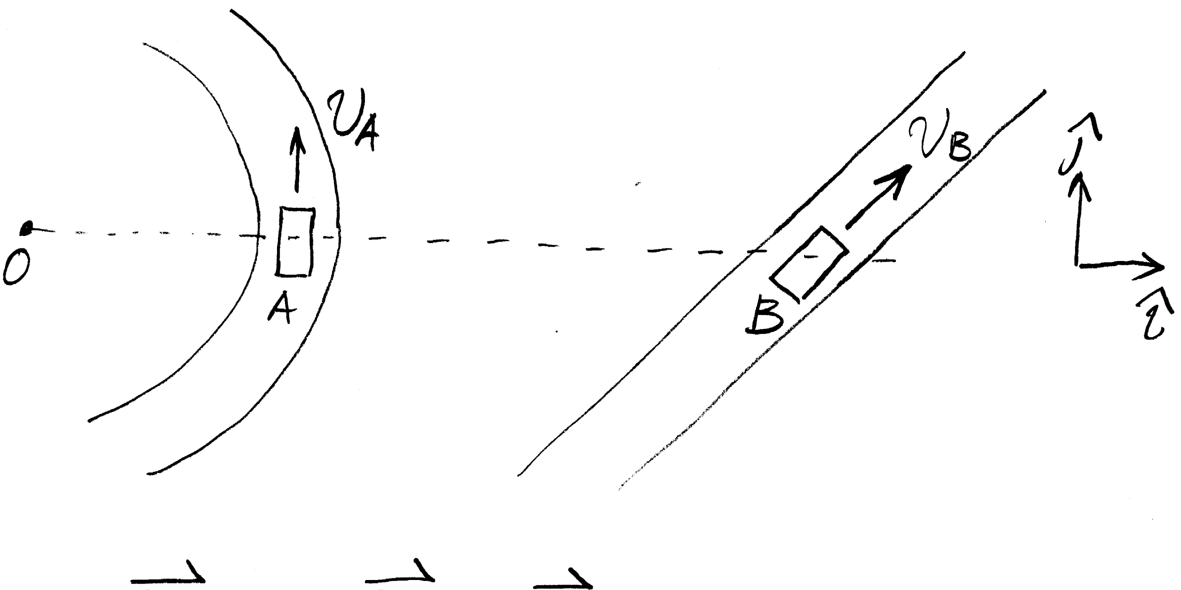
ENGRD/TAM 203 Spring 2006

HW5 (Assigned Feb 7, due on Feb 14)

Solution by Dennis Yang

2.5.3 Two cars are passing by each other, moving in the direction illustrated. Car A is moving at a constant 30 mph around a circle with radius 100 ft, and Car B is moving at a constant 60 mph in the direction $0.5\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}$. What are the velocity and acceleration of Car A with respect to Car B?

$(\vec{r}_{A/O} = 100\,\text{ft}\,\hat{\imath}, \ \vec{r}_{B/A} = 200\,\text{ft}\,\hat{\imath})$



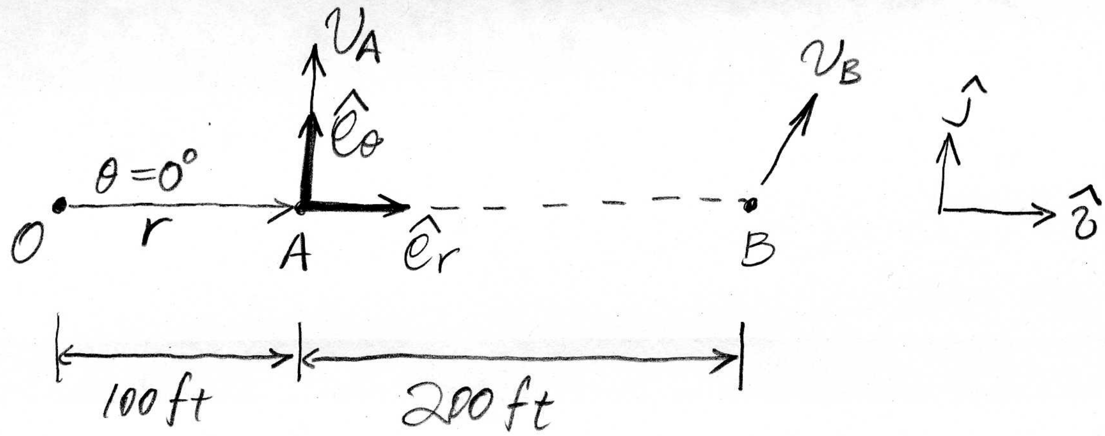
Solution

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

$$= 30\,\text{mph}\,\hat{\jmath} - 60\,\text{mph}\left(0.5\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}\right)$$

$$= -60\,\text{mph}\cdot 0.5\,\hat{\imath} + \left(30\,\text{mph} - 60\,\text{mph}\frac{\sqrt{3}}{2}\right)\hat{\jmath}$$

$$\boxed{\approx -30\,\text{mph}\,\hat{\imath} - 21.96\,\text{mph}\,\hat{\jmath}}$$



at $\theta = 0°$, $\hat{e}_r = \hat{i}$, $\hat{e}_\theta = \hat{j}$.

$$\vec{v}_A = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$= r\dot{\theta}\hat{e}_\theta \quad (\text{since } \dot{r} = 0 \text{ ft/s by } r \equiv 100 \text{ ft})$$

$$\Rightarrow \|\vec{v}_A\| = |r\dot{\theta}| = 100 \text{ ft} |\dot{\theta}| = 30 \text{ mph} \cdot 1.4667 \frac{\text{ft/s}}{\text{mph}}$$

$$\Rightarrow \dot{\theta} = \frac{30 \text{ mph}}{100 \text{ ft}} \cdot 1.4667 \frac{\text{ft/s}}{\text{mph}} = 0.44 \text{ rad/s}$$

and $\ddot{\theta} = 0 \text{ rad/s}^2$

$$\vec{a}_A = (\cancelto{0 \text{ ft/s}^2 \text{ since } \dot{r} = 0 \text{ ft/s}}{\ddot{r}} - r\dot{\theta}^2)\hat{e}_r + (2\cancelto{0 \text{ ft/s}}{\dot{r}}\dot{\theta} + r\cancelto{0 \text{ rad/s}^2}{\ddot{\theta}})\hat{e}_\theta$$

$$= -r\dot{\theta}^2\hat{e}_r = -r\dot{\theta}^2\hat{i}$$

$$= -(100 \text{ ft})(0.44 \text{ rad/s})^2\hat{i}$$

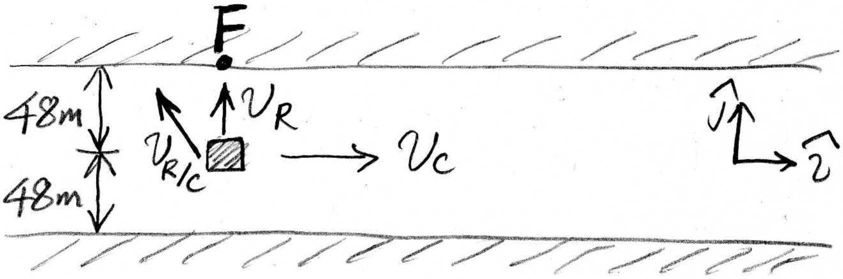
$$= -19.36 \text{ ft/s}^2\hat{i}$$

$\vec{a}_B = 0$ since Car B is moving at constant speed along a strait line.

$$\boxed{\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B = -19.36 \text{ ft/s}^2\hat{i}}$$

☐

2.5.8 You're out paddling a raft R in the middle of the river and hear a friend (located at F) call from the shore. In what direction should you paddle to arrive at F in 2 minutes? (the river's current has a velocity 1 m/s $\hat{\imath}$)



Solution

$\vec{V_C}$ ~ the velocity of the river current.

$\vec{V_R}$ ~ the velocity of the raft.

$\vec{V_{R/C}}$ ~ the relative velocity of the raft with respect to the river current.

assume: $\vec{V_{R/C}} = V_x \hat{\imath} + V_y \hat{\jmath}$

we know that $\vec{V_R} = \dfrac{48\,m}{2\,min \cdot (60\,\frac{s}{min})} \hat{\jmath} = 0.4\ m/s\ \hat{\jmath}$

and $\vec{V_C} = 1\ m/s\ \hat{\imath}$

Thus, $\vec{v}_R = \vec{v}_c + \vec{v}_{R/c}$ implies that

$$0.4\,\mathrm{m/s}\,\hat{j} = 1\,\mathrm{m/s}\,\hat{i} + (v_x\hat{i} + v_y\hat{j})$$

$$\Rightarrow 0.4\,\mathrm{m/s}\,\hat{j} = (1\,\mathrm{m/s} + v_x)\,\hat{i} + v_y\,\hat{j} \qquad (*)$$

$$(*)\cdot\hat{i} \Rightarrow 0.4\,\mathrm{m/s}\,\hat{j}\cdot\hat{i} = (1\,\mathrm{m/s} + v_x)\,\hat{i}\cdot\hat{i} + v_y\,\hat{j}\cdot\hat{i}$$

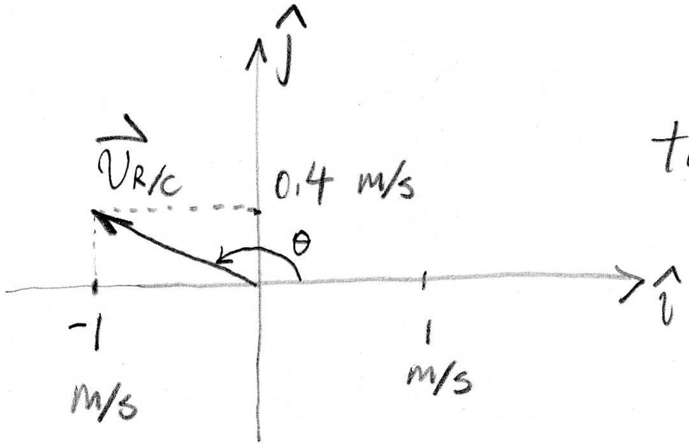
$$\Rightarrow \quad 0\,\mathrm{m/s} = 1\,\mathrm{m/s} + v_x$$

$$\Rightarrow \quad \boxed{v_x = -1\,\mathrm{m/s}}$$

$$(*)\cdot\hat{j} \Rightarrow 0.4\,\mathrm{m/s}\,\hat{j}\cdot\hat{j} = (1\,\mathrm{m/s} + v_x)\,\hat{i}\cdot\hat{j} + v_y\,\hat{j}\cdot\hat{j}$$

$$\Rightarrow \quad \boxed{0.4\,\mathrm{m/s} = v_y}$$

Thus, $\underline{\underline{\vec{v}_{R/c} = -1\,\mathrm{m/s}\,\hat{i} + 0.4\,\mathrm{m/s}\,\hat{j}}}$

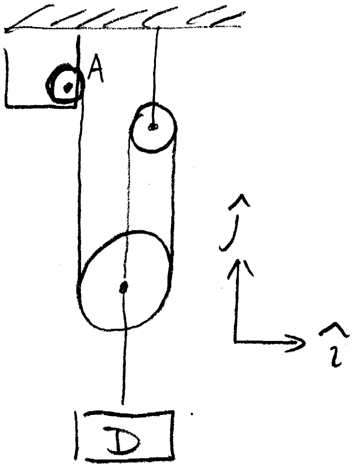


$$\tan\theta = \frac{0.4\,\mathrm{m/s}}{-1\,\mathrm{m/s}} = -0.4$$

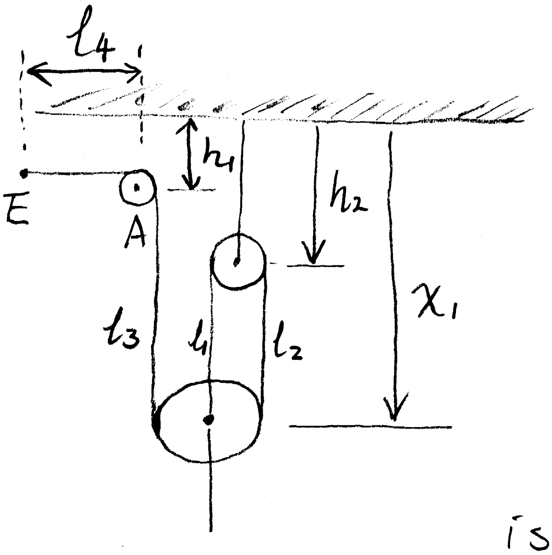
$$\Rightarrow \boxed{\theta \approx 158.2^\circ}$$

□

2.5.14 A motor at A reels in the illustrated rope at 0.4 m/s. What is the absolute velocity of Block D?



Solution



As illustrated in the plot on the left, when the motor at A reels, it is the same as pulling the end E to the left (at 0.4 m/s).

The total length of the rope is $L = l_1 + l_2 + l_3 + l_4$, where L is fixed and

$$l_1 = l_2 = x_1 - \underset{\substack{\uparrow \\ \text{a constant}}}{h_2}, \quad l_3 = x_1 - \underset{\substack{\uparrow \\ \text{a constant too!}}}{h_1}$$

Thus, $L = (x_1 - h_2) + (x_1 - h_2) + (x_1 - h_1) + l_4$

$$\Longrightarrow L = 3x_1 - 2h_2 - h_1 + l_4$$

$$\Rightarrow \quad \cancelto{0}{\dot{L}} = 3\dot{x}_1 - \cancelto{0}{2\dot{h}_2} - \cancelto{0}{\dot{h}_1} + \dot{l}_4$$

$$\Rightarrow \quad 3\dot{x}_1 = -\dot{l}_4$$

$$\Rightarrow \quad \dot{x}_1 = -\frac{1}{3}\dot{l}_4 = -\frac{1}{3} \times 0.4 \, m/s = -\frac{0.4}{3} \, m/s$$

Thus, $v_D = \dot{x}_1 (-\hat{j}) = -\frac{0.4}{3} \, m/s \, (-\hat{j})$

$$= \frac{0.4}{3} \, m/s \, \hat{j}$$

$$v_D \approx 0.1333 \, m/s \, \hat{j}$$

□