Chapter 4

Principles of Walking and Running

T. McGeer

Contents

Symbols

Roman

c	axial position of mass centre
H	angular momentum
K	spring stiffness
l	leg length
m	mass
R	foot radius
r_{gyr}	radius of gyration
SR	specific resistance, Eq. (13)
T	torque
V	linear velocity
W	work
w	offset from link axis to mass centre

Greek

α_0	leg angle at support transfer
β	torso inclination
γ	slope, positive downhill
γ_g	slope for gravity-powered walking
$\Delta\theta$	rotation from surface normal, positive forward
$\theta_{C,T}$	stance leg angle at take-off
θ_{C*}	peak stance leg angle
η	coefficient of restitution (Eq. (12))
τ	dimensionless time $t\sqrt{g/l}$
τ_0	step period
τ_b	flight time
τ_c	contact time
Ω	angular speed
ω_F	swing pendulum frequency

Advances in Comparative and Environmental Physiology, Vol. 11

Subscripts

$-$	immediately before support transfer	C	stance leg
$+$	immediately after support transfer	F	swing leg
0	steady cycle conditions	H	at the hip

1 Why Legs?

In the early days of air travel, detractors used to argue that, if God had meant us to fly, he would have given us wings. Had he meant us to roll, he might also have given us wheels – but instead we, along with the great preponderance of land animals great and small, have wound up travelling on legs. Given that situation, it is of course interesting to ask how legs work, and that will be our main concern in this chapter. But throughout we should also wonder, why legs at all? We will make some preliminary observations now, and return to this later question after some study of leg dynamics.

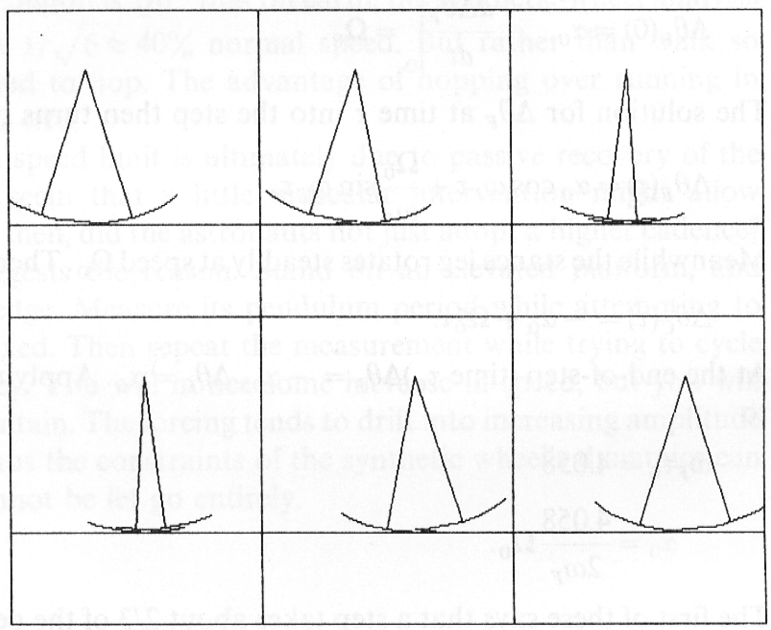
Whatever God's intentions might have been, there is no contesting that the wheel is perfect for travel on smooth terrain. Nature's failure to use it might be explained most easily by the biological impossibility of a continuously rotating joint. But that explanation is not completely satisfactory. It seems that without drastic redesign our bodies could *be* wheels, rolling about via a series of somersaults, or perhaps rotating around our long axes as might a child playing on a hillside. Alternatively, rather than either roll or walk, we could slither like a snake, an approach which, while distasteful from the theological point of view, nevertheless has a certain technical appeal. In particular, rolling and (to a lesser extent) slithering are steady motions, like those of an aeroplane, a boat, a tank, and most other man-made vehicles. To an engineer, steady motion is natural, and so legged locomotion, with its elaborate pattern of eternal disequilibrium, by contrast seems contrived and inelegant. In this light, the question "Why legs?" becomes compelling indeed.

Our first topic will suggest an answer. We will begin by demonstrating that rolling and walking are actually more alike than one might at first think. The demonstration will show that walking is not a contrived product of elaborate motor control, but rather a natural motion which needs no forcing at all. This is the essence of passive dynamic walking, which will be our theme through much of the chapter. We will move from a simple "synthetic wheel" to models having human-like form and gait. Each model will be capable of unforced locomotion, but we will also discuss how the passive motion can be pumped and modulated to produce dextrous behaviour. Later we will turn to running and to quadrupedal locomotion.

2 The Synthetic Wheel

Let us construct an elementary biped. The components are two spokes from a wagon wheel, and their accompanying pieces of rim (Fig. 1). These will be hinged at a hip joint, where we will also put a "payload" mass. The payload will be chosen large enough to put the overall mass centre practically coincident with the hip, just as in the original wagon wheel. Then if one leg is put on the ground and given a push, it will roll forward at constant speed. Meanwhile an infinitesimal shortening of the other leg will keep it clear of the ground, and so leave it dangling freely from the hip.

Now follow this assembly through the motion shown in Fig. 1. The legs begin at equal and opposite angles from the vertical, and with rotational speeds matched (i.e. rotating as if they were fixed in a complete wheel). From that point the stance



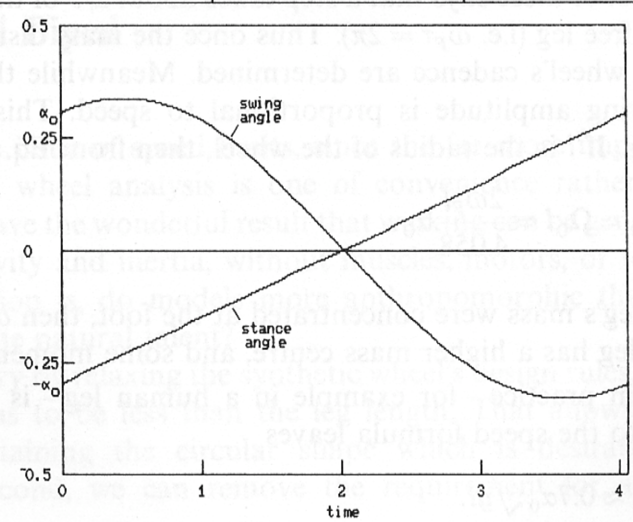


Fig. 1. The simplest of walking models is the synthetic wheel, a biped with straight legs and semicircular feet. The stance leg rolls forward steadily like a spoke in a wheel, while the free leg swings ahead like a pendulum. Support is transferred between legs when their speeds and angles match. The cycle is naturally stable and will repeat continuously, thus synthesising the motion of an ordinary wheel. Here the timescale is normalised by the swing pendulum frequency ω_F

leg continues to roll steadily, but the free leg swings along the sinusoidal trajectory of an unforced pendulum. This leads in time to a very convenient situation: the legs' speeds again become equal just when that their angles are opposite to those at the beginning of the step. If at that point the stance leg is slightly shortened, and the swing leg slightly lengthened, then support will transfer seamlessly from one leg to the next. A new step will begin, just like the last, and so on and on, the model thus rolling (or walking) continuously just like an ordinary wheel.

The synthetic wheel's cadence is determined by the pendulum frequency of its legs, say ω_F. Thus the equation of motion for the swing leg is

$$\frac{d^2\Delta\theta_F}{dt^2} + \omega_F^2\Delta\theta_F = 0, \tag{1}$$

where $\Delta\theta_F$ is the angle from the vertical. We make the approximation that $\sin\Delta\theta_F \approx \Delta\theta_F$, which certainly holds in walking. This equation has to be solved with initial conditions

$$\Delta\theta_F(0) = \alpha_0 \qquad \left.\frac{d\Delta\theta_F}{dt}\right|_0 = \Omega_0. \tag{2}$$

The solution for $\Delta\theta_F$ at time τ into the step then turns out to be

$$\Delta\theta_F(\tau) = \alpha_0 \cos\omega_F\tau + \frac{\Omega_0}{\omega_F}\sin\omega_F\tau. \tag{3}$$

Meanwhile the stance leg rotates steadily at speed Ω_0. Therefore its angle at time τ is

$$\Delta\theta_C(\tau) = -\alpha_0 + \Omega_0\tau. \tag{4}$$

At the end-of-step (time τ_0)$\Delta\theta_F = -\alpha_0$, $\Delta\theta_C = \alpha_0$. Applying these conditions leads to

$$\omega_F\tau_0 = 4.058 \tag{5}$$

$$\alpha_0 = \frac{4.058}{2\omega_F}\Omega_0. \tag{6}$$

The first of these says that a step takes about 2/3 of the period for a full oscillation of the free leg (i.e. $\omega_F\tau = 2\pi$). Thus once the mass distribution is specified, ω_F and so the wheel's cadence are determined. Meanwhile the second formula says that the swing amplitude is proportional to speed. This limits maximum speed as follows. If l is the radius of the wheel, then from Eq. (6)

$$V_0 = \Omega_0 l = \frac{2\omega_F l}{4.058}\alpha_0. \tag{7}$$

If the leg's mass were concentrated at the foot, then ω_F would be $\sqrt{g/l}$. However a real leg has a higher mass centre, and some moment of inertia as well. The net result in practice – for example in a human leg – is that $\omega_F \approx 1.4\sqrt{g/l}$. Putting this into the speed formula leaves

$$V_0 \approx 0.7\alpha_0\sqrt{gl}. \tag{8}$$

Humans seem to find walking uncomfortable with α_0 greater than about 0.4. Taking this as an upper bound implies that

$$V_{0\max} \approx 0.3\sqrt{gl}. \tag{9}$$

Notice that it would be natural in this analysis to normalise speed by $\sqrt{gl}$. Following the long-standing terminology of ship designers, the speed thus normalised is often called a Froude number. On the other hand, it is also common practice to define the Froude number as V^2/gl, so the student must be wary. Leaving the semantic issue aside, however, the essential point is that the speed of a synthetic wheel scales with $\sqrt{gl}$. If similar dynamics hold in nature, then a similar scaling law should be found. Alexander and Jayes (1983) have presented a substantial body of evidence that $\sqrt{l}$ speed scaling does indeed hold, from shrews through to elephants and possibly beyond, up to very large dinosaurs (Alexander 1976). Scaling with gravity is harder to explore, but there is striking evidence from the manned moon missions. Astronauts reported a sensation of slow motion, and indeed since gravity on the moon is $1/6^{\text{th}}$ that on earth, the synthetic-wheel analysis would predict walking at $1/\sqrt{6} \approx 40\%$ normal speed. But rather than walk so slowly, astronauts preferred to hop. The advantage of hopping over running in low-g is discussed in Sect. 4.10.

The synthetic wheel's speed limit is ultimately due to passive recovery of the swing leg, and it might seem that a little muscular intervention might allow accelerated motion. Why, then, did the astronauts not just adopt a higher cadence? A simple experiment suggests the reason. Stand on an elevated platform, and dangle one leg over the edge. Measure its pendulum period while attempting to keep the hip muscles relaxed. Then repeat the measurement while trying to cycle the leg at higher frequency. You will notice some increase in speed, but you will also find it difficult to maintain. The forcing tends to drift into increasing amplitude rather than frequency. Thus the constraints of the synthetic wheel's dynamics can be loosened, but they cannot be let go entirely.

3 The Straight-Legged Biped

One ought not dwell on the issue of speed limits, since the far more important implication of the synthetic wheel analysis is one of convenience rather than constraint. That is, we now have the wonderful result that walking can be generated purely by interaction of gravity and inertia, without muscles, motors, or forcing of any kind. The key question is, do models more anthropomorphic than the synthetic wheel have the same natural talent?

We can pursue the inquiry by relaxing the synthetic wheel's design rules. First, we can allow the foot radius to be less than the leg length. That allows for a natural foot size, while retaining the circular shape which is desirable for mathematical simplicity. Second, we can remove the requirement for a large

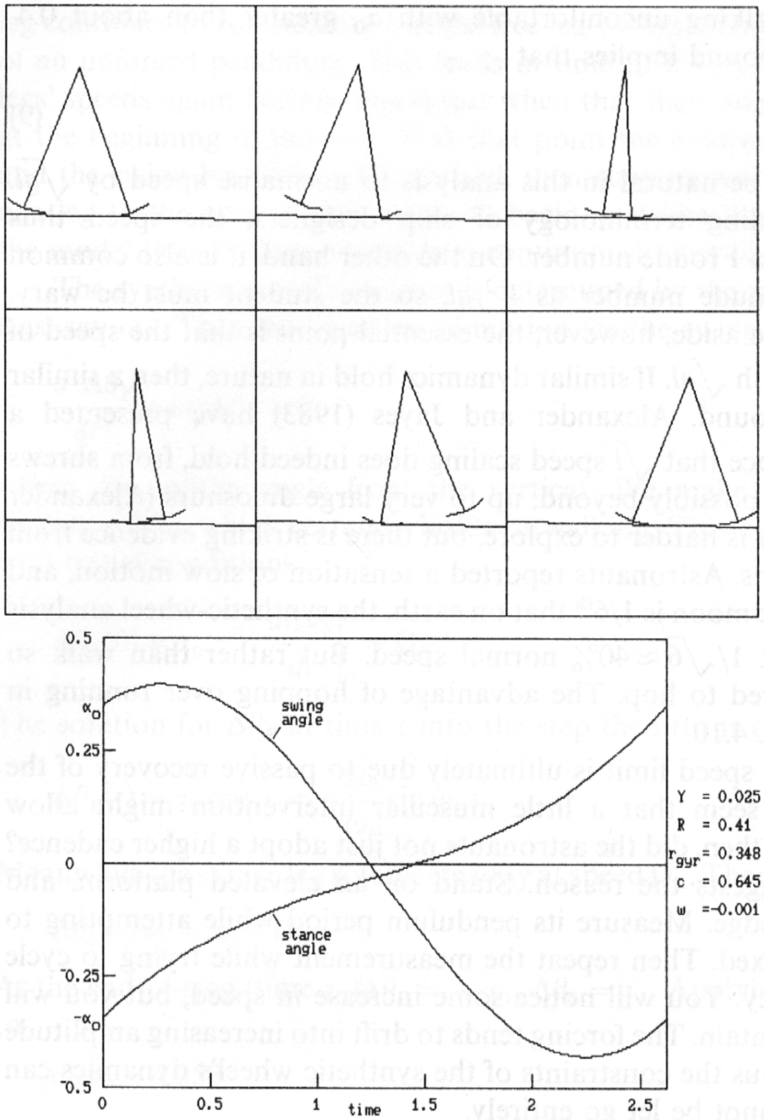
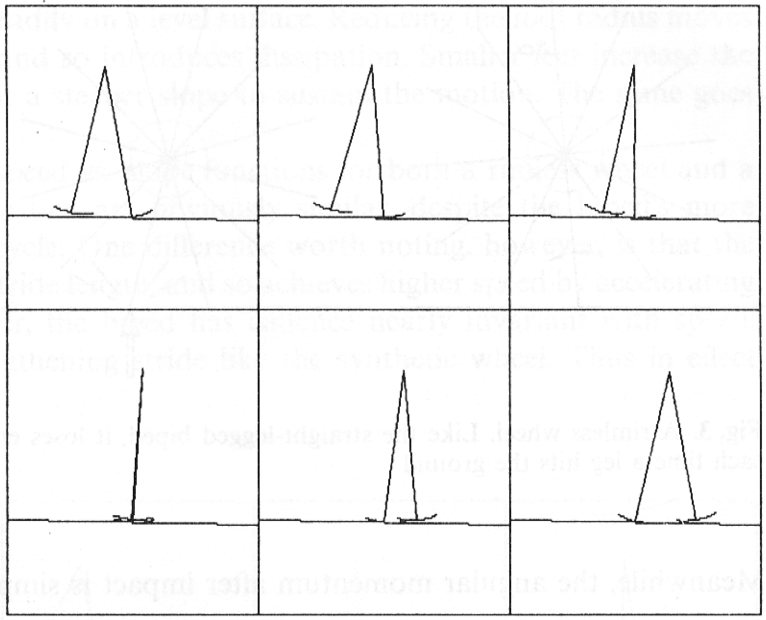


Fig. 2a. Modifying the synthetic wheel for a more anthropomorphic mass distribution and foot size produces a straight-legged biped. This model dissipates energy on each heel strike, but can make up for the loss by walking on a downhill slope (2.5% in this case). The timescale is normalised by $\sqrt{g/l}$, and angles are measured from the surface normal. The *table at right* lists parameters used in the gait calculation, which in this example are for one of our robots (McGeer 1990b)

payload, so that the legs are allowed an appreciable fraction of the total model mass. It turns out that this generalisation from the synthetic wheel, which we call a straight-legged biped, is capable of passive walking over a wide range of parametric variations (McGeer 1990b). Figure 2a shows an example. It has some new dynamics: because of the smaller feet, the stance leg is an inverted pendulum rather than a wheel, and so accelerates and decelerates through the step rather than maintaining a steady rotation. Furthermore, without the large hip mass there is inertial coupling between the legs. These effects complicate calculation of the passive gait (McGeer 1991), but the important point is that despite the complications the gait remains obviously similar to that of the synthetic wheel.



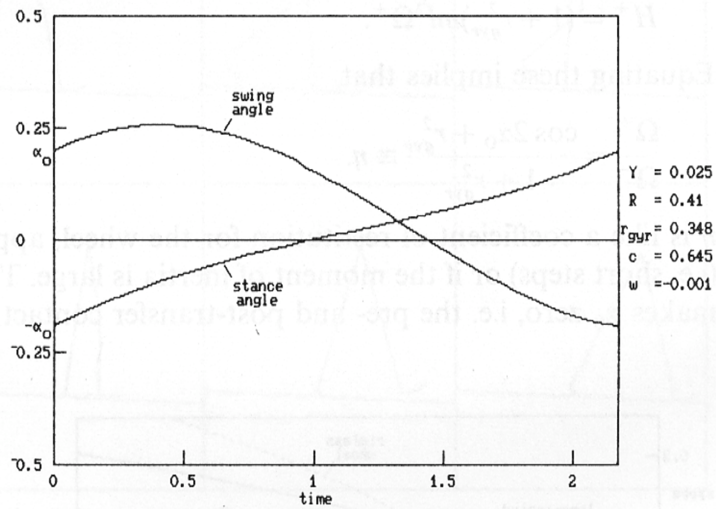


Fig. 2b. A straight-legged biped actually has two passive gaits. On the same 2.5% slope the model could walk in either of the gaits shown; the choice depends upon how the model is started

There is, however, one key difference: the straight-legged biped has to walk downhill to keep its passive cycle going. The energy gained in descent balances energy lost at each heel strike. The loss mechanism is most clearly illustrated by the rimless wheel of Margaria (1976) (Fig. 3). For analytical purposes we suppose that each time one of its legs strikes the ground, the foot is brought to rest instantaneously. No torque can be generated about the point of contact, so angular momentum must be the same before and after impact. This condition implies a drop in rolling speed, as follows. Before impact the rotational speed of the wheel is Ω^-, and the translational speed of the hip has a component $\Omega^- l \cos 2\alpha_0$ normal to the forward leg. The pre-impact angular momentum is therefore

$$H^- = ml(l\Omega^- \cos 2\alpha_0) + (ml^2 r_{gyr}^2)\Omega^-$$
$$= (\cos 2\alpha_0 + r_{gyr}^2) ml^2 \Omega^-. \tag{10}$$

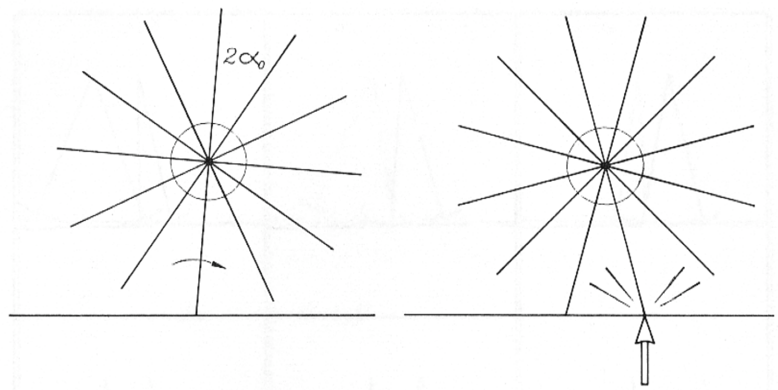


Fig. 3. A rimless wheel. Like the straight-legged biped, it loses energy in a heel strike impulse each time a leg hits the ground

Meanwhile, the angular momentum after impact is simply

$$H^+ = (1 + r_{gyr}^2) ml^2 \Omega^+. \tag{11}$$

Equating these implies that

$$\frac{\Omega^+}{\Omega^-} = \frac{\cos 2\alpha_0 + r_{gyr}^2}{1 + r_{gyr}^2} \equiv \eta. \tag{12}$$

η is like a coefficient of restitution for the wheel, approaching unity if α_0 is small (i.e. short steps) or if the moment of inertia is large. The synthetic wheel effectively makes α_0 zero, i.e. the pre- and post-transfer contact points are coincident; hence

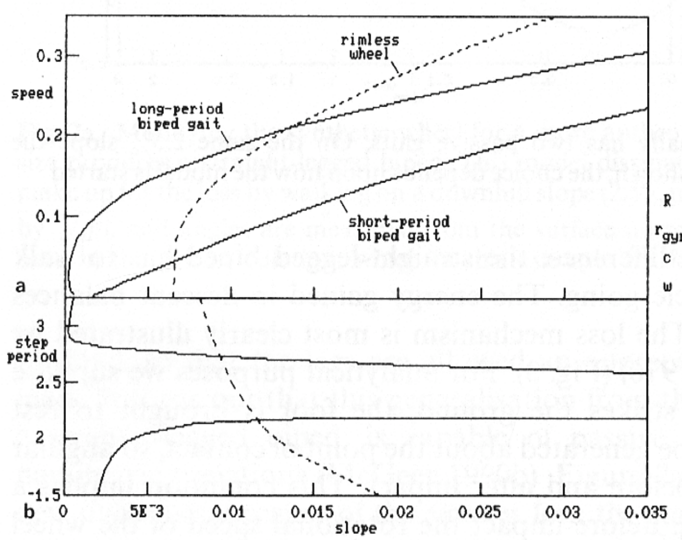


Fig. 4. **a** Increasing the downhill grade increases the speed of both a straight-legged biped and a rimless wheel. The rimless wheel in this example has $\alpha_0 = 0.2$, $r_{gyr} = 0.5$. Speed is in units of $\sqrt{gl}$. **b** For the biped, as for a synthetic wheel, cadence is nearly constant while stride length changes with speed. However, for a rimless wheel step length is fixed, so speed is changed by varying cadence

it has $\eta = 1$ and can roll steadily on a level surface. Reducing the foot radius moves the contact points apart, and so introduces dissipation. Smaller feet increase the energy loss, and so call for a steeper slope to sustain the motion. The same goes for higher speeds.

Figure 4a shows the speed vs. slope functions for both a rimless wheel and a straight-legged biped. The two are obviously similar, despite the biped's more complicated locomotion cycle. One difference worth noting, however, is that the rimless wheel has a fixed stride length, and so achieves higher speed by accelerating cadence (Fig. 4b). However, the biped has cadence nearly invariant with speed, and so accelerates by lengthening stride like the synthetic wheel. Thus in effect

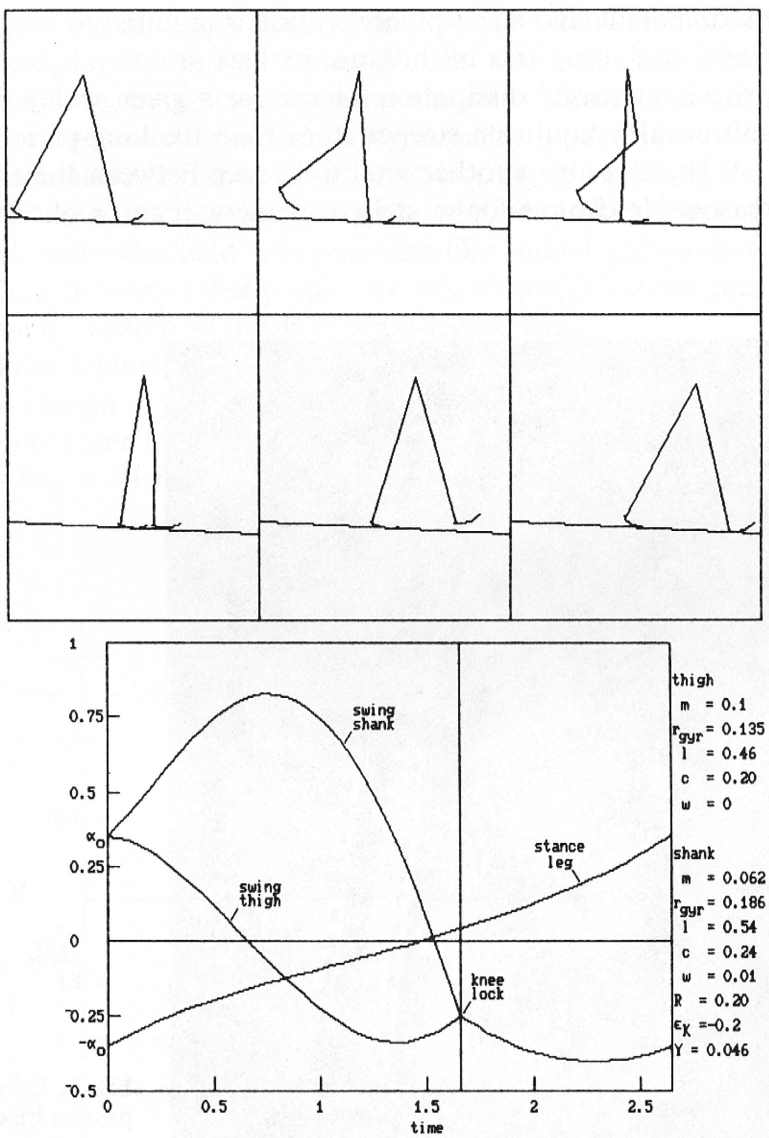


Fig. 5. A biped with knee joints also has passive walking cycles. In this example the leg parameters are roughly anthropomorphic, and the slope is 4.6% downhill. $\varepsilon_K < 0$ indicates that the foot is mounted forward on the leg, as shown in the cartoon

the straight-legged biped combines the synthetic wheel's dynamics with the rimless wheel's energetics.

Figure 4a indicates as well that a straight-legged biped has two possible speeds on any slope. Depending upon the initial conditions, it might adopt the "long period" gait, which we showed in Fig. 2a, or a "short-period" alternative, which is shown in Fig. 2b. Passive walking models seem always to have these paired gaits (unless they have no cyclic gait at all, which is also a possibility). The synthetic wheel provides another example of this rule, and also serves to illustrate that its long-period gait (that shown in Fig. 1) is likely to be preferred. The short-period alternative is realised by starting the step as in Fig. 1, but transferring support when the legs first reach equal and opposite angles (i.e. when $\omega_F\tau_0 = \pi$) rather than waiting until the speeds match as well ($\omega_F\tau_0 = 4.058$). That leaves the foot hitting the ground with substantial forward motion, rather than with no relative motion at all, and a sharp deceleration is required to bring it to rest. The synthetic wheel can shrug this off because its legs are very light, but for a real biped the price is increased dissipation. Hence for a given walking speed, the short-period gait usually requires a steeper slope than the long-period gait.

There is also another vital difference between the two gaits: stability. Given reasonable choices for model parameters, it turns out that if starting conditions

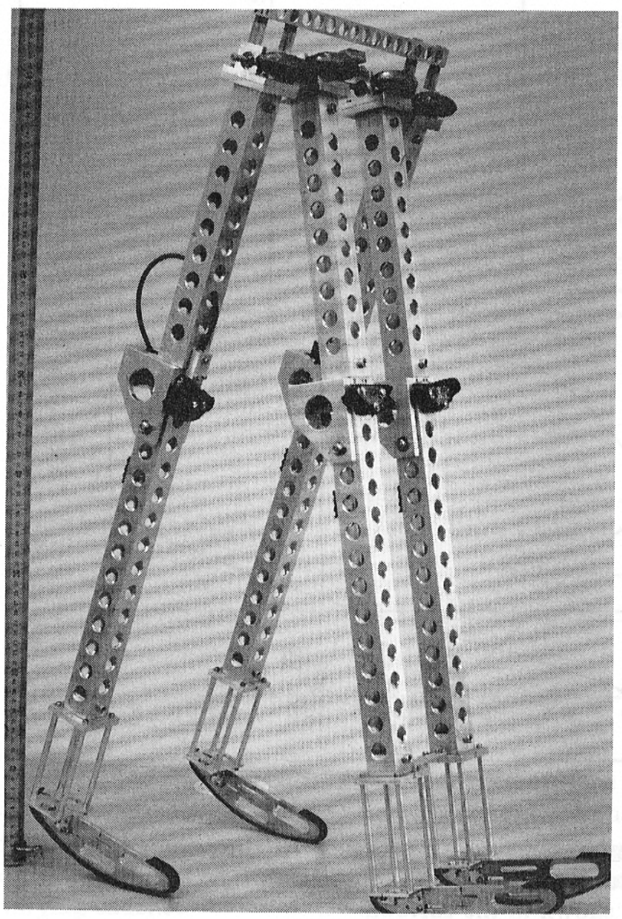


Fig. 6. Dynamite, an experimental passive biped with knees. To keep its motion confined to the longitudinal plane it actually has four legs, which are connected in pairs like crutches. Leg lengths is 80 cm, and mass 6.2 kg

are close to, but not right on, the trajectories of the long-period gait, then over a few steps the model will usually settle into the steady cycle (McGeer 1991). However, if started with conditions only infinitesimally different from those on the short-period gait, the model will almost invariably take a few steps and then collapse. Consequently, in experiments with straight-legged models we have seen only the occasional fleeting glimpse of short-period walking. This gait is not, however, just a curiousity. We will discuss redeeming features below.

4 The Knee-Jointed Biped

Moving on from the straight-legged biped, the next modification to make toward a more natural model is the introduction of knees. We will allow the knees free rotation in the flexural direction, but prevent hyperextention with a mechanical stop. The potential for passive recovery of the swing leg in such a model was noted several years ago by Mochon and McMahon (1980), and recently in exploring the dynamics further I have calculated complete passive gaits (McGeer 1990c). Figure 5 shows an example, which was calculated using human-like model parameters. Passive walking is possible with many other parameter sets, although the designer does not have quite as much freedom as in the straight-legged case.

For experiments on knee-jointed walking we built a model named Dynamite, which is shown in Fig. 6. Design of the hyperextention stops was an important detail. We wanted the knee to reach full extention inelastically after flexing through midstride, as indicated in Fig. 5. There was, however, a tendency to bounce back

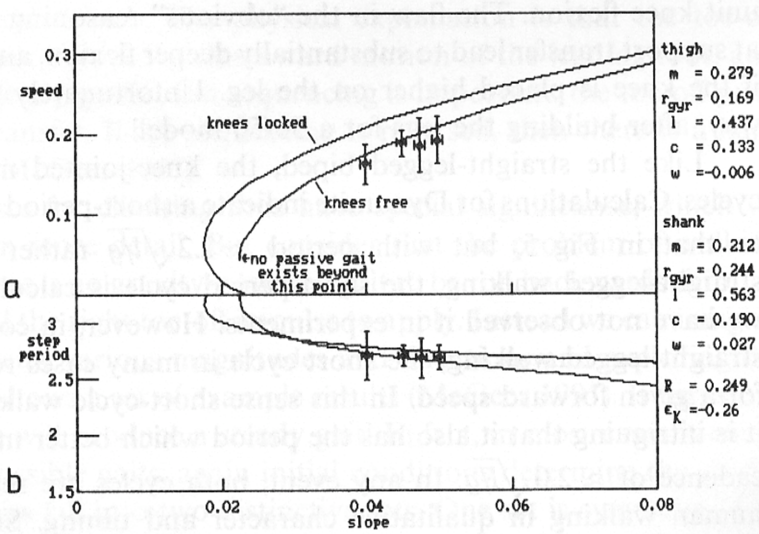


Fig. 7a, b. Speed and step period vs. slope for Dynamite in both knee-jointed and straight-legged gaits. *Continuous curves* show calculations for long-period passive walking. *Circles with uncertainty bars* show experimental measurements of the knee-jointed gait into flexion; to prevent this, we equipped each knee with a "debouncer" made with a suction cup and a carefully sized leak.

Figure 7a shows walking speeds predicted and measured for Dynamite as a function of slope. The plot also shows performance calculated for the same model in straight-legged walking, i.e. with knees locked. (For straight-legged experiments we put the machine on a chequerboard patern of tiles; these prevented scuffing of the swing foot at midstride, i.e. where $|\Delta\theta_F| \leqq |\Delta\theta_C|$ in Fig. 2a). Slightly steeper slopes are required in knee-jointed walking, because in that case there is energy dissipation not only at support transfer, but also when the swing knee locks at full extention. However, the important point is that despite the new dynamics of the knees-free gait, the motion is quite similar to straight-legged walking, which in turn is quite similar to rolling of the synthetic and rimless wheels. Thus one can be reassured that insight derived from the simpler models is applicable in more complicated circumstances.

Notice that the knee-jointed model, like a human, has its foot placed forward on the leg. The synthetic wheel, by contrast, has symmetric feet, and also (in the long cycle) a symmetric gait. That is, if you were shown a film of the synthetic wheel, you could not tell whether the projector was running backwards or forwards. On the other hand, the asymmetry in the knee-jointed models produces an asymmetric gait. (When the knees are locked the asymmetry manifests itself as very martial goose stepping). The asymmetric gait has a harder heel strike, and so requires generally steeper slopes to sustain the walking cycle. This is undesirable, but the asymmetric foot design remains attractive because it ensures that the contact force vector passes in front of the knee during stance. The contact force thus holds the stance leg locked in the extended position.

A subtler point of anthropomorphic design that becomes important in passive walking is the ratio of thigh and shank lengths. Natural proportions are about $0.46l$ in the thigh and $0.54l$ in the shank (Chapman and Caldwell 1983), as opposed to the "obvious" choice of 50/50 which would maximise swing foot clearance per unit knee flexion. The flaw in the "obvious" reasoning is that rotations induced at support transfer lead to substantially deeper flexion, and so better foot clearance, if the knee is placed higher on the leg. Unfortunately we learned this the hard way, after building the legs for a 50/50 model!

Like the straight-legged biped, the knee-jointed model has paired passive cycles. Calculations for Dynamite indicate a short-period cycle qualitatively similar to that in Fig. 5, but with period $\approx 2.2\sqrt{l/g}$ rather than $\approx 2.6\sqrt{l/g}$. As in straight-legged walking, the short-period cycle is calculated to be unstable, and we have not observed it in experiments. However, in contrast to the situation in straight-legged walking, the short cycle in many cases requires a shallower slope for a given forward speed. In this sense short-cycle walking is more efficient, and it is intriguing that it also has the period which better matches my own preferred cadence of $\approx 2.0\sqrt{l/g}$. In any event, both cycles are reasonably consistent with human walking in qualitative character and timing. Since the human leg also shares the features of foot design and shank/thigh proportions that the passive model indicates are desirable, it seems reasonable to infer that evolution has been sensitive to the potential of passive dynamics.

5 Pumping the Natural Dynamics

Models with passive dynamics are not restricted to walking downhill. Gait can also be sustained on level and uphill grades so long as energy is supplied by an appropriate mechanism. One such mechanism can be appreciated easily by the following argument from symmetry. We mentioned that if you were shown film of the synthetic wheel, you could not tell whether the projector was running forwards or backwards. What about film of a straight-legged biped? In that case perhaps you could tell, because if the project were running backwards then the model would appear to be climbing rather than descending. But on the other hand, coult it not be that the model really was climbing? The necessary energy would be supplied by a series of "toe-off" impulses, each the mirror image of the heel strike impulse in gravity-powered walking. This would be perfectly admissable dynamically; moreover, it is what people actually do through *dorsi-plantarflexion* of the ankle in the latter part of stance.

Of course humans have a knee-jointed rather than straight-legged cycle, and therefore would fail the "projector test" for symmetry. Furthermore, humans prolong the plantarflexion impulse over a finite interval, overlapping into a double-support phase during which both feet are in contact. However, we will restrict attention here to straight-legged models, with true impulses delivered instantaneously at support transfer. The approximation, while liberal, is not too worrisome in view of the similarities between the various walking models, and in any case it will certainly afford conceptual and mathematical simplification.

So let us pursue the symmetry argument further. A remarkable implication is that a "symmetric" climb – i.e. a time-reversed gravity-powered descent – is perfectly efficient. All of the energy supplied is stored in raising the centre of mass. This very happy situation arises because there is no toe-off impulse in a gravity-powered descent; hence there can be no heel strike impulse (and therefore no dissipation) in a symmetric climb. The physical explanation is that the toe-off push from the trailing foot arrests the downward motion of the leading foot, and so makes heel strike perfectly gentle. But sequencing is important: the toe-off push must precede support transfer. If the sequencing is reversed, then there is a hard and dissipative impact (McGeer 1991).

"Symmetric" impulses for climbing thus have special significance, which we shall presently discuss in more detail. But consider first the problem of walking on level ground. Here there is no gravity-powered gait to be run backwards through a projector, and so reveal the right toe-off impulse to apply. Instead we can imagine simply trying impulses with various magnitudes and directions and exploring the consequences. Figure 8 shows a set of example results (McGeer 1991). It turns out that almost any impulse will produce a steady gait! In fact, in most cases it could produce either of two possible gaits; again initial conditions determine the choice between them. These gaits fall into two distinctive sets. One set is symmetric, like long-cycle rolling of the synthetic wheel; energy is supplied by the toe-off impulse, and dissipated in a mirror-image impulse at heel strike. The period of this gait decreases as the toe-off impulse is inclined forward. The other set is asymmetric,

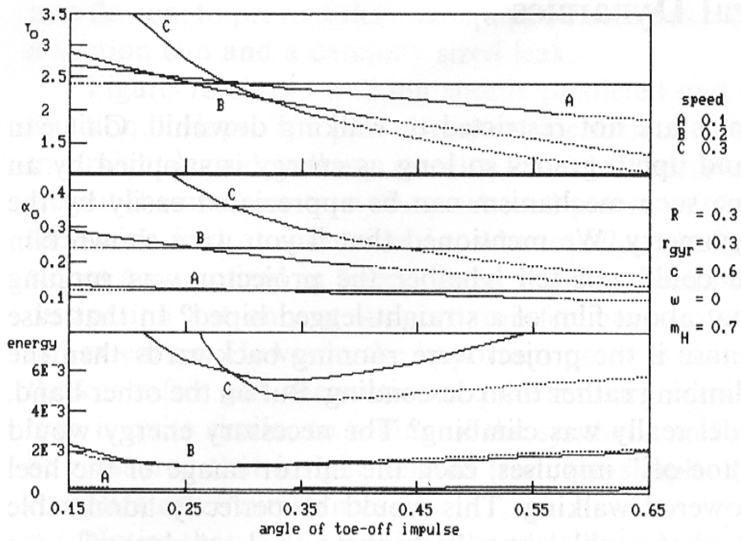


Fig. 8. The natural walking cycle can be pumped by an impulse from the trailing leg each time it leaves the ground. Here the resulting gaits are calculated, as a function of impulse angle, for a straight-legged biped walking on a level surface. Increasing impulse energy (here normalised by *mgl*) generally increases the steady walking speed. Meanwhile for an impulse of given angle and energy, the mathematics indicate a pair of possible gaits. Where *broken lines* are plotted the shorter-period gait of this pair is physically inadmissable, as explained in the text. Otherwise the choice between them is determined by starting conditions. Notice that one of the gaits has step period independent of impulse angle and energy

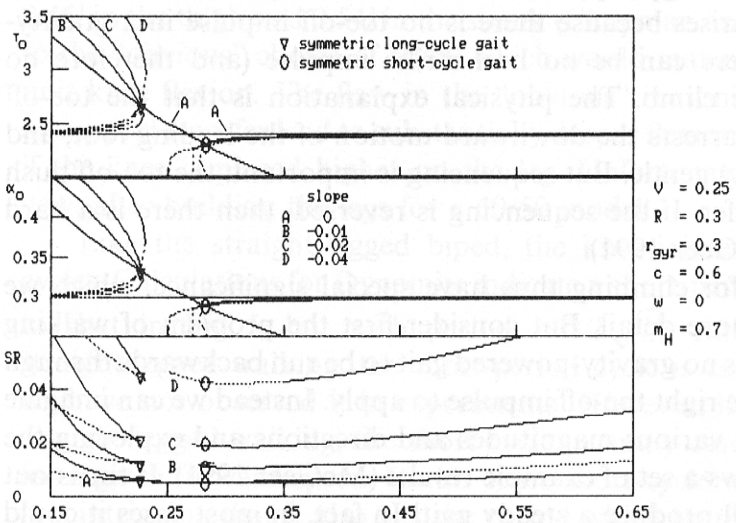


Fig. 9. Another plot of gait parameters as a function of impulse angle, but in this case with speed fixed at $0.25\sqrt{gl}$ and the slope increasing from zero to 4% uphill. The "symmetric" gaits are explained in the text. Energy input here is measured by specific resistance. $SR = -\gamma$ if there is no dissipation. $m_H = 0.7$ indicates that 70% of the model's mass is concentrated in "payload" at the hip

like short-cycle rolling of the synthetic wheel. The period of this gait, as of the corresponding synthetic wheel gait, is exactly half the swing pendulum period, completely independent of impulse magnitude and direction!

Given a good pair of legs, then, it is hard to go wrong. Get them off to a good start, and you can keep them going by pushing in almost any reasonable direction. The harder the push, the faster they go. Analysis does reveal some guidelines for refining the technique; we can observe, for example, that very steep or very shallow pushes will produce gaits with needlessly high dissipation. Stability is also an issue. This depends upon how the toe-off impulses vary during transients, but it is fair to say that, as in gravity-powered walking, the longer-period gaits are typically stable, and the shorter-period gaits unstable. The shorter-period gaits have in addition a more serious problem: often they are not just unstable, but physically inadmissable. Where Fig. 8 shows broken lines, a toe-off impulse strong enough to sustain the gait would make the model jump off the ground (For analytical purposes support transfer is still specified to occur immediately after impulse application, but physically this would require a downward pull on the forward foot.) Notice that selection of the long-period gait would imply symmetric walking if the impulses were nearly vertical, and asymmetric walking if the impulses were inclined further forward.

Let us now return to climbing. Figure 9 shows another exploration of impulse angles, but in this case for climbing various slopes at a specified speed of $0.25\sqrt{gl}$. In gravity-powered walking this speed is achieved on a downhill slope γ_g slightly steeper than 2%, and the angle of the heel strike impulse then turns out to be 0.227 (backward from the surface normal) in the long-period gait, and 0.283 in the short-period. The climbs with mirror-image impulses are indicated on the figure. These have the distinctive feature that their cadences and step lengths are invariant with slope. Climbing could also be pumped by impulses at other angles, but notice that as the slope increases from zero a forbidden zone develops in which either the mathematics produce no solutions for steady walking, or the solutions are physically inadmissable. The symmetric impulses are included in this category on slopes steeper than $-\gamma_g$, as can be appreciated by recalling that on slope $-\gamma_g$ the impulse is just sufficient to arrest the downward motion of the forward foot. The stronger impulse required on a steeper slope would therefore launch the forward foot upward. It follow that the impulse must be applied with increasing care on steep slopes, and moreover that some dissipation must be accepted. The latter point is indicated on the plot by the specific resistance

$$SR \equiv \frac{\text{mechanical work done}}{\text{weight} \times \text{distance travelled}}. \tag{13}$$

If all the work goes into raising the mass centre, then $SR = -\gamma$. This value is reached in a symmetric climbing gait, but otherwise dissipation at heel strike causes SR to rise above the minimum value.

These limitations are inconsequential so long as the slope is shallow, but for steep slopes on is obliged to find better techniques for pumping the walking cycle. Some of these take advantage of the torso.

6 Role of the Torso

We can introduce the torso with another argument from symmetry, provided that we first consider how to manage a steep descent. Ordinary passive walking in steep descent is not possible because the leg angles (α_0) would become unreasonably large. Thus the problem is to keep the cycle amplitude under control. One effective method is to put a braking torque on the stance leg, just as you would brake a wheel going downhill. Of course to apply a torque you need something to react against, and a backward-leaning torso is ideal.

The necessary recline is easily estimated for a torso on a synthetic wheel. Imagine holding a mass m_T, centred a distance c_T from the hip, at an angle β from the vertical. This calls for a torque about the hip of

$$T_H = -m_T g c_T \sin\beta. \tag{14}$$

If this torque is held through one step, then the work done by the reaction on the stance leg is

$$W_H = -\int_{-\alpha_0}^{\alpha_0} T_H d\theta_C = 2 m_T g c_T \sin\beta\ \alpha_0. \tag{15}$$

Meanwhile, the energy gained in descent is

$$E = 2mgl\alpha_0\gamma. \tag{16}$$

Matching the two implies that the torso should recline backward at

$$\beta = \sin^{-1}\left(\frac{-ml\gamma}{m_T c_T}\right). \tag{17}$$

Notice that β is independent of step length.

This analysis is not exact, since the hip torque has to vary somewhat through the step to compensate for accelerations. (The wheel no longer rotates uniformly, because its mass centre is no longer coincident with the hip). However Eq. (17) remains a good approximation, and holds also for bipeds with dissipative heel strike so long as γ is replaced by $(\gamma - \gamma_g)$ (McGeer 1991).

That being said, let us reverse the projector on a torso-braked descent. The time-mirror then reveals another perfectly efficient climb, with the torso bent purposefully forward, and energy supplied by a combination of symmetric impulses and hip torque. Moreover, as in Fig. 9, this symmetric motion is an opening to a whole spectrum of torso-mediated (but nonsymmetric) climbing gaits. All have the advantage that the major energy input is distributed uniformly through the motion, rather than being concentrated in large cyclic bursts.

Limitations, however, remain. For a human $m_T \approx 0.7m$ and $c_T \approx 0.2l$, so according to Eq. (17) the torso has to lean about seven degrees for each degree of descent. At this rate one soon runs out of inclination, no matter how purposeful one's bearing. Thus while a torso expands the range of travel, it still cannot provide the answer to stairs and other steep grades. Yet another pumping method is required.

7 Leg Length Variation

In addition to difficulties of energy supply, steep slopes present an elementary geometric problem. A biped having legs of equal length, as in Fig. 2*a*, cannot stand statically on a slope exceeding α_0, since that would place its mass centre outside the base of support. The obvious recourse in this situation is to lengthen the downhill leg, and shorten the uphill leg. Similarly it would seem appropriate when walking on steep slopes to vary leg lengths cyclically. Thus in a climb, for example, one could lengthen the stance leg to $l+\Delta l$ while passing through midstride, and simultaneously shorten the swing leg to $l-\Delta l$. But lengthening the stance leg implies raising the centre of mass, and so entails some energy input. Initially this is stored as potential energy, but it is converted to kinetic as the model tips forward into support transfer. This process has the effect of pumping the cycle, so we can add leg length variation to hip torque and toe-off impulses as a third method of energy supply.

An estimate of the length variation required to climb a given slope is given by

$$\underbrace{mg\,2\alpha_0 l(\gamma_g-\gamma)}_{\text{energy required for climbing}} \approx \underbrace{mg\,2\Delta l}_{\text{energy input by raising mass centre}} \Rightarrow \frac{\Delta l}{l} \approx \alpha_0(\gamma_g-\gamma).$$

This ignores a small change in kinetic energy associated with the length adjustment, and also variations in heel strike dissipation. However, the approximation is nevertheless very good for a rimless wheel pumped by length cycling, holding over the full range from level grades to stairs (McGeer 1991). Similar accuracy can be expected for a biped.

Of course, length cycling can be used in combination with the other pumping methods, and in fact our trusty reversible projector reveals that this is the best strategy. Imagine a pair of legs walking downhill, maintaining speed by dissipating some energy in a heel strike impulse, and the rest in shortening of the stance leg at midstride. Now play the motion backward, and again you see a perfectly efficient climb. Energy is supplied by a combination of toe-off pulsing and stance leg lengthening, and it all goes into raising the mass centre since the heel strike is dissipation-free.

8 Running

Gravity-powered walking leads naturally to the study of slope variation, and we have pursued this study to the point that the techniques outlined above are sufficient to negotiate any slope within reason. Now let us talk about speed.

We have indicated that a synthetic wheel, because of constraints on α_0 and swing frequency, is limited to speeds below about $0.3\sqrt{gl}$ [Eq. (9)]. The straight-legged and knee-jointed models have the same limitation. Furthermore, because these models have small feet, their mass centres follow a convex trajectory during stance; this introduces an independent speed limit due to centrifugal effect. The rimless wheel demonstrates the limit most clearly. Its hub follows a circular arc centred at the foot. Centrifugal effect therefore lightens the contact force on the foot, and when the rotational speed reaches $\sqrt{gl}$ the contact force goes to zero. You can actually feel this effect if you try to walk unusually fast: you reach a speed at which pushing is no longer possible.

To get around these speed limits, you run. Compression of the stance leg puts the mass centre on a concave rather than convex trajectory; leaping through the air eliminates the α_0 constraint; and inter-leg torque application accelerates the swing motion. (Actually, the leap can disappear in unusual situations such as running in tight circles, but the concave trajectory remains (McMahon et al. 1987).) With these ideas in mind, consider a straight-legged biped with springs: one spring in each leg, which can compress telescopically, and another at the hip, which provides the torque necessary for accelerated swinging. Two modes of motion are easily visualised, and might be called bouncing and scissoring. To start the bouncing motion, you would hold the legs vertically and drop the model on to a flat floor. In an ideal world tipping disturbances would be excluded and the rebounds would be perfectly elastic (which implies frictionless springs and massless lower legs), so bouncing would go on indefinitely. Meanwhile, to start the scissoring motion, you would first pull the legs apart and then drop the model. Given exquisite hip bearings, the subsequent back-and-forth swinging would also persist indefinitely, or at least until a foot hit the ground. But what would happen then? One possibility is that you would suddenly have a broken pile of formerly exquisite bearings, massless lower legs, and frictionless springs. However, another possibility, which would emerge if the model landed with an appropriate set of speeds and angles, in *passive dynamic running*.

Figure 10 illustrates an example cycle (McGeer 1990a). The motion is essentially bouncing and scissoring in synchrony. In fact, the cadence indicates that bouncing hardly disturbs the scissor action at all. With the hip stiffness and leg inertia used in this example, the scissor period in free all would be $2.57\sqrt{l/g}$, while in running it is $2.52\sqrt{l/g}$ (i.e. twice the step period in Fig. 10). (There would, however, be a catastrophic disturbance to the cycle if the swing foot were allowed to scuff at midstance. To avoid this problem the swing leg must be shortened at least as much as the stance leg compresses).

Now consider speed control and speed limits. Speed is set by the amplitude of the bouncing and scissoring motions. Figure 11 shows an example. Notice that scissor amplitude (indicated by the leg angles at take-off) and bounce height (indicated by flight time) are directly related; in the steady cycle you cannot change one without changing the other. (Of course you could do in a transient, and what happens then is a question of stability. It turns out that high speeds and cadences lead to passively stable cycles, while low speeds and cadences call for active stabilisation (McGeer 1990a).) The step period remains nearly equal to half the free-fall scissor period throughout the speed range, so that as the flight time goes up, the contact time goes down. Long-term vertical equilibrium requires that the

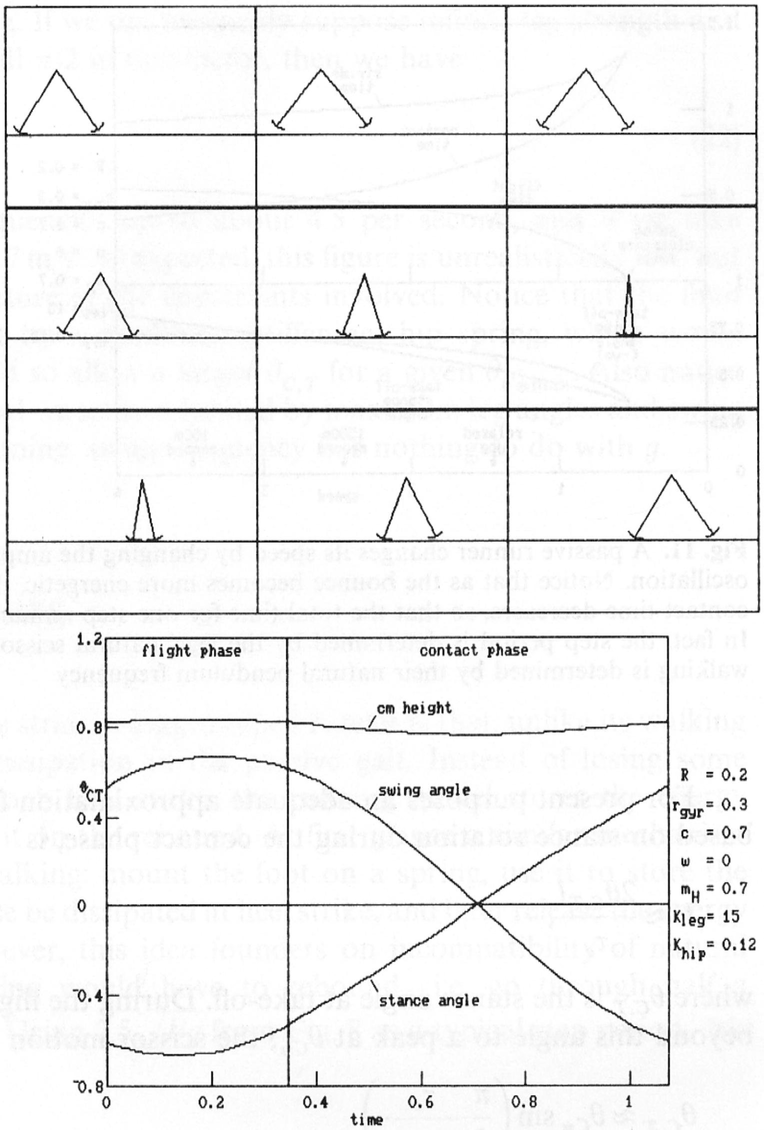


Fig. 10. Passive dynamic running. Telescoping spring are put into the legs of the straight-legged biped, and a torsional spring is introduced around the hip joint. (Leg spring stiffness is in units of *mg/l*; hip spring stiffness in units of *mgl*.) In its natural running cycle the model bounces on the legs in alternation while they swing back and forth in a scissoring motion. No energy is dissipated during the cycle; it is simply exchanged between kinetic, gravitational, and elastic stores. The speed in this example is $1.4\sqrt{gl}$, a jogging pace

upward force averaged over one cycle be mg, and hence that the upward force averaged over the contact phase be $mg(\tau_c + \tau_b)/\tau_c$. Declining contact time at fixed cadence therefore implies increasing loads in the stance spring. At the same time, increasing scissor amplitude implies increasing loads on the hip spring. In view of these effects one can see an upper speed limit developing as follows. (Incidentally, the lower speed limit shown in the figure can be eliminated by increasing stance stiffness, which increases flight time.)

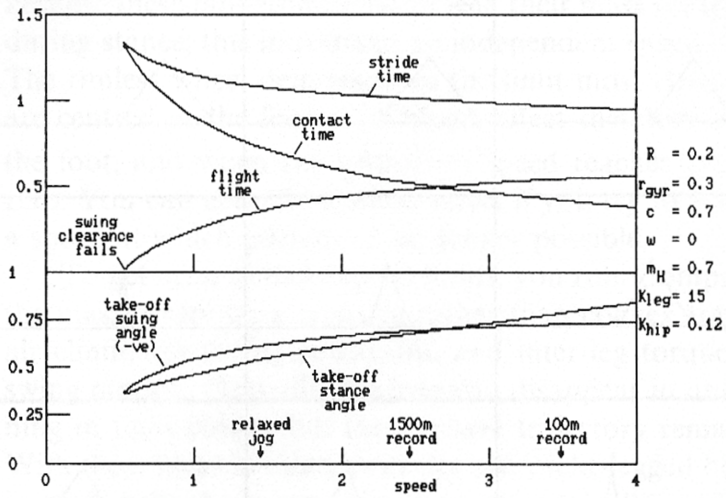


Fig. 11. A passive runner changes its speed by changing the amplitude of its bounce-and-scissor oscillation. Notice that as the bounce becomes more energetic, the flight time increases but the contact time decreases, so that the total time for one step remains nearly independent of speed. In fact, the step period is determined by the legs' natural scissor frequency, just as cadence in walking is determined by their natural pendulum frequency

For present purposes an adequate approximation for average forward speed, based on stance rotation during the contact phase, is

$$V \approx \frac{2\theta_{C,T} l}{\tau_c}, \tag{18}$$

where $\theta_{C,T}$ is the stance angle at take-off. During the flight phase the leg continues beyond this angle to a peak at θ_{C*}; the scissor motion is nearly sinusoidal, so

$$\theta_{C,T} \approx \theta_{C*} \sin\left(\frac{\pi}{2}\frac{\tau_c}{\tau_c + \tau_b}\right). \tag{19}$$

Let us also write τ_c as

$$\tau_c = \frac{\tau_c}{\tau_c + \tau_b}(\tau_c + \tau_b), \tag{20}$$

then

$$V \approx \frac{2l\theta_{C*}}{\tau_c + \tau_b} \frac{\sin \frac{\pi}{2}\frac{\tau_c}{\tau_c + \tau_b}}{\frac{\tau_c}{\tau_c + \tau_b}}. \tag{21}$$

Maximum speed calls for maximum θ_{C*}, which in animals is limited by the construction of the hip joint, and for minimum $\tau_c + \tau_b$, while is limited by hip stiffness. In addition, the second quotient here reaches a maximum value of $\pi/2$ when $\tau_c = 0$; thus τ_c should be made as small as possible, limited as noted above by strength in the stance leg. If we optimistically suppose infinite leg strength and stiffness, and so take the full $\pi/2$ in this factor, then we have

$$V_{\max} \approx \frac{\pi l \theta_{C*\max}}{(\tau_c + \tau_b)_{\min}}. \tag{22}$$

Sprinters achieve step frequencies up to about 4.5 per second, and if we take $\theta_{C*\max} \approx 3\pi/8$ then $V_{\max} \approx 17\,\text{m/s}$. As expected, this figure is unrealistically fast, but it serves to indicate the nature of the constraints involved. Notice that the limit could be raised somewhat by a nonlinear (stiffening) hip spring, which would flatten the scissor peaks and so allow a larger $\theta_{C,T}$ for a given $\theta_{C*\max}$. Also notice that as in the synthetic wheel we remain limited by maximum leg angles and swing frequency. However, in running, swing frequency has nothing to do with g.

9 Energetics of Running

A remarkable feature of the straight-legged biped runner is that, unlike its walking counterpart, it has zero dissipation in the passive gait. Instead of losing some energy in an impulse at each heel strike, the running model stores the energy elastically, and withdraws it on the rebound. At first glance a similar mechanism would seem feasible for walking: mount the foot on a spring, use it to store the energy that would otherwise be dissipated at heel strike, and later release the energy in a toe-off impulse. However, this idea founders on incompatibility of natural frequencies. The foot spring would have to rebound – i.e. go through half a cycle—in one step period. Using $2.5\sqrt{l/g}$ from Fig. 8 as a typical step period, this implies that

$$\pi\sqrt{\frac{m}{k}} = 2.5\sqrt{\frac{l}{g}} \Rightarrow \frac{mg}{kl} = 0.63. \tag{23}$$

But mg/kl is the fractional spring compression under static load! 63% of leg length seems a bit excessive. A stiffer spring is required to get a more reasonable compression, and a tuned rebound is then possible only with a shorter contact time. That, in turn, implies a running gait.

Why, then, do we not prefer always to run? The reason, of course, is that we are not made of such ideal stuff as the passive model. When running we must continually supply energy to three major consumers: heel strike dissipation, aerodynamic drag, and "active" synthesis of spring action.

The heel strike dissipation occurs because our knees hinge rather than compress telescopically, and consequently allow impulses to propagate up the leg. An estimate of the energy loss can be made if we suppose that a running version of our knee-jointed biped would land with angles and speeds similar to those in the cycle of Fig. 10. Applying conservation of angular momentum about each joint and the point of contact (just as in walking analysis) then determines the heel strike impulse and the drop in kinetic energy. The knee angle at landing is an important parameter; 25° flexure is reasonable, and in that case the impulse turns out to be nearly vertical with magnitude $0.07m\sqrt{gl}$, and the associated energy loss is $0.013mgl$. (Flexing the knee another 10° would cut the loss to $0.005mgl$). Dividing this by the distance travelled between heel strikes gives the contribution to specific resistance, which in this case is about 0.008. In sprinting ($V = 3.5\sqrt{gl}$) the figure would rise to something like 0.03. (Incidentally jogging shoes and fatty foot pads cushion heel strike, so that the landing impulse is distributed over a period of order $0.1\sqrt{l/g}$, but the energy is lost just as irretrievably as in an instantaneous impulse).

Aerodynamic drag has been estimated experimentally and theoretically by various investigators, e.g. (Hill 1927; Pugh 1971). Its contribution to SR is about 0.01 in jogging, and rises quadratically with speed to about 0.06 in sprinting.

A further expenditure of energy is required to generate spring action, which while entirely passive in our model, in animals is done by a combination of active muscles and passive tendons. Ker et al. (1987) estimate (from force plate and film records) that during the first part of stance a 70-kg jogger removes $\approx 100\,J$ of potential and kinetic energy from the centre of mass, replacing it on the rebound. People seem to store somewhat more than half of this in stance-spring-like elements: $\approx 17\,J$ in bending the arch of the foot; $\approx 35\,J$ in stretching the Achilles tendon, and some significant (but as yet unknown) amounts in tendons of other ankle extensors and, presumably, knee extensors as well. The rest is dissipated in muscle and must be regenerated from chemical reserves. Unfortunately, an estimate of this component of the bouncing action is not available, nor do I know of any data on the balance between passive springs and active muscles in scissoring. It is noteworthy, however, that Alexander (1988a) has found in specialised runners, such as horses, long leg tendons in series with short or even rudimentary muscles. These make spring action almost fully passive. But if for more modestly equipped humans we allow only a 50% passive return, then the input required for the cycle of Fig. 10 would be $\approx 0.14mgl$, and in sprinting $\approx 0.42mgl$. Adding in the work required to balance aerodynamic drag and heel strike losses brings the SR up to 0.10 in jogging and 0.5 in sprinting. These figures are much less attractive than for walking (Fig. 9), and the associated power requirements ($SR \times mgV$) are formidable: $0.2mg\sqrt{gl}$ for jogging and $1.8mg\sqrt{gl}$ for sprinting, as compared with $0.0013mg\sqrt{gl}$ for walking at $V = 0.25\sqrt{gl}$. Thus we "discover" that running is hard work – but perhaps we knew that already!

Regardless of how the work is distributed between muscles and tendons, the important point as far as the dynamics are concerned is that both types of tissue behave like springs. Alexander (1988a) has made the case for passive springiness in tendon; McMahon (1990), based on a variety of evidence (Hoffer and Andreassen (1978), Houk (1979), Cavagna (1970), McMahon and Greene (1979)) has argued that muscles appear springy when controlled by spinal reflexes, and that the leg as a whole, when tested dynamically, exhibits characteristic spring properties. The message of the passive model is that these properties alone are sufficient for running; no higher motor control is required. Of course adjustments must be made for variations in speed and terrain, but as in walking these can rely on pumping the underlying oscillation (McGeer 1990a). Again it seems that, for running as for walking, we are well equipped to take advantage of the dynamic possibilities of legs.

10 Why Not Hop?

Bounce-and-scissor running as in Fig. 10 is not limited to bipeds. An analogous cycle in a monoped has been calculated by Thompson and Raibert (1990), with the scissor oscillation between the leg and a torso. Why should we not locomote in this way, using our legs in unison as does a kangaroo? Part of the answer is that by counter-oscillating our legs we prevent annoying bobbing of the torso. Furthermore, passive hopping is unstable, and therefore generally more difficult to control than running. But above all, hopping in practice requires more effort: each step is a full scissor cycle, not just half, and the bounce must be sufficiently energetic to provide the extra flight time.

However, if one's legs are sufficiently strong that bounce height is not a problem, then hopping promises very high speed. Our analysis of maximum running speed (Pugh 1971) suggested that beyond a point extra leg strength could not do you any good; however, this was because we specified a sinusoidal scissor action. As an alternative, imagine hopping with a completely different protocol for hip torque – in fact that used by Raibert (1986) for hopping robots. You would start by pushing forward and upward into a long flight phase, applying (active) torque through the push in such a way that both torso and legs left the ground with zero rotational speed. After take-off you would torque the legs around into the appropriate alignment for landing, and then relax the hip for the duration of the flight phase. Upon landing you would go through a normal rebound, again torquing as necessary to reach the same take-off conditions as on the previous stride. Hip strength is immaterial to such a cycle; maximum speed would be determined only by how high and how long you could jump. Under normal circumstances people cannot jump with enough energy to make this strategy worthwhile, but on the moon, where apparent leg strength increases sixfold, hopping seems to be an attractive proposition.

11 Quadruped Gaits

So far we have concentrated on bipeds, which has been selfishly neglectful of the majority interest. Our only companions in the bipedal camp are birds (and then only when they are not out using their God-given wings). The rest of legged mammals and reptiles are quadrupeds. From our point of view this arrangement needlessly ties up the hands, but its aficionados might point out the merits of a statically stable trunk and reduced loads on the legs. Furthermore, they have the upper hand in speed: a cheetah outpaces an ostrich, despite its shorter legs. But above all, the sheer weight of numbers invites study of quadruped locomotion.

The obvious introductory model might be called the synthetic bicycle, i.e. a pair of synthetic wheels connected by a frame. Each wheel would have exactly the same gait as in the bipedal case, so all the analysis of Sect. 2 holds without modification. In fact, all of our biped models can be connected in this way, again without changing the cyclic gaits provided that fore and hind legs start in-phase. Pairing could be between legs on the same side, in which case the gait would be called a pace, or between opposite sides, which would produce a trot. Of these options, the trot seems more attractive, since it minimises rolling moments when the model is supported on two feet.

Of course, fore and hind legs might not start in-phase. Then new dynamics would develop in pitching of the frame, and in accommodating accelerations of the hips. (The "front biped" might be past midstance and trying to accelerate when the "hind biped" was coming up to midstance and trying to decelerate). The consequences for passive locomotion have not been investigated. It is intriguing, however, that while for running most quadrupeds prefer the trot, for walking the most popular gait is the amble, with front and hind bipeds 90° out-of-phase. Analysis might prove this to be the more stable gait, but one can imagine alternative explanations. While the amble causes pitching of the body, it also reduces vertical motion at the mass centre. Also it may confer some advantage in lateral balance, through coupling from the longitudinal dynamics.

Trotting, pacing, and ambling, then, are the analogues of bipedal walking and running; trotting and pacing quadrupeds are essentially bipeds in formation, while amblers are similar but have some additional dynamics. What about analogues of bipedal hopping? These are the pronk, in which all four legs work in-phase, and the bound, in which the fore and hind bipeds are 180° out-of-phase. Pronking and bounding might be produced by scissor springs acting between paired legs and a rigid frame. In bounding there is also the possibility of scissoring by bending the back; an advantage is that this strategy would effectively lengthen the legs, and so allow a small increase in speed (moderated by some reduction in scissor frequency). It also seems that in practice the back makes a superior passive spring. Alexander (1988b) has identified passive structures in the back of the deer, particularly the aponeurosis (an expanded tendon) of the longissimus muscle, that he calculates would provide most of the spring action necessary for passive scissoring. Similar structures apparently are not present in the hips, or at best have limited effectiveness. Data of Fedak et al. (1982) indicate that the scissor motion in trotting becomes asymmetric and therefore inherently dissipative at high speed. Consequently, bounding should be more efficient than trotting above some characteristic speed, and more efficient than pronking in all conditions. Alexander and Jayes (1983) have observed that virtually all quadrupeds do indeed stop trotting at speeds between $1.4\sqrt{gl}$ and $1.7\sqrt{gl}$, and use a bound-like gait to go faster. Hoyt and Taylor (1981) have verified that the switch minimises oxygen consumption per unit distance travelled.

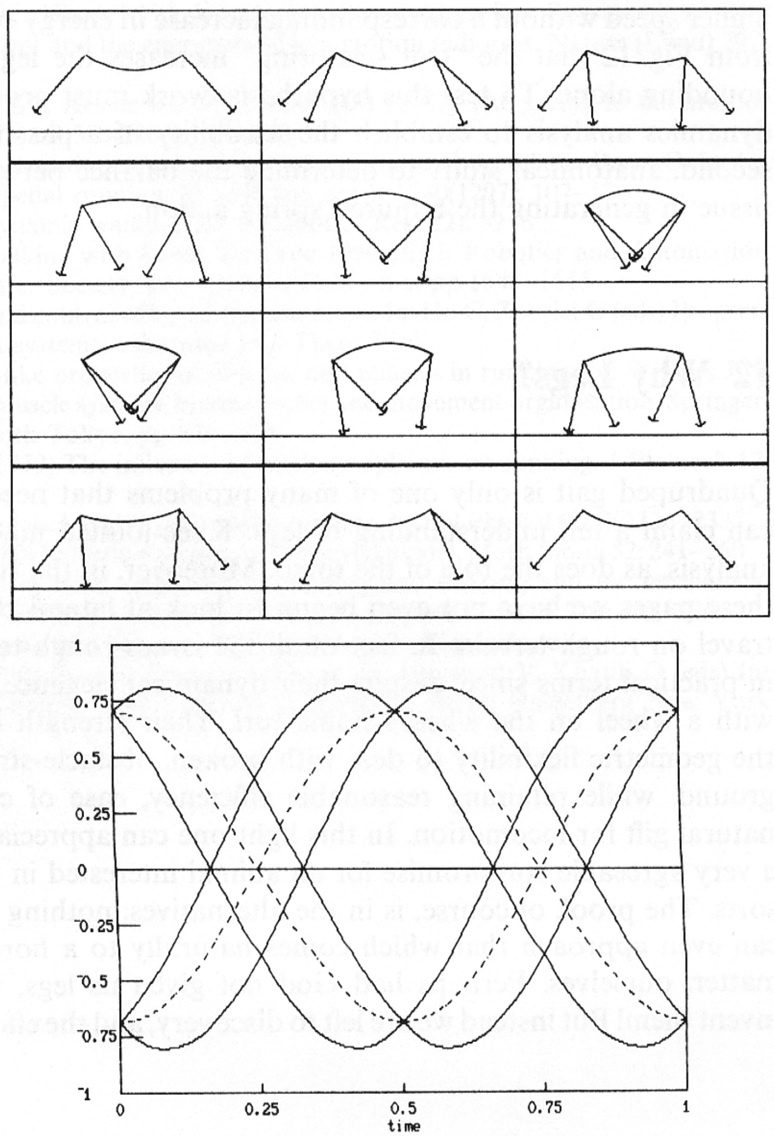


Fig. 12. Almost all quadrupeds gallop at high speeds. A galloping gait might be constructed by superposing two passive scissor motions. The first is scissoring of the front and hind hips, i.e. a sinusoidal variation of their equilibrium positions (*broken lines*). The second is scissoring of the legs about those equilibrium positions. Time here is normalised by the stride period

However, most animals do not abandon the trot entirely at high speed; instead they superpose it upon bounding, as indicated in Fig. 12. Thus each pair of legs scissors out-of-phase as in trotting, but about a median position that itself swings back and forth as in bounding. The result is a gallop. I think it likely that galloping could be sustained by a passive quadruped model, with springs in the hip, back, and legs. However, such a model has not been investigated, so Fig. 12, unlike our other plots, is the result of speculation rather than rigorous analysis. Still, the hypothesis suggests why galloping should be preferred over bounding. Since trotting is apparently efficient at moderate amplitudes (i.e. at speeds up to $\approx 1.4\sqrt{gl}$), it would seem that superposing this motion on bounding should offer higher speed without a corresponding increase in energy consumption. Thus notice from Fig. 12 that the "trot scissoring" increases the leg motion beyond that in bounding alone. To test this hypothesis, work must proceed on two fronts: first, dynamics analysis to establish the feasibility of a passive galloping model, and second, anatomical study to determine the balance between muscles and passive tissue in generating the required spring action.

12 Why Legs?

Quadruped gait is only one of many problems that need more study before we can claim a full understanding of legs. Knee-jointed models also require further analysis, as does the role of the torso. Moreover, in the two-dimensional world of these pages we have not even begun to look at lateral balance and steering, nor travel on rough terrain. In fact of all the issues rough terrain is most significant in practical terms since, despite their dynamical elegance, legs can never compete with a wheel on the wheel's home turf. Their strength is rather that they have the geometric flexibility to deal with broken, obstacle-strewn, and steeply sloped ground, while retaining reasonable efficiency, ease of control, and the wheel's natural gift for locomotion. In this light one can appreciate why legs should offer a very agreeable compromise for an animal interested in travel over terrain of all sorts. The proof, of course, is in the alternatives; nothing in the engineer's garage can even approach that which comes naturally to a horse, or a cat, or for that matter, ourselves. Perhaps had God not given us legs, we should have had to invent them! But instead we are left to discovery, and the effort is only beginning.

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