PASSIVE WALKING WITH KNEES

Tad McGeer
School of Engineering Science
Simon Fraser University
Burnaby, British Columbia, Canada V5A 1S6
TMCG@SFU.MAILNET, Tad_McGeer@cc.sfu.ca

Abstract

Passive dynamic walking, a phenomenon originally described for bipeds having straight legs, also works with knees. Thus given only a downhill slope as a source of energy, a human-like pair of legs will settle into a natural gait generated by passive interaction of gravity and inertia. No muscular input is required. The physics is much the same as in straight-legged walking, but the knee-jointed form has two advantages. First, it offers a simple solution to the problem of foot clearance during the recovery phase. Second, in some cases it is more stable.

Straight legs vs knees

Walking can be a natural mode of a simple mechanical device; it does not require the continuous forcing which is used almost invariably in robot design. This is the essence of ***passive dynamic walking***, which I reported at the 1989 Robotics & Automation Conference [McGeer 89]. Similar ideas have recently been developed for hopping and running as well [Thompson 89], [McGeer 90*b*]. The simplest demonstration is provided by a mechanism consisting only of two rigid legs hinged at the hip. Such a device will walk all by itself on a downhill slope. The same mechanism can also be made to walk on the level and uphill by "pumping" the passive motion, and the gait can be modulated to vary footfalls from one step to the next. Thus the passive walking effect can serve as a foundation for dextrous and efficient legged locomotion.

Here we add knees to the catalogue of design options. Our motivation is twofold. First, knees make the gait more anthropomorphic, which is aesthetically pleasing and also useful for study of walking in nature. Second, knees solve the problem of toe stubbing in the so-called recovery phase, when the free leg is brought forward in preparation for the next step. Our straight-legged biped (described at the 1989 conference) had to clear its feet actively during recovery, using small retraction motors in each leg; these substantially complicated an otherwise passive design, and moreover used a lot of energy. The machine needed only about 0.2J per step (obtained by going downhill) to keep the passive cycle going; meanwhile to motors consumed 3J per step! By comparison foot clearance by knee flexure offers a solution that is passive, efficient, and reliable.

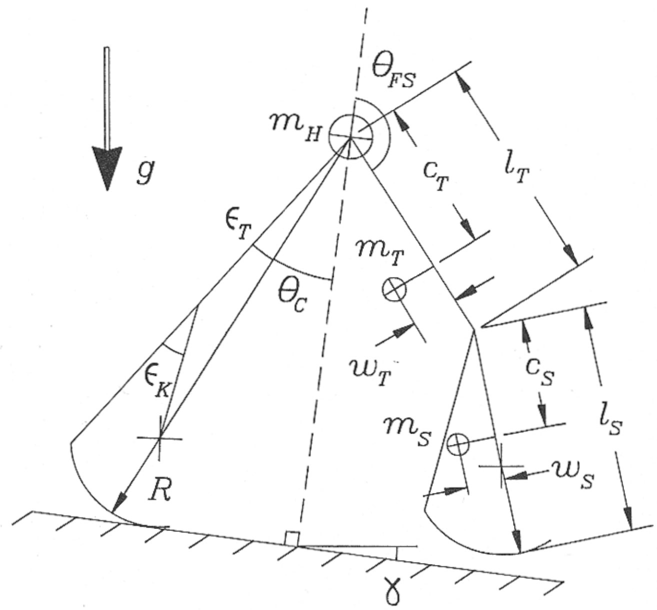

Figure 1: *A chain of four rigid, inanimate links will walk all by itself down a shallow incline. The motion is two-dimensional (i.e. confined to the plane of this drawing), but quite human-like (cf. figure 2). Links are connected by pin joints, with mechanical stops at the knees to prevent hyperextension. Here the stance knee is shown locked against its stop. Thigh and shank have inertias $m_T r_{gyr_T}$ and $m_S r_{gyr_S}$ about their respective mass centres, and the model can also include a point mass at the hip. Feet are semicircular. Many models of this general form will walk passively; geometry and mass properties are broadly adjustable.*

The model and its cycles

Figure 1 shows our model for knee-jointed walking. We confine attention to the simplest example, with no source of energy other than a downhill slope, and only a point mass at the hip to represent a torso. All of our calculations are cast in dimensionless terms, with overall mass m, extended leg length l, and gravity g providing the base units.

At first glance one might think such a contraption rather unlikely to walk all by itself. However upon more careful study [Mochon 80] found that it was capable of so-called *ballistic walking*. That is, if the limbs were given appropriate speeds and angles at the start-of-step, then they would swing passively through to heel strike in a perfectly natural style. We have found that impulse generated at heel strike can then regenerate the start-of-step conditions, thus completing a passive cycle.

Figure 2 shows an example. The step begins and ends with both feet in contact and both legs extended, which in the plot is indicated by equal and opposite leg angles. (These angles are measured from the hip to the centres of curvature of the feet. They differ by ϵ_T (figure 1) from the angles from hip to knees, which we have shown in the cartoon.) Both the midstride swing knee lock, and the end-of-step heel strike, are treated as impulsive, inelastic collisions. Therefore both produce instantaneous changes in link speeds, and in particular heel strike changes the speeds back to their start-of-step values. Then the new step begins, so to speak, right on the heels of its predecessor (*cf.* figure 9).

It may seem remarkable that such a simple mechanism should be capable of such an elaborate, and cyclic, behaviour. If so, then it must be even more remarkable that the cycle of figure 2 is not the only possibility. The same machine, on the same slope, can also walk passively using the gait of figure 3. (Straight-legged bipeds have similarly paired alternatives [McGeer 90*c*].) Of the two choices the shorter-period gaits are generally less attractive, mainly because they are almost always unstable. Furthermore they are often (though not in this example) less efficient than the longer-period gaits, that is, they need a steeper slope for the same forward speed.

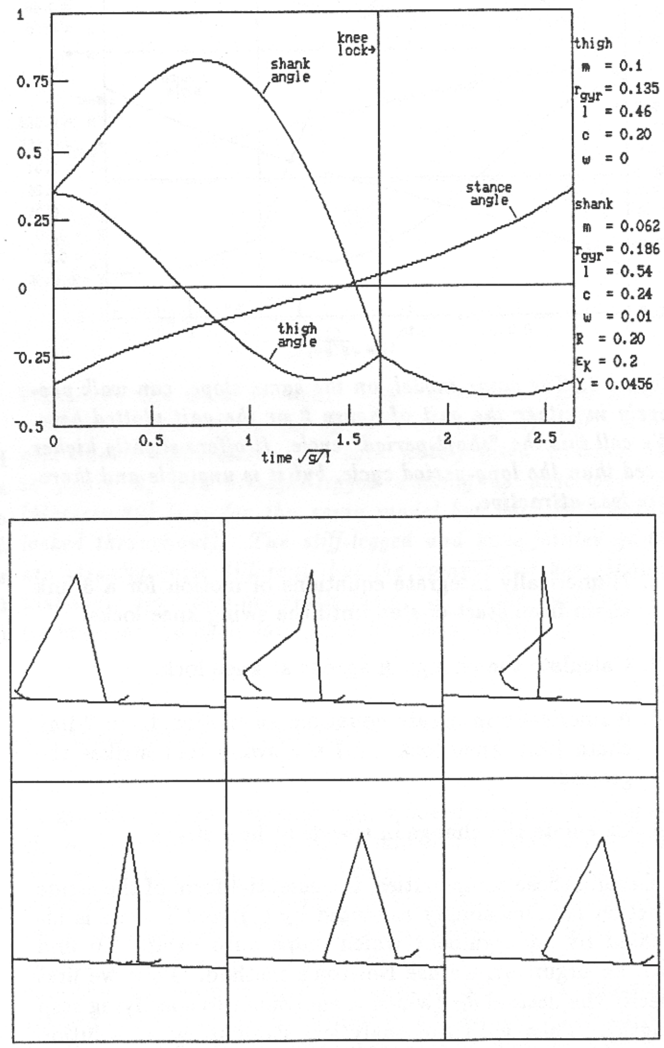

Figure 2: *A typical passive cycle for the model of figure 1. Angles in the plot are relative to the surface normal, which is tilted at 4.6% from the vertical. Thigh and shank parameters are comparable to those of a human [Chapman 83], with 68% of the overall mass concentrated at the hip. Time is in units of $\sqrt{l/g}$, l being the length of the extended leg and g the gravitational acceleration. The stance leg stays extended through the step, while the swing leg flexes through midstance and so clears the swing foot. Then it returns to full extention, hits the stop inelastically, and remains locked until the foot strikes the ground. The impulse at foot strike changes the links speeds back to their start-of-step values, and the cycle repeats.*

Calculating the cyclic motion

Steps such as those plotted in figures 2 and 3 have four independent initial conditions: the stance angle θ_C, and the rotational speeds of the stance leg (Ω_C), swing thigh (Ω_{FT}), and swing shank (Ω_{FS}). Together with the slope γ these determine the initial conditions for the next step, through some *stride function* $\vec{S}$:

$$\begin{bmatrix} \theta_C \\ \Omega_C \\ \Omega_{FT} \\ \Omega_{FS} \end{bmatrix}_{k+1} = \vec{S}(\theta_{C_k}, \Omega_{C_k}, \Omega_{FT_k}, \Omega_{FS_k}, \gamma) \qquad (1)$$

$\vec{S}$ is a fairly complicated operator; it involves integration of equations of motion from start- to end-of-step, combined with calculation of the impulsive speed changes at knee lock and heel strike. For straight-legged walking these procedures can be done analytically, since the equations of motion are amenable to linearisation. ([McGeer 89] summarises the derivation.) However in a knee-jointed step, as indicated in figure 2, the swing shank in particular reaches large angles and speeds. Hence the nonlinear dynamic effects must be retained, and $\vec{S}$ must be evaluated numerically as follows:

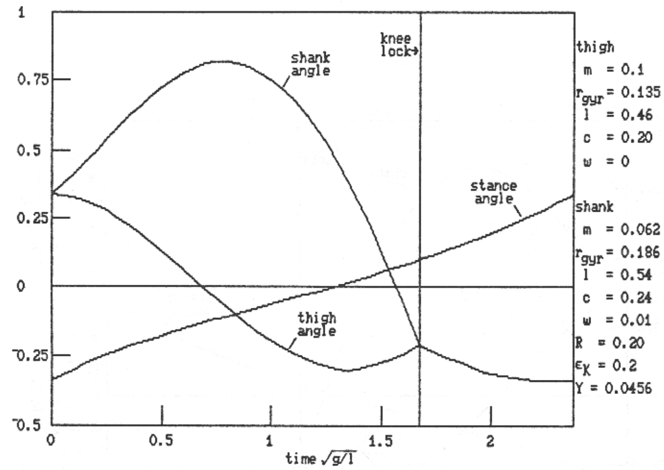

Figure 3: *The same model, on the same slope, can walk passively in either the gait of figure 2 or the gait plotted here. We call this the "short-period" cycle. It offers slightly higher speed than the long-period cycle, but it is unstable and therefore less attractive.*

1. Numerically integrate equations of motion for a 3-link chain from start-of-step until the swing knee locks.
2. Calculate the change in speeds at knee lock.
3. Numerically integrate equations of motion for a 2-link chain from knee lock until the swing foot strikes the ground.
4. Calculate the change in speeds at heel strike.

Despite these complexities, the essential form of the stride function remains simply as stated by (1), and a cycle is indicated by an argument which maps onto itself. To find such an argument we use Newton's method. Thus we first specify the desired θ_C (which is equivalent to specifying step length). Then with the analytical straight-legged solution for guidance [McGeer 90*a*], we "guess" appropriate values for the link speeds and slope, and evaluate $\vec{S}$. In general it produces an output different from the input. Newton's method therefore calls for an adjustment satisfying

$$\begin{bmatrix} \theta_C \\ \Omega_C \\ \Omega_{FT} \\ \Omega_{FS} \end{bmatrix} + \begin{bmatrix} \Delta\theta_C \\ \Delta\Omega_C \\ \Delta\Omega_{FT} \\ \Delta\Omega_{FS} \end{bmatrix} \approx \vec{S}(\theta_C, \Omega_C, \Omega_{FT}, \Omega_{FS}, \gamma) +$$

$$\left[\frac{\partial \vec{S}}{\partial \Omega_C} \;\; \frac{\partial \vec{S}}{\partial \Omega_{FT}} \;\; \frac{\partial \vec{S}}{\partial \Omega_{FS}} \;\; \frac{\partial \vec{S}}{\partial \gamma} \right] \begin{bmatrix} \Delta\Omega_C \\ \Delta\Omega_{FT} \\ \Delta\Omega_{FS} \\ \Delta\gamma \end{bmatrix} \qquad (2)$$

We evaluate the gradients numerically, and then solve for a new set of initial speeds and slope. *If* a cyclic gait exists, then this procedure converges rapidly, usually to 5 significant figures in each variable within half a dozen iterations.

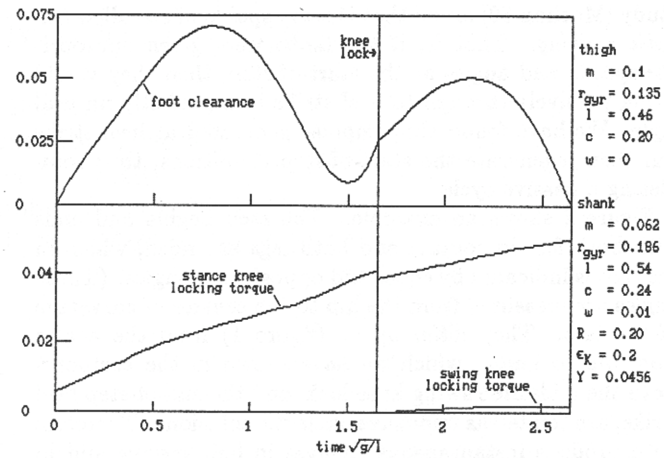

Figure 4: *Swing foot clearance and knee torques during the step of figure 2. Foot clearance is expressed as a fraction of extended leg length. The foot has a close call at midstride, but humans seem to manage with a similarly slim margin. Meanwhile naturally arising torques keep the knees locked against their stops during the appropriate parts of the step. Torque is in units of mgl, m being the total mass of the model.*

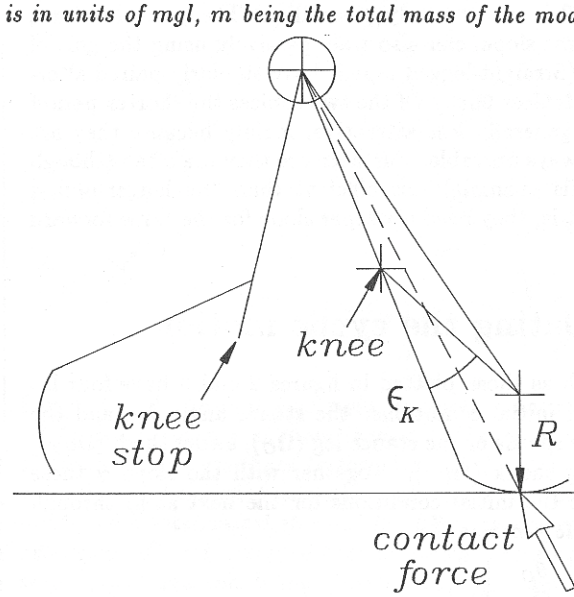

Figure 5: *The locked knee angle ϵ_K is an important design parameter. The force on the stance foot is directed roughly from the contact point to the overall mass centre, which is usually near the hip. This vector must pass in front of the knee if the contact force is to keep the stance leg locked against its stop. Hence the foot's centre of curvature must be placed well in front of the knee, which calls for a human-like geometry. Notice that as the step proceeds the force vector moves forward, and so the locking torque increases. Hence the worst case for design arises at the start of a long step.*

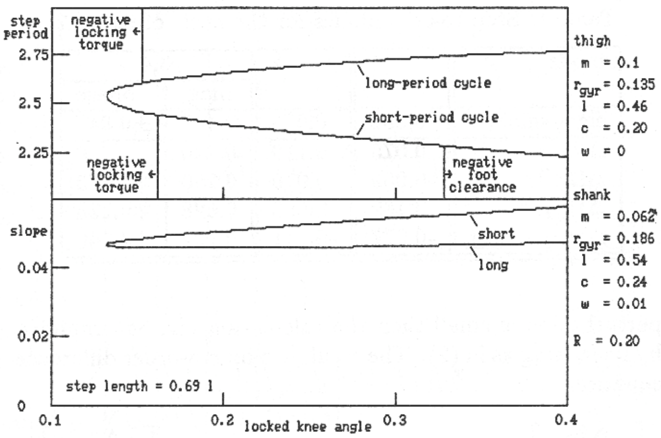

Figure 6: *The need for an anthropomorhic foot and shank is illustrated by studying models with various knee-locking angles. If the locking angle is made smaller (i.e. feet further back) then the contact force tends to flex rather than extend the stance knee. With further reduction the long- and short-period gaits merge and "pinch off"; models with feet further aft cannot walk passively. On the other hand models with feet too far forward would have trouble with toe-stubbing, particularly in the short-period gait.*

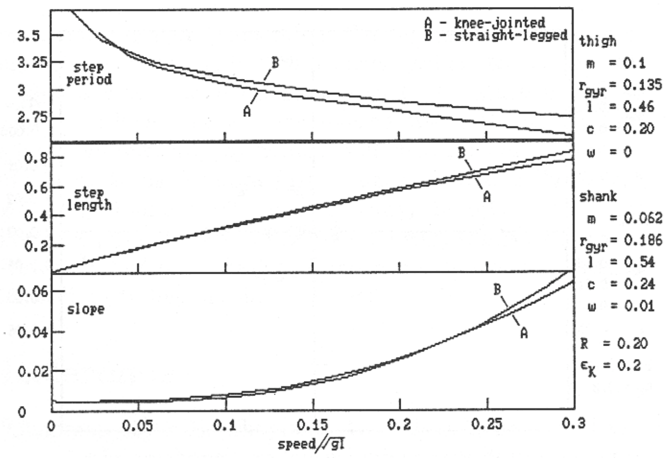

Figure 7: *To walk at higher speed a passive biped needs a steeper slope. Here the speed is in units of $\sqrt{gl}$. The plot shows calculations for both knee-jointed walking as in figure 2, and straight-legged walking as described by [McGeer 89] (i.e. for the same model but with both knees locked throughout). The stiff-legged and knee-jointed gaits are visually quite different, but the comparison here shows that their dynamics are really the same. (Note that there is no knee-jointed cycle for speeds less than $0.03\sqrt{gl}$.)*

Foot clearance and knee locking

The practical motivation for knees is swing foot clearance, and figure 4 illustrates the level of clearance offered by the passive gait. Also shown are the torques on the knee joints while the legs are extended. In this example both foot clearance and knee torques are satisfactory, but some care in foot design is required to achieve this result. In particular, here the foot radius is $0.2l$; a larger value would reduce foot clearance. (If for example the centre of curvature were coincindent with the knee, then flexure wouldn't lift the foot at all.) Thus knees only prevent toe-stubbing if the feet are not too large. Also, here the centre of curvature is placed well in front of the knee ($\epsilon_K = 0.2$). This is necessary for passive locking of the stance leg; moving the foot back would make the passive torque flexural (as suggested by figure 5) and so call for active intervention to prevent the leg from buckling. Thus an anthropomorphically asymmetric foot is obligatory for passive walking; the heel musn't stick out as far as the toe!

Actually in evaluating the stride function, our numerical integrator simply follows the procedure outlined in the preceding section, stopping only when the stance and swing angles become equal and opposite. Thus it doesn't care whether the foot clearance or locking torques become negative during the stride. But although we can therefore search, usually we cannot find cyclic gaits for models beset by such problems. Figure 6 provides an illustration: if the feet are moved back on the shank, the stance knee goes into flexure and must be locked actively. If the feet are moved further back, cyclic solutions vanish. Increasing foot radius has a similar effect.

Speed and efficiency

In gravity-powered walking higher speed requires a steeper slope. Figure 7 shows the quantitative relationship for our example model. Also shown is the behaviour of the same model in straight-legged walking (as calculated by the method of [McGeer 90*a*]). The two gaits have remarkably similar performance, which leads to the satisfying conclusion that the simpler straight-legged model goes a long way toward explaining the behaviour of a knee-jointed machine. By the same token it seems reasonable to suppose that our relatively simple knee-jointed model has much to say about walking in nature.

One might also conclude from the comparison that knee-jointed and straight-legged machines are equally efficient. However that is not quite true. A knee-jointed machine has an additional mechanism for energy dissipation, namely the collision at swing knee lock. In our example here its effects are modest, but they would be larger with different parameter choices (especially a smaller hip mass). Moreover a knee-jointed machine is constrained to use some combination of small foot radius and forward foot displacement in order to generate a positive locking torque. Either leads to a relatively hard and therefore dissipative heel strike [McGeer 90*a*]. A straight-legged designer is free of these requirements, and so can build a model with higher "fundamental" efficiency. But on the other hand he must provide for active foot clearance; in our experience this becomes so expensive in practice that knee joints win the day.

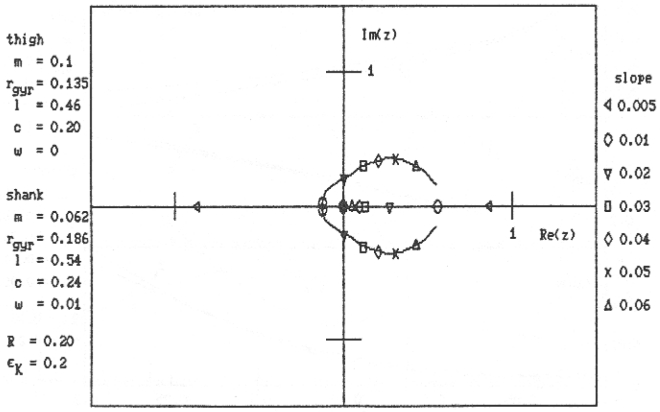

Figure 8: *Fully passive walking requires not only existence of a repetitive cycle, but also stability. Analysis of small perturbations on the cyclic motion leads to a 4^{th} order difference equation; here a locus of eigenvalues is shown for the model of figure 7. All four eigenvalues have magnitude less than unity throughout the speed range, so the model is always stable in "long-period" walking. (However the same model is unstable in "short-period" walking, and some other designs are unstable in both long and short gaits.)*

Comparison with human walking

The gait of figure 2 is obviously anthropomorphic. However the timescale is slower. My own steps usually have period less than $2.0\sqrt{l/g}$, whereas the passive model has a short period of $2.4\sqrt{l/g}$ and a long period of $2.7\sqrt{l/g}$. These figures would be slower still with an extended torso rather than a point hip mass [McGeer 90*c*]. On the other hand, [McGeer 90*c*] has shown that it can be attractive to use higher cadence when the cyclic gait is "pumped" rather than sustained by descending a slope. This accounts for some of the discrepancy between man and gravity-powered model; another portion is likely due to torsional stiffness in man's joints. You can feel an elastic effect as you stretch a knee to full extension, or swing your legs apart at the hip; [Cohen 85] and [Mansour 86] offer quantitative measurements of the torques involved. This elasticity increases the legs' pendulum frequencies, and so cadence in walking. However (at least in the straight-legged case) it also causes a harder and more dissipative heel strike. A free-jointed machine, achieving the same speed with longer steps but slower cadence, would be more efficient. But then human legs, presumably, are not made just for walking.

Stability

Practical passive walking requires not only existence of a cyclic motion, but also robustness of the cycle with respect to perturbations. The response to some perturbation can be followed from step to step using the stride function (1). If the perturbation is small then the calculation can be simplified by linearising as in (2). The result is a fourth-order difference equation:

$$\begin{bmatrix} \Delta\theta_C \\ \Delta\Omega_C \\ \Delta\Omega_{FT} \\ \Delta\Omega_{FS} \end{bmatrix}_{k+1} \approx \begin{bmatrix} \dfrac{\partial\vec{S}}{\partial\theta_C} & \dfrac{\partial\vec{S}}{\partial\Omega_C} & \dfrac{\partial\vec{S}}{\partial\Omega_{FT}} & \dfrac{\partial\vec{S}}{\partial\Omega_{FS}} \end{bmatrix} \begin{bmatrix} \Delta\theta_C \\ \Delta\Omega_C \\ \Delta\Omega_{FT} \\ \Delta\Omega_{FS} \end{bmatrix}_k \quad (3)$$

where 'Δ' indicates a perturbation from the steady-cycle initial conditions. Stability can be assessed by calculating eigenvalues and eigenvectors of the gradient matrix (which is itself found by numerical differentiation of $\vec{S}$).

Table 1: Step-to-step modes for the cycle of figure 2

mode	1	2	3,4	
			mag.	phase
eigenvalue, z	-0.001	0.073	0.447	$\pm$0.947
$\Delta\theta_C$	0.075	0.119	0.270	0
$\Delta\Omega_C$	-0.059	-0.079	0.200	$\mp$2.573
$\Delta\Omega_{FT}$	0.189	0.688	0.628	$\mp$0.323
$\Delta\Omega_{FS}$	-0.977	-0.711	0.702	$\pm$3.140

Table 1 lists the resulting "step-to-step" modes for the long-period cycle of figure 2. Perturbations in any mode scale over k steps in proportion to z^k. Here all four eigenvalues have magnitude less than unity, so the cycle is stable. In fact while the knee-jointed model might appear rickety by comparison with its straight-legged counterpart, its recovery from perturbations is actually more rapid. (In fact this particular model walking straight-legged – *i.e.* with knees locked throughout – would have an *unstable* eigenvalue at $z = -1.27$!) Furthermore knee-jointed walking remains rapidly convergent over a wide range of speeds (figure 8).

Further inspection of table 1 offers some insight into transient behaviour. If a step starts with (small) perturbations in Ω_{FT} and Ω_{FS} only, and not in the stance leg, then only the first two modes will be appreciably excited. Since these have very small eigenvalues, such perturbations are essentially eliminated in only one step. However if the perturbations include θ_C (*i.e.* wrong step length for the slope in use) or Ω_C (*i.e.* wrong speed), then the slower complex mode is excited. The subsequent recovery oscillates over several steps.

Large perturbations

Linearised stability analysis provides excellent information on the rate of recovery from perturbations, but none at all on the magnitude of perturbation which can be tolerated. To probe the limits one must resort to "exact" step-to-step evaluation of the stride function following perturbations of various size. Figure 9 shows two examples, in which the initial perturbations are scaled in proportion to the complex eigenvector of table 1. (However the perturbations are not "small", so the transient is not as predicted by the linearised

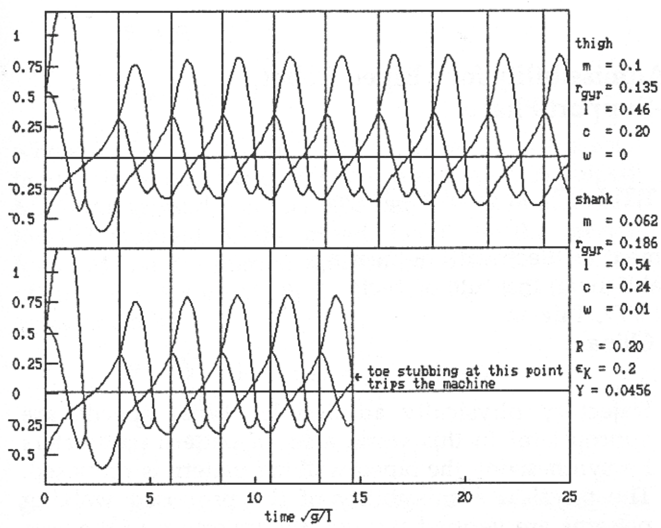

Figure 9: ***While passive walking is stable with respect to small perturbations, it can tolerate only a limited amount of abuse. These examples show leg angle transients following disturbances on the "long period" cycle (cf. figure 2 for labelling of the curves). In the first example the model recovers after a few steps. In the second, which begins with a slightly larger disturbance, the model teeters for a few steps and then collapses ignominiously on its face!***

stride function.) The machine is able to recover from fairly large disturbances, but not so large as a practical robot would likely have to handle in the wild. Active stabilisation would enhance robustness; some examples (for passive running) are given by [McGeer 90*b*].

Experiments

At the time of writing we are completing a machine for experiments in knee-jointed walking. The machine has about 4kg total mass, and 70cm leg length. Motion is kept two-dimensional by building the legs in crutch-like pairs. Piston-type oil dashpots are used for the knee stops; these ensure an inelastic collision when the swing knee locks. (The shank would bounce off a plain mechanical stop.) This part of the design requires some care, but otherwise the machine is elementary. Its simplicity, combined with the similarity between knee-jointed walking and the already-proven straight-legged form, favour a successful result. But still, as we mentioned earlier, intuition does not immediately suggest that such a contraption will walk by itself. Seeing is believing.

Nature knows best

People encountering our original straight-legged biped would often ask, "Why doesn't it have knees?" We always replied that they were not necessary; we could telescope the legs rather than fold them, and so simplify analysis as well as mechanical design. Thus might the engineer's sleek machinery soon dispense with nature's awkward contrivances.

But a closer comparison reveals that nature is not so easily outdone. Legged locomotion itself, though at first glance unimpressive by comparison with the smooth rolling of wheels, turns out upon inspection to be an equally elegant way of getting around. One is steady, the other cyclic, but both are pure physics in action. Now so it seems with knees. How many more examples of nature's dynamical sensitivity lie waiting to beguile the engineer?

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