Your Name: STAFF

"SOLUTIONS"

TA name and section time: ____________

T&AM 203 Prelim 2

Tuesday April 18, 2006, 2006

Draft April 18, 2006

3 problems, 25 points each, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problems.

b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- reasonable justification, enough to distinguish an informed answer from a guess, is given; you clearly state any reasonable assumptions if a problem seems **poorly defined**;
- work is I.) neat,
  II.) clear, and
  III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	4:	/25
Problem	5:	/25
Problem	6:	/25

$$\underline{\mathbf{v}}_B = \underline{\mathbf{v}}_A + \underline{\boldsymbol{\omega}} \times \underline{\mathbf{r}}_{B/A} + \underline{\mathbf{v}}_{rel} \qquad \underline{\mathbf{a}}_B = \underline{\mathbf{a}}_A + \underline{\boldsymbol{\omega}} \times (\underline{\boldsymbol{\omega}} \times \underline{\mathbf{r}}_{B/A}) + \dot{\underline{\boldsymbol{\omega}}} \times \underline{\mathbf{r}}_{B/A} + 2\,\underline{\boldsymbol{\omega}} \times \underline{\mathbf{v}}_{rel} + \underline{\mathbf{a}}_{rel}$$

$$\frac{1}{\rho} = \frac{y''}{(1+y'^2)^{3/2}}$$

ρ = radius of curvature

1) (29 pt) A car with mass m_1 moving at v_1 crashes into the rear of a stationary car with mass m_2 and sticks to it. The duration of the crash is Δt after which the cars move together. Give all answers in terms of m_1, m_2, v_1 and Δt. [Later work may not be graded if it depends on incorrect earlier work].

a) (15 points) Please re-read the rules at the front of the exam. How fast are the cars moving after the crash?

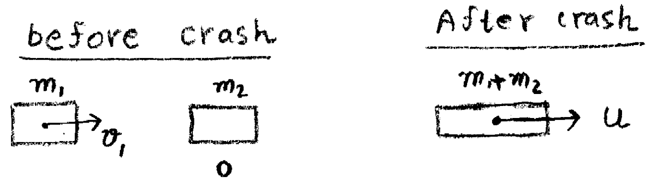
b) (5 points) What is F (the force of car 2 on car 1) during the crash, assuming F is constant in this time interval.

c) (3 points) Given v_1 and m_1 consider a range of cars that might be hit by car 1. For what mass m_2 car is its acceleration during the crash the biggest compared to all other possible cars? (Answers of the form $m_2 \to 0$ or $m_2 \to \infty$ are acceptable.)

d) (3 points) Like part (c), for what mass m_2 car is its final kinetic energy maximum (that is, more than the kinetic energy of any other car with a different m_2)?

e) (3 points) Like parts (c) and (d) for what mass m_2 car is the total crash energy dissipation maximum (that is, more dissipation than for all other m_2)?

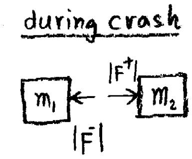
a)



Taking m_1, m_2 as system

momentum before crash = momentum after crash

$$\Rightarrow \quad m_1 v_1 + 0 = (m_1 + m_2)u \quad \Rightarrow \quad \boxed{u = \frac{m_1 v_1}{m_1 + m_2}}$$



b) For mass m_1

$$I = F\Delta t = \Delta P = m_1 u - m_1 v_1 = \frac{m_1^2 v_1}{m_1 + m_2} - m_1 v_1 = -\frac{m_1 m_2 V_1}{m_1 + m_2}$$

$$\boxed{F = -\frac{m_1 m_2 V_1}{(m_1 + m_2)\Delta t}}$$

$\because |F^-| = |F^+|$

c) acceleration of m_2 during crash $= \dfrac{F^+}{m_2} = \dfrac{m_1 V_1}{(m_1 + m_2)\Delta t}$ this is max when $m_1 + m_2$ is min

$\Rightarrow \boxed{m_2 \to 0}$

d) Final KE of $m_2 = \dfrac{m_2 u^2}{2} = \dfrac{1}{2} m_2 \left(\dfrac{m_1 v_1}{m_1 + m_2}\right)^2 = \dfrac{m_1^2 v_1^2}{2}\left[\dfrac{m_2}{(m_1 + m_2)^2}\right]$

for max KE $\quad \dfrac{d(KE)}{dm_2} = 0 \quad \Rightarrow \quad (m_1 + m_2)^2 - 2(m_1 + m_2) = 0$

$\Rightarrow \quad (m_1 + m_2)(m_2 - m_1) = 0$

$\Rightarrow \quad \boxed{m_2 = m_1}$

e) Total crash energy dissipation $= \text{Initial}_{KE} - \text{Final}_{KE} = \frac{1}{2} m_1 v_1^2 - \frac{1}{2}(m_1 + m_2)u^2$

$= \frac{1}{2} m_1 v_1^2 - \frac{1}{2}(m_1 + m_2)\left(\dfrac{m_1 v_1}{m_1 + m_2}\right)^2$

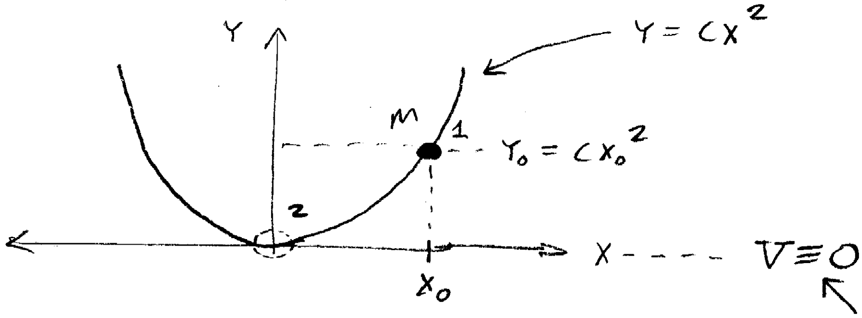
for max dissipation

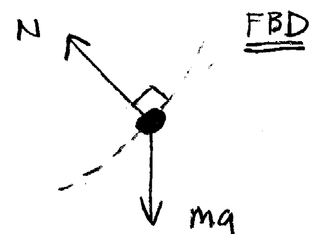
$\dfrac{1}{2} \dfrac{m_1^2 v_1^2}{m_1 + m_2}$ is min $\Rightarrow$ $m_1 + m_2$ is max

$\Rightarrow \boxed{m_2 \to \infty}$

2) (27 pt) A particle with mass m slides with no friction in a parabolic trough that is described with the equation $y = cx^2$. Equivalently you could think of a bead on a wire. Gravity g points in the negative y direction. The bead is released from rest at $x = x_0$. Find the force of the trough/wire on the mass/bead when it reaches $x = y = 0$. Answer in terms of some or all of x_0, c, g, m, $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.

The problem is set-up as



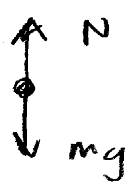


There is NO FRICTION. Use CONSERVATION OF ENERGY.

$$T_1 + V_1 = T_2 + V_2 \Rightarrow 0 + \cancel{m}g(cx_0^2) = \frac{1}{2}\cancel{m}V_2^2$$

$(x,y) = (x_0, cx_0^2)$ $\quad$ $(x,y) = (0,0)$ $\quad \Rightarrow \boxed{V_2 = x_0\sqrt{2gc}}$

When $x = 0, y = 0$ the FBD is



$$\Rightarrow (\Sigma F) \cdot \hat{n} = \boxed{N - mg = m\frac{V_2^2}{\rho|_{x=0}}}$$

Solving for N gives

$$N = mg + m\frac{2gcx_0^2}{\rho|_{x=0}} = \boxed{mg\left(1 + \frac{2cx_0^2}{\rho|_{x=0}}\right)}$$

Finally from the equation on the front page of the test

$$\rho|_{x=0} = \left.\frac{(1 + y'^2)^{3/2}}{y''}\right|_{x=0} = \frac{(1 + 0)^{3/2}}{2c} = \boxed{\frac{1}{2c}}$$

Thus

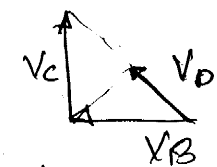
$$\boxed{N = mg(1 + 4c^2x_0^2)}$$

3) (27 pt) A motor drives link $\mathcal{A}$ at given constant $\omega_{\mathcal{A}}$. All three links are equal length ℓ. All questions concern the velocities and accelerations when the system is passing through the configuration shown.

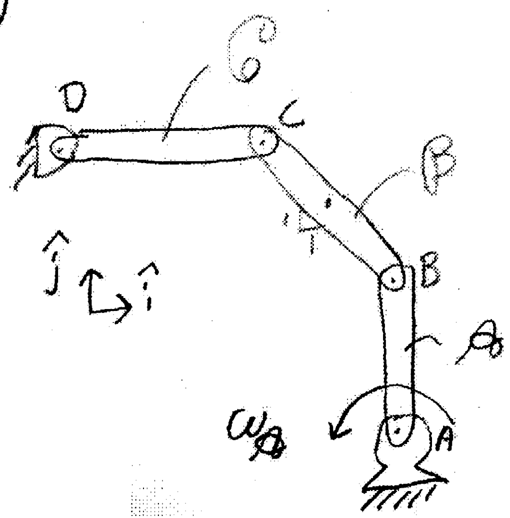
a) (9 points) What is the angular velocity of link $\mathcal{B}$.

b) (8 points) On figure (b) draw in, as accurately as you can, the velocities of points B, C and D. The velocity of point B is drawn for you. This problem will be graded independently of problem (a) and your reasoning can be based on equations or any thing else.

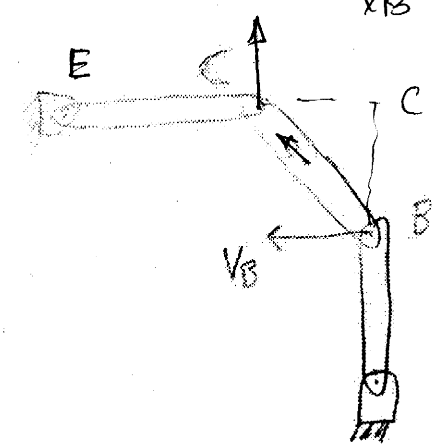
c) (8 points) Write out, but do not solve, one or more vector equations from which you could find the angular acceleration of $\mathcal{B}$. Clearly indicate which terms are known and which unknown in your equation(s) and explain how the number of equations match the number of unknowns. Expressions like $\underline{\mathbf{r}}_{B/A}$ should be evaluated in terms of ℓ, $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.



a)



b)



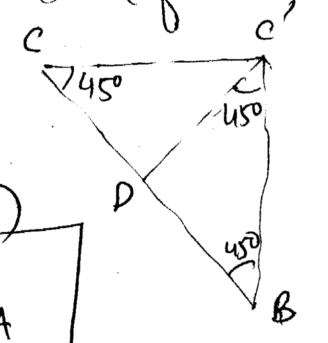
Method 2

V_B is given to be towards left.

C can only move up or down. If it moves down the rod CB needs to compress or buckle which is not possible (rigid body).

Hence V_C is upward.

Drawing perpendiculars to V_C and V_B we get c' to be the instantaneous center of rotation



$r_{cc'} = r_{c'B} = \ell/\sqrt{2}$

$V_c = \omega_B r_{cc'} = \omega_B r_{c'B} = V_B$

Part (a)

$\Rightarrow \boxed{\omega_B = \frac{V_B}{r_{c'B}} = \frac{\omega_A \ell}{\ell/\sqrt{2}} = \sqrt{2}\,\omega_A}$

$$\boxed{V_B = \omega_A l}$$ (rotation about point A)

V_D needs to be perpendicular to ~~$r_{c/D}$~~ $\vec{r}_{c'/D}$ hence it is along the rod BC.

$$V_D = \omega_B r_{C'/D} = \sqrt{2}\,\omega_A \cdot \frac{l}{2}$$

$$\boxed{V_D = \frac{\omega_A l}{\sqrt{2}}}$$

$$\boxed{V_C = V_B = \omega_A l}$$

~~$a_E \; a_C = a_0 + a_{C/E} + a_{B/C} + a_{B/A}$~~

$$\underline{a}_E \Rightarrow \underline{0} = \cancelto{0}{\underline{a}_A} + \underline{a}_{B/A} + \underline{a}_{C/B} + \underline{a}_{E/C}$$

$\underline{\alpha}_A \times \underline{r}_{B/A} + \underline{\omega}_A \times (\underline{\omega}_A \times \underline{r}_{B/A})$

$\underline{\alpha}_B \times \underline{r}_{B/A} + \underline{\omega}_B \times (\underline{\omega}_B \times \underline{r}_{C/B})$

to be determined

$\underline{\alpha}_C \times \underline{r}_{E/C} + \underline{\omega}_C \times (\underline{\omega}_C \times \underline{r}_{E/C})$

$\underline{r}_{B/A} = l\hat{j}$

$\underline{r}_{C/B} = \frac{l}{\sqrt{2}}(-\hat{i} + \hat{j})$

$\underline{r}_{D/C} = -l\hat{i}$

$\underline{\alpha}_A = 0$

hence we get 2 eqns for two unknowns α_B and α_C