ENGRD/TAM 203 Spring 2006

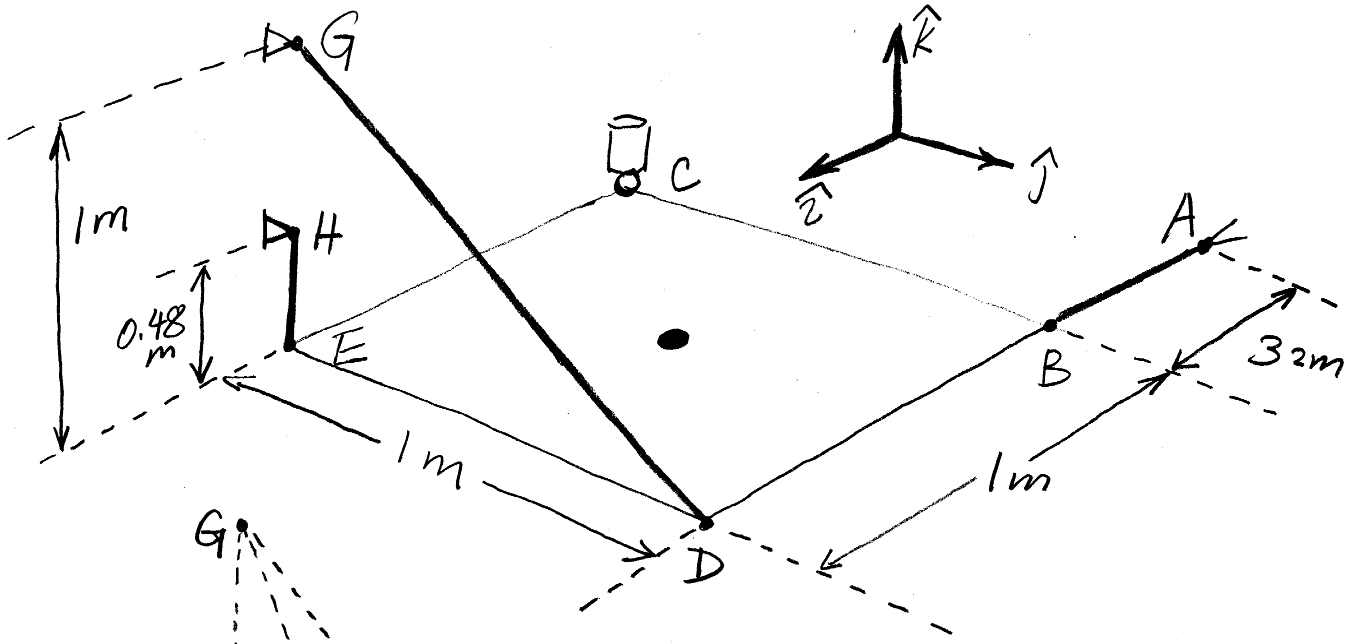
HW6 (Assigned on Feb. 9, due on Feb. 16)

Solution by Dennis Yang

RP 4.87

A uniform 5 kg shelf is supported at one corner with a ball and socket joint and the other three corners with strings. At the moment of interest the shelf is at rest. Gravity acts in the $-\hat{k}$ direction. The shelf is in the x-y plane.

Find the tension in cable AB only.



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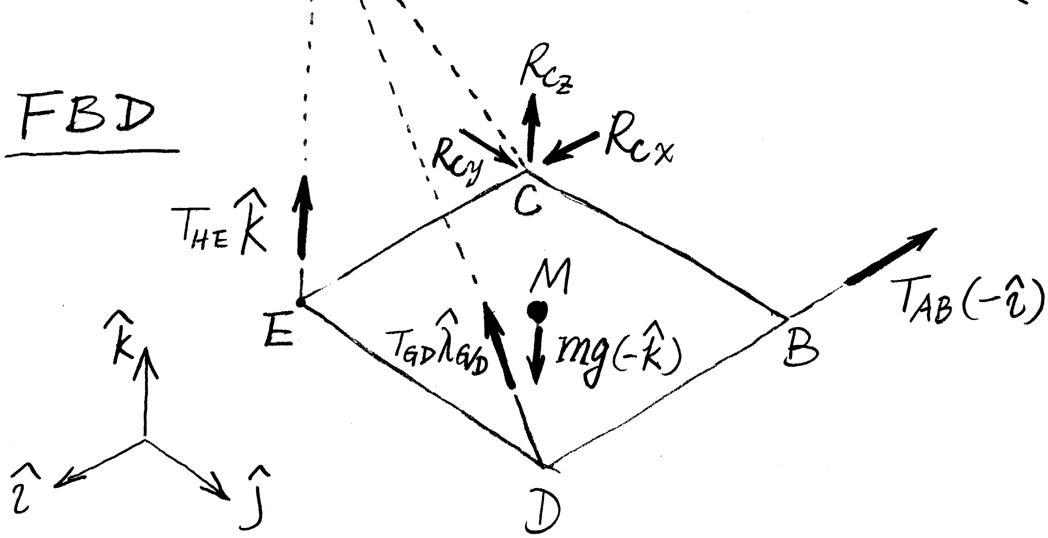


Solution Our goal is to find the tension in cable AB only. Thus, we should observe that the reaction force at C (i.e., $R_{Cx}\hat{\imath}+R_{Cy}\hat{\jmath}+R_{Cz}\hat{k}$), $T_{HE}\hat{k}$ (the force on the shelf from cable HE), and $T_{GD}\hat{\lambda}_{GD}$ (the force on the shelf from cable GD), all of them pass through line GC.

Thus, the moments produced by them about axis $\frac{\vec{r}_{G/C}}{\|\vec{r}_{G/C}\|}$ is ZERO!

Since the shelf is at rest, we have

$$\sum_i \vec{M}_{i/C} = \vec{0} \Rightarrow \underbrace{\left\{\sum_i \vec{M}_{i/C}\right\}\cdot \frac{\vec{r}_{G/C}}{\|\vec{r}_{G/C}\|}}_{\text{total moments about axis } \frac{\vec{r}_{G/C}}{\|\vec{r}_{G/C}\|}} = 0$$

$$\Longrightarrow \left\{\vec{r}_{B/C}\times T_{AB}(-\hat{\imath}) + \vec{r}_{M/C}\times mg(-\hat{k})\right\}\cdot \frac{\vec{r}_{G/C}}{\|\vec{r}_{G/C}\|} = 0$$

$$\Longrightarrow \left\{1m\hat{\jmath}\times T_{AB}(-\hat{\imath}) + \left(\tfrac{1}{2}m\hat{\imath}+\tfrac{1}{2}m\hat{\jmath}\right)\times mg(-\hat{k})\right\}\cdot \frac{1m\hat{\imath}+1m\hat{k}}{\sqrt{(1m)^2+(1m)^2}} = 0$$

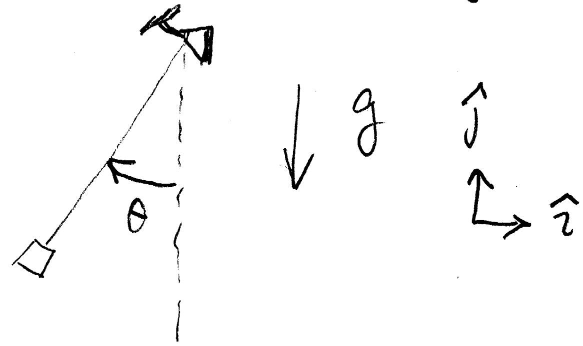
$$\Longrightarrow \left\{1m\cdot T_{AB}\hat{k} + \tfrac{1}{2}m\cdot mg\hat{\jmath} + \tfrac{1}{2}m\cdot mg(-\hat{\imath})\right\}\cdot(1m\hat{\imath}+1m\hat{k}) = 0$$

$$\Longrightarrow 1m^2\, T_{AB} - \tfrac{1}{2}m^2\cdot mg = 0 \Rightarrow T_{AB} = \tfrac{1}{2}mg = \tfrac{1}{2}\cdot 5kg\cdot 9.81\,N/kg$$

$$\Longrightarrow \boxed{T_{AB} = 24.525\ N}$$

□

3.1.3 A car is breaking at constant acceleration, causing fuzzy dice hanging from the mirror to take the position shown. Approximate the fuzzy dice as a lumped mass m and solve for the steady-state value of θ if the car is decelerating at $0.2g$.



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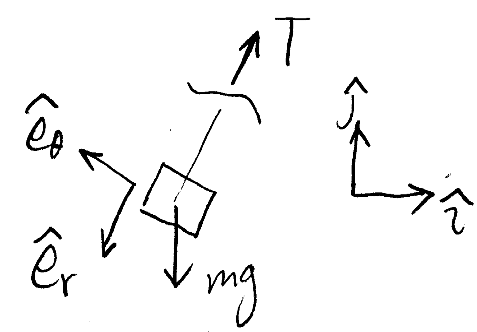
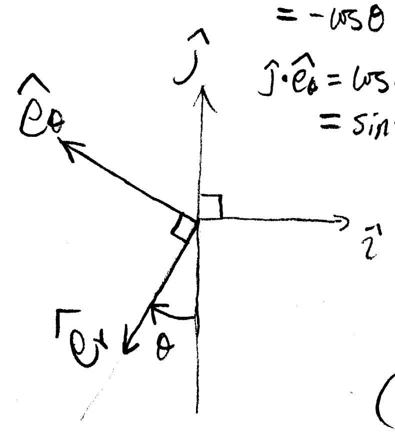


$$\hat{\imath}\cdot\hat{e}_\theta = \cos(90^\circ-\theta+90^\circ) = -\cos\theta$$

$$\hat{\jmath}\cdot\hat{e}_\theta = \cos(90^\circ-\theta) = \sin\theta$$



Solution

$$\sum_i \vec{F}_i = m\vec{a} \Longrightarrow mg(-\hat{\jmath}) + T(-\hat{e}_r) = m(0.2g)\,\hat{\imath} \qquad (*)$$

Since we don't care what T is, we can dot $(*)$ with $\hat{e}_\theta$ to get rid of $T(-\hat{e}_r)$, that is

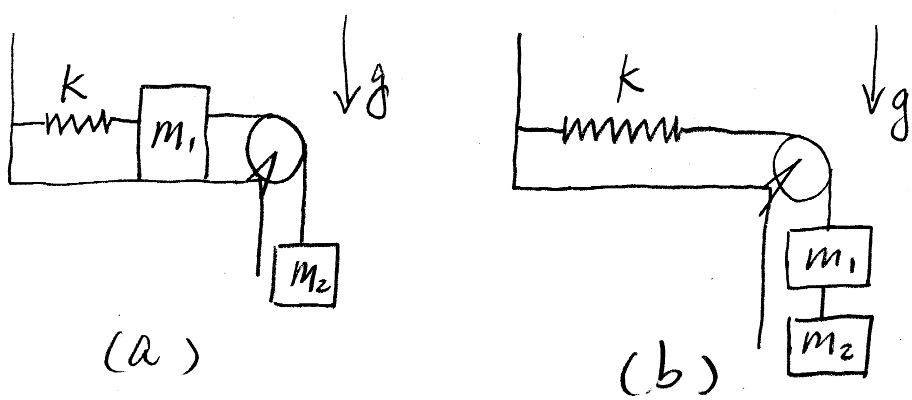
$$(*)\cdot\hat{e}_\theta \Longrightarrow mg(-\hat{\jmath})\cdot\hat{e}_\theta + T(-\hat{e}_r)\cdot\hat{e}_\theta = m(0.2g)\,\hat{\imath}\cdot\hat{e}_\theta$$

$$\Longrightarrow -mg\sin\theta = 0.2mg(-\cos\theta)$$

$$\Longrightarrow \tan\theta = 0.2 \Longrightarrow \boxed{\theta \approx 11.3^\circ}$$

$\square$

3.1.22 Assume no friction, massless pulley P, and inextensible rope. The spring is initially unstretched. For senarios (a) & (b), determine what extension of the spring is necessary to support a static equilibrium of the system. Then assume that m_2 is pulled downward 0.01 m and released. What is the acceleration of m_2 at the time of release?



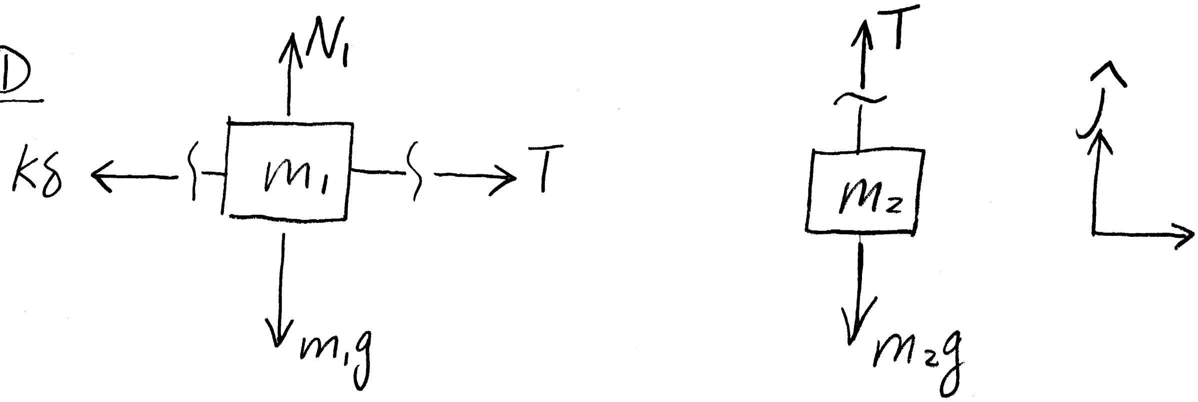
$k = 1000$ N/m

$m_1 = 10$ kg

$m_2 = 20$ kg

Solution (a)

FBD



Note: δ is the extension of the spring. Since the pulley is round, frictionless, and massless the tensions on both sides of the pulley are equal.

When the system is at static equilibrium

for m_1, $\sum_i \vec{F}_i = \vec{0} \Rightarrow T\hat{i} + k\delta(-\hat{i}) + N_1\hat{j} + m_1 g(-\hat{j}) = \vec{0}$ (1)

$(1)\cdot\hat{i} \Rightarrow T\hat{i}\cdot\hat{i} + k\delta(-\hat{i})\cdot\hat{i} + N_1\hat{j}\cdot\hat{i} + mg(-\hat{j})\cdot\hat{i} = \vec{0}\cdot\hat{i}$

$\Rightarrow T - k\delta = 0 \Rightarrow T = k\delta$ (3)

for m_2, $\sum_i \vec{F}_i = \vec{0} \Rightarrow T\hat{j} + m_2 g(-\hat{j}) = \vec{0}$ (2)

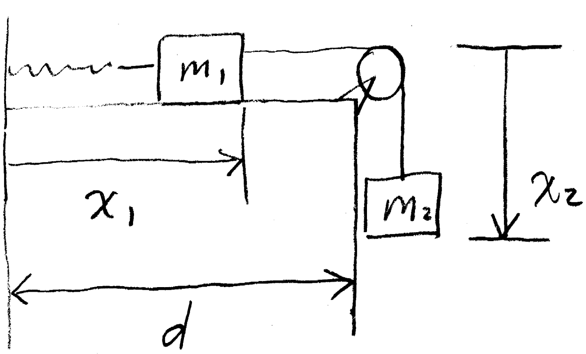
$(2)\cdot\hat{j} \Rightarrow T\hat{j}\cdot\hat{j} + m_2 g(-\hat{j})\cdot\hat{j} = \vec{0}\cdot\hat{j}$

$\Rightarrow T - m_2 g = 0 \Rightarrow T = m_2 g$ (4)

$(3),(4) \Rightarrow k\delta = m_2 g \Rightarrow \delta = \frac{m_2 g}{k}$

Thus, $\delta = \frac{20\,kg \cdot 9.81\,N/kg}{1000\,N/m} = \underline{0.1962\,m}$

Next, when m_2 is pulled downward 0.01 m further



let L be the total length of the rope. Then we have

$L = (d + x_2) - x_1$

$\xrightarrow[\text{$L, d$ are constant}]{} \cancel{\dot{L}}^{0} = \cancel{\dot{d}}^{0} + \dot{x}_2 - \dot{x}_1$

$\Rightarrow 0 = \dot{x}_2 - \dot{x}_1 \Rightarrow \underline{\dot{x}_1 = \dot{x}_2}$

Now for m_1, $\sum_i \vec{F}_i = m_1 \vec{a}_1$

$$\Longrightarrow T\hat{\imath} + K\delta(-\hat{\imath}) + N_1\hat{\jmath} + mg(-\hat{\jmath}) = m_1\ddot{x}_1\hat{\imath} \quad (5)$$

$$(5)\cdot\hat{\imath} \Rightarrow T\hat{\imath}\cdot\hat{\imath} + K\delta(-\hat{\imath})\cdot\hat{\imath} + N_1\hat{\jmath}\cdot\hat{\imath} + mg(-\hat{\jmath})\cdot\hat{\imath} = m_1\ddot{x}_1\hat{\imath}\cdot\hat{\imath}$$

$$\Rightarrow T - K\delta = m_1\ddot{x}_1 \quad (6)$$ (Note: now $\underline{\delta = 0.1962\,m + 0.01\,m}$)

for m_2, $\sum_i \vec{F}_i = m_2\vec{a}_2$

$$\Longrightarrow T\hat{\jmath} + m_2 g(-\hat{\jmath}) = m_2\ddot{x}_2(-\hat{\jmath}) \quad (7)$$

$$(7)\cdot\hat{\jmath} \Rightarrow T\hat{\jmath}\cdot\hat{\jmath} + m_2 g(-\hat{\jmath})\cdot\hat{\jmath} = m_2\ddot{x}_2(-\hat{\jmath})\cdot\hat{\jmath}$$

$$\Rightarrow T - m_2 g = -m_2\ddot{x}_2$$

$$\Rightarrow T = m_2 g - m_2\ddot{x}_2$$

Substitute this to (6):

$$(m_2 g - m_2\ddot{x}_2) - K\delta = m_1\ddot{x}_1$$

$$\Longrightarrow m_2 g - m_2\ddot{x}_1 - K\delta = m_1\ddot{x}_1$$

$\ddot{x}_1 = \ddot{x}_2$

$$\Longrightarrow \boxed{\ddot{x}_1 = \frac{m_2 g - K\delta}{m_1 + m_2}}$$

Thus, $$\ddot{x}_1 = \ddot{x}_2 = \frac{20\,kg \cdot 9.81\,N/kg - 1000\,N/m \cdot (0.1962\,m + 0.01\,m)}{10\,kg + 20\,kg}$$

$$= -0.3333\ m/s^2$$

$$\underline{\vec{a}_2 = -0.3333\ m/s^2\,(-\hat{\jmath}) = 0.3333\ m/s^2\,\hat{\jmath}}$$

□

Solution (b)

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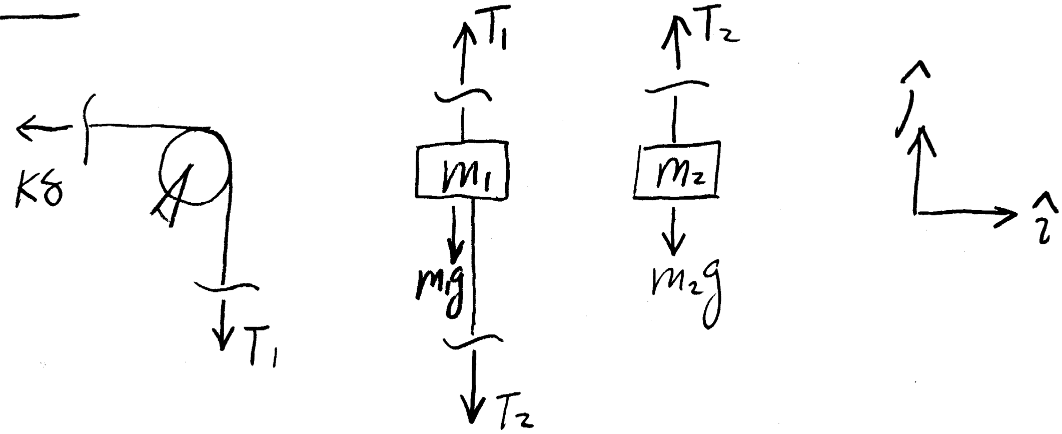


Note: δ is the extension of the spring.
Since the pulley is massless, the tension on both sides of the pulley are equal, i.e., $\underline{T_1 = K\delta}$.

When the system is at rest

for m_1, $\sum_i \vec{F}_i = \vec{0} \Rightarrow T_1\hat{j} + m_1 g(-\hat{j}) + T_2(-\hat{j}) = \vec{0}$ (1)

$(1)\cdot\hat{j} \Rightarrow T_1\hat{j}\cdot\hat{j} + m_1 g(-\hat{j})\cdot\hat{j} + T_2(-\hat{j})\cdot\hat{j} = \vec{0}\cdot\hat{j}$

$\Rightarrow T_1 - m_1 g - T_2 = 0$

$\Rightarrow K\delta = T_1 = m_1 g + T_2$ (2)

for m_2, $\sum_i \vec{F}_i = \vec{0} \Rightarrow T_2\hat{j} + m_2 g(-\hat{j}) = \vec{0}$ (3)

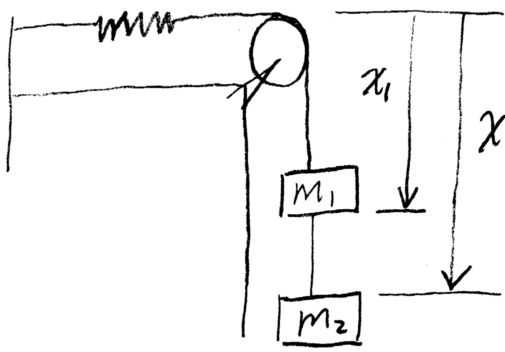
$(3)\cdot\hat{j} \Rightarrow T_2\hat{j}\cdot\hat{j} + m_2 g(-\hat{j})\cdot\hat{j} = \vec{0}\cdot\hat{j}$

$\Rightarrow T_2 - m_2 g = 0 \Rightarrow T_2 = m_2 g$ (4)

Substitute (4) into (2) $\Rightarrow K\delta = m_1 g + m_2 g$

$$\Rightarrow \delta = \frac{(m_1 + m_2)g}{K} = \frac{(10\,kg + 20\,kg)\cdot 9.81\,N/kg}{1000\,N/m} = \underline{0.2943\,m}$$

When m_2 is pulled downward 0.01 m further



Let L be the length of the rope btw m_1 and m_2. Then

$$L = x_2 - x_1$$

$$\Longrightarrow \underbrace{\ddot{L}}_{\text{L is constant}}{}^{0} = \ddot{x}_2 - \ddot{x}_1$$

$$\Longrightarrow \ddot{x}_1 = \ddot{x}_2$$

For m_1, $\sum_i \vec{F}_i = m_1 \vec{a}_1$

$$\Longrightarrow T_1 \hat{j} + m_1 g(-\hat{j}) + T_2(-\hat{j}) = m_1 \ddot{x}_1(-\hat{j}) \quad (5)$$

$$(5)\cdot\hat{j} \Rightarrow T_1 \hat{j}\cdot\hat{j} + m_1 g(-\hat{j})\cdot\hat{j} + T_2(-\hat{j})\cdot\hat{j} = m_1 \ddot{x}_1(-\hat{j})\cdot\hat{j}$$

$$\Rightarrow T_1 - m_1 g - T_2 = -m_1 \ddot{x}_1 \quad (7)$$

For m_2, $\sum_i \vec{F}_i = m_2 \vec{a}_2$

$$\Longrightarrow T_2 \hat{j} + m_2 g(-\hat{j}) = m_2 \ddot{x}_2(-\hat{j}) \quad (6)$$

$$(6)\cdot\hat{j} \Rightarrow T_2 \hat{j}\cdot\hat{j} + m_2 g(-\hat{j})\cdot\hat{j} = m_2 \ddot{x}_2(-\hat{j})\cdot\hat{j}$$

$$\Longrightarrow T_2 - m_2 g = -m_2 \ddot{x}_2$$

$$\Longrightarrow T_2 = m_2 g - m_2 \ddot{x}_2 \xrightarrow[\ddot{x}_1 = \ddot{x}_2]{} T_2 = m_2 g - m_2 \ddot{x}_1 \quad (8)$$

Substitute (8) to (7) $\Longrightarrow T_1 - m_1 g - (m_2 g - m_2 \ddot{x}_1) = -m_1 \ddot{x}_1$

$$\xrightarrow[T_1 = k\delta]{} \boxed{\ddot{x}_1 = -\frac{k\delta - (m_1 + m_2)g}{m_1 + m_2}}$$

Thus, $\ddot{x}_1 = \ddot{x}_2 = -\dfrac{1000\,N/m\,(0.2943\,m + 0.01\,m) - (10\,kg + 20\,kg)\cdot 9.81\,N/kg}{10\,kg + 20\,kg} = -0.3333\ m/s^2$

$$\vec{a}_2 = -0.3333\ m/s^2(-\hat{j}) = 0.3333\ m/s^2\,\hat{j}$$

$\square$

3.1.34 If a car decelerates at 1.1g, how many car length will it travel if it decelerates from 60 mph to zero? (the car length is 15 ft.)

Solution This is a 1-D ("straight line") motion problem. Since the acceleration is constant (1.1g) we have

$$v^2(t_2) - v^2(t_1) = 2a(x(t_2) - x(t_1))$$

$$v(t_1) = 60 \text{ mph} \approx 88 \text{ ft/s}$$

$$v(t_2) = 0$$

$$\Longrightarrow x(t_2) - x(t_1) = \frac{v^2(t_2) - v^2(t_1)}{2a}$$

$$= \frac{(88 \text{ ft/s})^2 - 0^2}{2 \cdot 1.1 \cdot 32.2 \text{ ft/s}^2}$$

$$\approx 109 \text{ ft}$$

$$\# \text{ of car length} = \frac{109 \text{ ft}}{15 \text{ ft}} \approx 7.3$$

$\square$