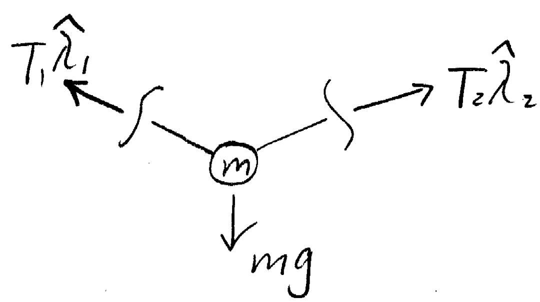
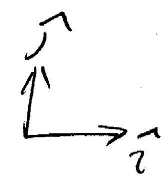
ENGRD/TAM 203
HW9 (Assigned on Feb 21, due on Feb 28)
Solution by Dennis Yang

3.2.18 A mass m is suspended as shown. At $t=0$ the string attached at B is cut.

a) What was the tension in the strings before the cut?

Solution FBD

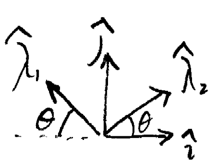




m is initially at rest: $\sum_i \vec{F}_i = \vec{0}$

$$\Longrightarrow \quad T_1\hat{\lambda}_1 + T_2\hat{\lambda}_2 + mg(-\hat{j}) = \vec{0} \qquad (*)$$

$$(*)\cdot\hat{i} \Longrightarrow \quad T_1\hat{\lambda}_1\cdot\hat{i} + T_2\hat{\lambda}_2\cdot\hat{i} + mg(-\hat{j})\cdot\hat{i} = \vec{0}\cdot\hat{i}$$



$$\Longrightarrow \quad T_1\cos(180^\circ-\theta) + T_2\cos\theta = 0$$

$$\Longrightarrow \quad -T_1\cos\theta + T_2\cos\theta = 0$$

$$\Longrightarrow \quad T_1 = T_2$$

$$(*)\cdot\hat{j} \Longrightarrow \quad T_1\hat{\lambda}_1\cdot\hat{j} + T_2\hat{\lambda}_2\cdot\hat{j} + mg(-\hat{j})\cdot\hat{j} = \vec{0}\cdot\hat{j}$$

$$\Longrightarrow \quad T_1\cos(90^\circ-\theta) + T_2\cos(90^\circ-\theta) - mg = 0$$

$$\underset{T_1=T_2}{\Longrightarrow} \quad 2T_1\sin\theta = mg \Longrightarrow \boxed{T_1 = T_2 = \frac{mg}{2\sin\theta}}$$

b) What is the velocity of the mass just after the cut?

Solution No infinitely large force

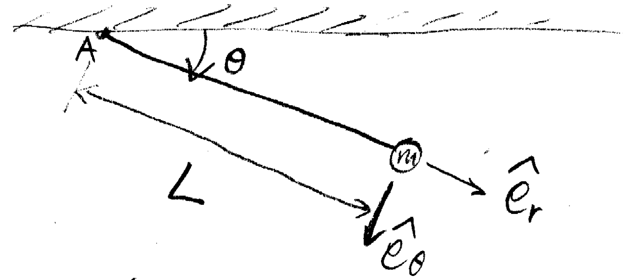
$\Longrightarrow$ No infinitely large acceleration

$\Longrightarrow$ the velocity of m is continuous

$\Longrightarrow$ the velocity of m is $\vec{0}$ just after the cut since m is at rest before the cut.

c) What is the direction of the mass's acceleration just after the cut?

Solution



$$\vec{a}_m = \vec{a}_{m/A} + \vec{a}_A \quad (\vec{a}_A = \vec{0} \text{ since } A \text{ is a fixed point!})$$

$$\Longrightarrow \vec{a}_m = \vec{a}_{m/A} = (\ddot{L} - L\dot{\theta}^2)\hat{e}_r + (L\ddot{\theta} + 2\dot{L}\dot{\theta})\hat{e}_\theta$$

L is constant $\Longrightarrow \dot{L} = \ddot{L} \equiv 0$ (all the time!)

the velocity of m is still $\vec{0}$ just after the cut

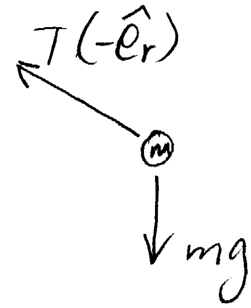
$\Longrightarrow \dot{\theta} = 0$ (at this moment!)

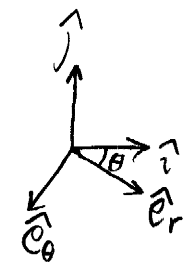
Thus, $\boxed{\vec{a}_m = L\ddot{\theta}\,\hat{e}_\theta}$ (The acceleration is in $\hat{e}_\theta$ direction)

d) What is the tension in the uncut string just after the cut? ($L = 2m$)

Solution

FBD:





$$\sum_i \vec{F}_i = m\vec{a}_m$$

$$\Rightarrow T(-\hat{e}_r) + mg(-\hat{j}) = m(L\ddot{\theta}\,\hat{e}_\theta) \quad (**)$$

$$(**)\cdot\hat{e}_r \Rightarrow T(-\hat{e}_r)\cdot\hat{e}_r + mg(-\hat{j})\cdot\hat{e}_r = mL\ddot{\theta}\,\hat{e}_\theta\cdot\hat{e}_r$$

$$\Rightarrow -T + mg\cos(90^\circ - \theta) = 0$$

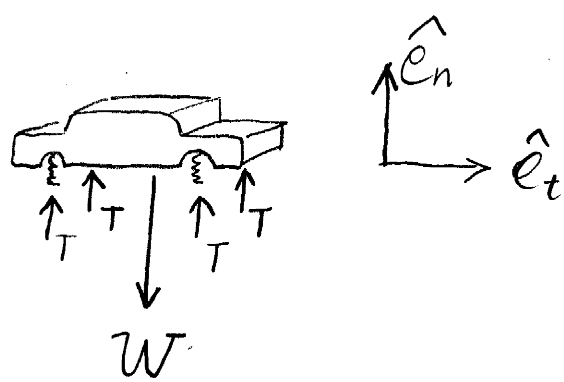
$$\Rightarrow -T + mg\sin\theta = 0$$

$$\Rightarrow \boxed{T = mg\sin\theta}$$

$\square$

3.3.10 A car is traveling at 20 mph and is at the bottom of a dip (with radius of curvature 50 ft). Determine the deflection of the four identical springs (under the car) from their unloaded state. The car's weight supported by the springs is 3600 lbf. The spring constant is 800 lbf/in.

Solution FBD



$$\sum_i \vec{F}_i = m\vec{a} \Longrightarrow 4\cdot T\hat{e}_n + W(-\hat{e}_n) = m\frac{v^2}{r_c}\hat{e}_n + m\overset{0}{\cancel{\dot{v}}}\hat{e}_t$$

$$\Longrightarrow (4T - W)\hat{e}_n = m\frac{v^2}{r_c}\hat{e}_n \quad (*)$$

$$(*)\cdot\hat{e}_n \Longrightarrow 4T - W = m\frac{v^2}{r_c}$$

$$\Longrightarrow T = \frac{1}{4}\left(m\frac{v^2}{r_c} + W\right)$$

Let δ be the deflection then, $\delta = \frac{T}{k} = \frac{1}{4k}\left(m\frac{v^2}{r_c} + W\right)$

$$V = 20\,\text{mph} \times 1.4667\,\frac{\text{ft/s}}{\text{mph}} \approx 29.3\ \text{ft/s}$$

$$m = \frac{3600\ \text{lbf}}{32.2\ \text{ft/s}^2} \approx 111.8\ \text{slg}$$

Thus, $$\delta = \frac{1}{4 \times 800\ \text{lbf/in}}\left(111.8\ \text{slg} \times \frac{(29.3\ \text{ft/s})^2}{50\ \text{ft}} + 3600\ \text{lbf}\right)$$

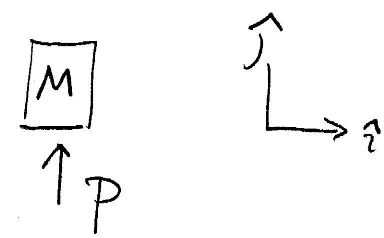
$$\boxed{\delta \approx 1.73\ \text{in}}$$

$\square$

3.5.1 As shown below, a wedge is struck by a 7 lbm maul. The maul is at 30 mph initially and comes to rest 0.21 s after the strike. What is the average force exerted by the wedge against the maul? (Ignore the linear impulse due to gravity)

Solution

FBD.



$$M\vec{v}(t_2) = M\vec{v}(t_1) + \vec{P}, \quad \vec{v}(t_2) = \vec{0}$$

$$\Rightarrow \quad \vec{0} = M\vec{v}(t_1) + \vec{P}$$

$$\Rightarrow \quad \vec{0} = M v(t_1)(-\hat{j}) + P\hat{j} \quad (*)$$

$$(*)\cdot\hat{j} \Rightarrow \quad \vec{0}\cdot\hat{j} = M v(t_1)(-\hat{j})\cdot\hat{j} + P\hat{j}\cdot\hat{j}$$

$$\Rightarrow \quad 0 = -M v(t_1) + P$$

$$\Rightarrow \quad P = M v(t_1)$$

$$\xrightarrow[P = F_{ave}\cdot\Delta t]{} \quad F_{ave} = \frac{M v(t_1)}{\Delta t} = \frac{\frac{7\ \text{lbm}}{32.2\ \text{ft/s}^2}\cdot 30\ \text{mph}\cdot 1.4667\ \frac{\text{ft/s}}{\text{mph}}}{0.21\ \text{s}}$$

$$\boxed{F_{ave} \approx 45.5\ \text{lbf}, \quad \vec{F}_{ave} = 45.5\ \text{lbf}\ \hat{j}}$$

□