Propulsive Efficiency of Rowing Oars

David S. Cabrera[1] Andy L. Ruina[2]

Department of Theoretical & Applied Mechanics
Biorobotics and Locomotion Laboratory
306 Kimball Hall, Cornell University
Ithaca, NY 14853
phone: (607) 255–7108, fax: (607) 255–2011
email: [1]dsc16@cornell.edu, [2]ruina@cornell.edu

submission date: February 13, 2006
keywords: work, dissipation, propellers
word count: 4500

Abstract

Is the common folklore, that oars are less efficient at propulsion than propellers, correct? Here we examine the propulsive efficiency of the oars used in competitive rowing. We take the propulsive efficiency η of rowing to be the ratio of the energetic benefit, the energy D_b dissipated by boat drag, to the energetic cost, the work W_r performed by the rower. Air drag is neglected as is the energetic cost of raising and lowering the oar out of and into the water. We calculate η first by directly using extensive data from an instrumented single scull and again using less data and extrapolating on the basis of a simple rowing model. From the data, we estimate that $\eta \approx 0.84$. That is, about 84% of the rower's energy dissipated during a stroke is due to boat drag and the remaining 16% of the energy dissipated is due to oar drag. The best marine propellers have efficiencies of about 80%. We also point out some subtleties in energetic calculations in rowing, discuss the essential differences between oars and propellers, and discuss how oars might be made still more efficient.

1 Introduction

Modern propellers are obviously an improvement over more-ancient oars and paddles in that they are compact and well suited to transmissions from rotary engines. It is thus commonly assumed that propellers are an improvement over oars in other regards, for example, in propulsive efficiency. Is this really so? Several authors have addressed the efficiency of oars using various definitions for efficiency. Alexander (1927), Affeld et al. (1993), Kleshnev (1999), Pendergast et al. (2003), and Atkinson (2004) sensibly define oar efficiency, η, as the ratio of work done by boat drag to the work done (discussed below) by the rowers. Meanwhile, Wellicome (1967) defines efficiency as the ratio of “useful” work (discussed later) done by the crew to the total crew work and Celentano et al. (1974) define efficiency as the ratio of crew work to the sum of crew work and oar drag dissipation. Unfortunately, as discussed below, there are problems with how these authors calculate the rowers' work.

Here we use the first definition of propulsive efficiency from above, the ratio of the boat drag dissipation to total rower work, but take some care in calculating the terms therein. We discuss the efficiencies of oars versus propellers and show, using a simple example, that oars can have arbitrarily close to perfect efficiency. We then review previous authors' calculations of rower work and discuss why these calculations are erroneous. Finally, we suggest how oar efficiency may be improved.

2 Methods

In this section we present our definition of propulsive efficiency and outline how we calculate it using data gathered from an instrumented single scull and also more indirectly with less data but using a simple rowing model (Cabrera et al., 2006).

2.1 Definition of Efficiency

We define the *energetic* efficiency of a rowing oar, ε, as the ratio of energetic benefit to the energy cost. The calculation is over a single stroke, assumed to be one of many strokes in a periodic sequence. At a chemical level, energy cost E is due to the work done by the forces the rower exerts on the boat and oars, the energy needed to overcome joint friction, and various energy costs associated with muscle function (e.g. the cost of maintaining muscle force, muscle shortening/lengthening, muscle activation, etc.). The energetic benefit, D_{b}, is the portion of E that goes into overcoming boat drag. This definition is equivalent to the definition of *drag* efficiency posed by Pendergast et al. (2003). In this paper, however, we only consider as a cost the mechanical work done by the forces that the rower applies to the boat and oars (neglecting the work of raising, lowering, and feathering the oars) and we denote this work by W_r. Thus, we consider *propulsive* efficiency, η, and write η as:

$$\eta = \frac{\text{benefit}}{\text{cost}} = \frac{\text{boat dissipation}}{\text{rower work}} = \frac{D_{\mathrm{b}}}{W_r}. \tag{1}$$

This definition is equivalent to the definition of *Froude* efficiency posed by Pendergast et al. (2003). Ultimately, we will calculate η from on-water data. In order to do this, we derive an expression for W_r in terms of variables for which we have data.

Consider the system consisting of the boat and oars (not including the people) as shown in Fig. 1. Energy balance (over any period of time) tells us that:

$$W_{\text{ext}} = \Delta E_K + \Delta E_P + W_{\text{int}}, \tag{2}$$

where W_{ext} is the work done on the system by forces external to the system, ΔE_K is the change in the system's kinetic energy, ΔE_P is the change in the system's gravitational potential energy, and W_{int} is internal work. External work (W_{ext}) consists of the work done by the rower (W_r) and the work done by hydrodynamic and aerodynamic forces on the boat (W_{b}) and oars (W_{O})[1] . These energy sources/sinks are depicted in Fig. 1. Internal work (W_{int}) consists of energy loss due to friction at the oarlock, etc. Note that $W_{\text{b}},\ W_{\text{O}} < 0$ and $W_r > 0$. We consider the motions of the system over a single stroke and assume that these motions are periodic. Therefore, we have $\Delta E_K = \Delta E_P = 0$ and the energy balance equation (Eq. (2)) becomes:

$$W_r + W_{\text{b}} + W_{\text{O}} = W_{\text{int}}. \tag{3}$$

We denote the magnitude of boat and oar dissipation by $D_{\text{b}} = |W_{\text{b}}|$ and $D_{\text{O}} = |W_{\text{O}}|$, respectively. Then, solving Eq. (3) for W_r and substituting into the efficiency equation (Eq. (1)) we get:

$$\eta = \frac{D_{\text{b}}}{W_{\text{int}} + D_{\text{b}} + D_{\text{O}}} = \frac{\text{boat dissipation}}{\text{total dissipation}}. \tag{4}$$

Equation (4) is a generic expression for propulsive efficiency, allowing for various energy losses and models of the blade-fluid interaction. For simplicity, we make the following assumptions:

1. pitching and yawing motions of the boat are negligible,
2. the oars have identical kinematics and inertial/geometric properties,
3. air resistance on the oars is negligible,
4. losses due to internal work (bearing friction, etc.) are negligible ($W_{\text{int}} = 0$),
5. boat drag (F_{boat}) is proportional to the square of boat velocity (v_{b}),
6. the resultant oar blade force is perpendicular to the blade (as shown in Fig. 2), and
7. the rotational dissipation, from torque on the oar blade multiplied by its angular velocity, is negligible (equivalently, we assume that for energetic purposes the forces of water on the oar are statically equivalent to a single force at the center of the oar blade).

[1] Note, these definitions of internal and external work are unrelated to "internal work" and "external work" as used in some of the biomechanics literature (e.g. in Cavagna et al. (1963)).

Using these assumptions, we write the works done by boat and oar drag as:

$$W_{\mathrm{b}} = -\int_0^T F_{\mathrm{boat}} v_{\mathrm{b}}\, dt = \int_0^T -C_1 v_{\mathrm{b}}^3\, dt, \tag{5}$$

$$W_{\mathrm{O}} = \int_0^T F_{\mathrm{oar}}(\mathbf{v}_{\mathrm{O}} \cdot \hat{\mathbf{e}}_\theta)\, dt = \int_0^T F_{\mathrm{oar}}(\ell\dot{\theta} + v_{\mathrm{b}}\cos\theta)\, dt, \tag{6}$$

where T is the stroke period, C_1 is the boat drag coefficient, F_{oar} is the sum of the forces on the oar blades, $\mathbf{v}_{\mathrm{O}}$ is the velocity of each oar blade, ℓ is the outboard oar length, θ is the oar angle, and $\dot{\theta}$ is the oar angular velocity (see Fig. 2 for a geometric schematic and free body diagrams of the boat-oars-rower system). Note that the negative sign in Eq. (5) is due to the fact that boat drag opposes boat velocity. Also note that, since we neglect air resistance the forces on the oar blades are zero during the recovery phase of the stroke (when the oars are in the air). Therefore, W_{O} is non-zero only during the drive phase (when the oars are in the water).

Incorporating Eqs. (5) and (6) into Eq. (4) and using $W_{\mathrm{int}} = 0$ then gives us:

$$\eta = \frac{\text{boat dissipation}}{\text{boat+oar dissipation}} = \frac{D_{\mathrm{b}}}{D_{\mathrm{b}} + D_{\mathrm{O}}}, \tag{7}$$

where

$$D_{\mathrm{b}} = |W_{\mathrm{b}}| = \left| -\int_0^T C_1 v_{\mathrm{b}}^3\, dt \right|, \tag{8}$$

$$D_{\mathrm{O}} = |W_{\mathrm{O}}| = \left| \int_{\tau_1}^{\tau_2} F_{\mathrm{oar}}(\ell\dot{\theta} + v_{\mathrm{b}}\cos\theta)\, dt \right|, \tag{9}$$

where τ_1 is the time of *catch* (when the oars go in the water) and τ_2 is the time of *release* (when the oars come out of the water). We will use Eq. (7) for calculating η.

2.2 Method I: Calculating η From On-water Data

We compute η using data collected from a heavyweight women's single scull (Cabrera et al., 2006). This data set includes measurements of the following functions of time: v_{b}, θ (and, thus, $\dot{\theta}$ and $\ddot{\theta}$), and F_{hand_θ} (the summed components of the oar handle forces perpendicular to the oars' axes). Since we do not have direct measurements of F_{oar}, we use the following expression for F_{oar} in terms of the measured variables, found by taking angular momentum balance for the oar about the oarlock:

$$F_{\mathrm{oar}} = \frac{s}{\ell} F_{\mathrm{hand}_\theta} + \frac{I_{\mathrm{G}} + m_{\mathrm{O}} d^2}{\ell}\ddot{\theta} + \frac{m_{\mathrm{O}} d}{\ell}\dot{v}_{\mathrm{b}}\cos\theta, \tag{10}$$

where s is the inboard oar length, I_{G} is the sum of the oars' moments of inertia about their centers of mass, m_{O} is the total oar mass, and d is the distance from the oarlock to the oar's center of mass. Note that we have modeled the oars as uniform, rigid rods as in Cabrera et al. (2006). Substituting the above expression for F_{oar} into Eq. (7) gives us the following expression we use to compute η_d, the efficiency calculated from the data:

$$\eta_d = \frac{D_{\mathrm{b}}}{D_{\mathrm{b}} + D_{\mathrm{O}_d}}, \tag{11}$$

where

$$D_{\mathrm{O}_d} = \left| \int_{\tau_1}^{\tau_2} \left(\frac{s}{\ell} F_{\mathrm{hand}_\theta} + \frac{I_{\mathrm{G}} + m_{\mathrm{O}} d^2}{\ell} \ddot{\theta} + \frac{m_{\mathrm{O}} d}{\ell} \dot{v}_{\mathrm{b}} \cos\theta \right) (\ell \dot{\theta} + v_{\mathrm{b}} \cos\theta) \; dt \right|. \tag{12}$$

We compute $\dot{v}_{\mathrm{b}}$, $\dot{\theta}$ and $\ddot{\theta}$ by fitting the data for $v_{\mathrm{b}}(t)$ and $\theta(t)$ with cubic splines and differentiating with respect to time. Alternatively, we could (but do not) use finite differencing or different representations of $v_{\mathrm{b}}(t)$ and $\theta(t)$ (e. g., a Fourier series or a high-order polynomial). Cubic splines worked for us.

Since we do not know the times of catch and release, we assume that the catch and release occur so that the oar blade force is zero just after catch and just before release. Using this assumption, we calculate the catch and release times, τ_1 and τ_2, by linearly interpolating $F_{\mathrm{oar}}(t)$ (from Eq. (10)) when a change in sign occurs and finding the time, τ, when $F_{\mathrm{oar}}(\tau) = 0$ (see Fig. 3).

The values of the physical constants (C_1, s, ℓ, I_{G}, m_{O}, and d) used are the same as those used in Cabrera et al. (2006) for a single sculler and are shown in Table 1. The constants s, ℓ, and m_{O} are measured with s and ℓ being modified to account for the distance from the end of the oar handle at which the rower applies a force and the distance from the tip of the oar blade at which the resultant blade force is assumed to act. The values of d and I_{G} follow from assuming the oar is a uniform rod. The value of C_1 is a function of water density (ρ), boat wetted area (A^*), and a shape-determined constant (C^*). Since we did not have direct measurements of A^* and C^*, we assumed geometric similarity of boats and used a value of C_1 obtained from drag test data for eights (Lazauskas, 1998) to compute C_1 for singles (see Cabrera et al. (2006)).

We approximate the integrals in Eq. (11) using the trapezoidal rule. The subintervals are of equal length ($\Delta t = T/50$) except in the evaluation of D_{O} where the first and last subintervals of $[\tau_1, \tau_2]$ have slightly smaller widths. The absolute numerical error associated with the trapezoidal rule is $O(\Delta t)^2$. The force on the oar blade, the boat drag force, the times of catch and release, and the inertial properties of the oars are sufficiently inaccurate that a more accurate method of approximating the integrals is not useful. A more accurate calculation of η could be made by knowing the kinematics of the oar blade more accurately; efficiency is determined largely by how much the blade slips.

2.3 Method II: Calculating η Using Our Model

In Cabrera et al. (2006), we presented a simple model of rowing and showed that this model is capable of accurately predicting documented data (see Fig. 2 for a schematic of the model). In that study, we determined the set of body position functions ($x_{\mathrm{B/F}}(t)$, $x_{\mathrm{S/B}}(t)$, $x_{\mathrm{H/S}}(t)$) and fore-aft oarlock position ($d_{\mathrm{L/F}}$) that best fit the data by minimizing a weighted sum of the squares of the differences between the simulated and measured variables. We use the results of the best fit simulations from that study, using two different force-velocity relationships to model the oar-fluid interaction, to compute η.

2.3.1 Method II.1: Using Model 1 of the Oar Blade Force

Model 1 is that of Pope (1973). Pope assumes that the magnitude of the oar blade force is proportional to the square of the component of blade velocity in the $\hat{\mathbf{e}}_\theta$-direction. Therefore, Pope has

$$F_{\mathrm{oar}} = C_2(\mathbf{v}_{\mathrm{O}} \cdot \hat{\mathbf{e}}_\theta)^2 = C_2(v_{\mathrm{b}} \cos\theta + \ell\dot{\theta})^2, \tag{13}$$

where C_2 is the blade drag coefficient. The value of C_2 (listed in Table 1) depends on ρ, blade area (A'), and a shape and depth-determined constant (C'). The value of ρ is assumed to be 1000 kg/m^3, A' was measured, and C' is determined using the results of drag tests performed on flat plates by Hoerner (1965).

Using Model 1, we obtain the following expression for efficiency, denoted by η_1:

$$\eta_1 = \frac{D_{\mathrm{b}}}{D_{\mathrm{b}} + D_{\mathrm{O}_1}}, \tag{14}$$

where

$$D_{\mathrm{O}_1} = \left| \int_{\tau_1}^{\tau_2} C_2(v_{\mathrm{b}} \cos\theta + \ell\dot{\theta})^3 \; dt \right|. \tag{15}$$

The integrands of the dissipation terms in Eq. 14 are the work rates, $\dot{W}_{\mathrm{b}}$ and $\dot{W}_{\mathrm{O}}$. Considering these quantities as ordinary differential equations with initial conditions $W_b(0) = 0$ J and $W_{\mathrm{O}}(0) = 0$ J, we compute W_{b} and W_{O}, the integrals in Eq. 14, using the same 4-stage, Runge-Kutta algorithm we used to solve the model governing equations as discussed in Cabrera et al. (2006).

For small oar angles ($\cos\theta \approx 1$) and assuming that boat speed and oar slip speed are approximately constant, the above expression for η_1 reduces to:

$$\eta_1 \approx \frac{1}{1 + \frac{C_2}{C_1}\left(\frac{|\mathbf{v}_{\mathrm{O}}|}{v_{\mathrm{b}}}\right)^3}. \tag{16}$$

We do not use the simplified form (Eq. (16)) in our calculations.

2.3.2 Method II.2: Using Model 2 of the Oar Blade Force

Model 2 is based on experiments and numerical simulations performed by Wang et al. (2004) on a robotic fly wing. The resultant oar blade force is decomposed into lift and drag components, F_L and F_D, where the drag force opposes the direction of blade velocity and lift is perpendicular to drag as shown in Fig. 4. (Note that the lift force does no work.) The magnitudes of the lift and drag forces are assumed to be quadratic in oar speed, $|\mathbf{v}_{\mathrm{O}}|$, as follows:

$$F_L = C_L|\mathbf{v}_{\mathrm{O}}|^2, \tag{17}$$

$$F_D = C_D|\mathbf{v}_{\mathrm{O}}|^2, \tag{18}$$

where C_L and C_D are the lift and drag coefficients, respectively, and are assumed to be functions of the angle of attack, ϕ (the angle between the blade velocity relative to the fluid and the $\hat{\mathbf{e}}_r$-direction as shown in Fig. 2). The relationships between C_L, C_D, and ϕ are shown in the lift-drag polar (the plot of C_L versus C_D) in Fig. 4 and are written as follows:

$$C_L = C_L^{\max} \sin 2\phi, \tag{19}$$

$$C_D = C_L^{\max}(1 - \cos 2\phi), \tag{20}$$

where $C_L^{\max}$ is a constant. We used $C_L^{\max} = C_2/2$ in the model calculations.

Using Model 2, we obtain the following expression for efficiency, denoted by η_2, after some simplification:

$$\eta_2 = \frac{D_{\mathrm{b}}}{D_{\mathrm{b}} + D_{\mathrm{O}_2}}, \tag{21}$$

where

$$D_{\mathrm{O}_2} = \left| \int_{\tau_1}^{\tau_2} 2C_L^{\max} |\mathbf{v}_{\mathrm{O}}| (\ell\dot{\theta} + v_{\mathrm{b}} \cos\theta)^2 \; dt \right|. \tag{22}$$

The integrals in Eq. (21) are calculated using the same procedure used to calculate the integrals in Eq. (14).

3 Results

3.1 Calculations Using Method I: Raw Data

Using the data, we calculate the energy dissipation due to boat drag for a women's heavyweight single sculler over a single 1.95 second stroke to be $D_{\mathrm{b}} = 487$ J, the energy dissipation due to oar drag to be $|W_{\mathrm{O}}| = 96$ J, and a propulsive efficiency of $\eta_d = 0.84$. That is, about 84% of the total energy dissipated during a stroke is due to boat drag and the remaining 16% is due to oar drag. The oar blade force, blade velocity, and oar dissipation rate are shown by the solid lines in Fig. 5. Note that, presumably because of modelling errors, there is a brief period at the end of the drive phase when the oar dissipation rate is slightly negative (Fig. 5c,f), implying that energy is being pumped into the system. We neglect this ≈ -1 J in our energy integral.

3.2 Calculations Using Method II.1: Incomplete Data

Here we use compute efficiency from best fits of the rowing model to raw data using Model 1 of the blade force. Using our nominally chosen value of C_2, we calculate $D_{\mathrm{b}} = 483$ J, $|W_{\mathrm{O}}| = 158$ J, and an efficiency of $\eta_1 = 0.75$. The predicted blade force, blade velocity, and oar dissipation rate are shown by the dashed lines in Fig. 5a–c. The predicted boat drag dissipation agrees well with the value of $D_{\mathrm{b}} = 487$ J obtained from the data calculations above (less than 1% difference). However, the predicted oar dissipation (158 J) is 65% greater than the dissipation calculated from the data (96 J). As seen in Fig. 5, the blade force is well-predicted (Fig. 5a) but the blade velocity is not (Fig. 5b), presumably, due to the lack of fit in θ (and, thus, $\dot{\theta}$ and $\ddot{\theta}$).

In Cabrera et al. (2006), we showed that selecting a value of oar drag coefficient C_2 that is 2.4 times as large as the nominal value used above produces a better fit of our model to the data, especially in the fit to oar angle. From the best fit simulation using Model 1 and this new value of the oar drag constant ($C_2^* = 2.4C_2$), we calculate $D_\mathrm{b} = 493$ J, $D_o = 101$ J, and $\eta_1 = 0.84$ which agree more favorably with the values obtained from the data (less than 2% difference in boat dissipation and less than 6% difference in oar dissipation). Plots of the blade force, blade velocity, and oar dissipation rate are shown by dashed-dotted lines in Fig. 5a–c. We see a better prediction in the blade velocity (Fig. 5b) and, thus, a better prediction of oar dissipation rate (Fig. 5c).

In Wang (2005), it is noted that in a study of the transient motion of a plate accelerated from rest to a constant velocity, the maximum lift force is up to 50% greater than the average lift force during steady-state. This study suggests that a larger value of C_2 than the nominally chosen values be used in our model. Although the study does not account for the even larger value of C_2 used above (2.4 times the nominal value), it is noted that the study does not account for surface effects which tend to increase drag (Hoerner, 1965).

3.3 Calculations Using Method II.2: Incomplete Data

Here we use compute efficiency from best fits of the rowing model to raw data using Model 2 of the blade force. We calculate $D_\mathrm{b} = 489$ J, $D_o = 118$ J, and $\eta_2 = 0.81$. Plots of the blade force, oar velocity, and oar dissipation rate are shown by dashed lines in Fig. 5d–f. Again, the predicted boat dissipation (489 J) agrees well the dissipation calculated from the data (487 J), less than a 1% difference. The predicted oar dissipation (118 J) is 23% greater than the dissipation calculated from the data, although the difference is not as marked as the difference seen above when using Model 1 and the nominally chosen value of C_2.

Although not shown in Cabrera et al. (2006), we determined that selecting a value of $C_L^{\max}$ that is 4/3 times as large as the nominal value produced a better fit of our model to the data. From the best fit simulation using Model 2 and this new value of the maximum lift coefficient ($C_L^{\max *} = (4/3)C_L^{\max}$), we calculate $D_\mathrm{b} = 492$ J, $D_o = 97$ J, and $\eta_2 = 0.84$. Plots of the blade force, blade velocity, and oar dissipation rates are shown by dashed-dotted lines in Fig. 5d–f.

See Table 2 for a summary of the above results.

4 Discussion

4.1 Oar vs. Propeller Efficiency

Oar dissipation comes from its slip velocity (motion through the water orthogonal to the blade) multiplied by the oar force. The useful work of the oar comes from the boat velocity multiplied by the oar force. Thus, for a given boat speed, oar efficiency is maximized by minimizing the oar slip. Small slip velocities (and, thus, oar efficiencies arbitrarily close to 1) can be generated for any oar, no matter how crudely shaped, by moving the blade slowly through the water. Although a consequence of moving the oar slowly is that the generated thrust at the blade is very small (and, thus, not a very useful thing to do), it is theoretically possible to obtain near perfect efficiency without having to create a special blade geometry.

To make this idea definite, imagine riding on a steadily moving 100 meter long barge as shown in Fig. 6. Dip an oar of arbitrary shape into the water near the bow of the barge. Then walk slowly to the stern, walking a shade faster than the rate at which the barge is moving so that you walk the 100 meter length of the barge as the barge advances forward 99 meters. Say the force on the oar, and thus the reaction force of your feet on the barge, is 1 Newton. The oar will slip in the water 1 meter while the barge advances 99 meters. The oar dissipation is 1 Joule, the work you have done on the barge and oar is 100 Joules, and the useful work (barge displacement times propulsive force) is 99 Joules. The propulsive efficiency of the oar is thus 99%. Of course, as noted, getting oars of this efficiency to actually move a boat would require thousands of rowers or gigantic oars, and these in turn would engender other costs. But, fundamentally, oar efficiency can be driven arbitrarily close to 1 without sophisticated oar design. In our more complex model, say, this result would be obtained by making the oar drag coefficient arbitrarily large.

Propellers, on the other hand, cannot so easily have such high efficiencies. From blade element theory (see Dommasch (1953), for example), the forces on a propeller blade are modeled like an airfoil. The resultant force on each spanwise element of the blade consists of a lift and a drag force. By convention, the lift force does no work so dissipation is solely due to drag. Therefore, to minimize blade dissipation (and, thus, maximize efficiency), blade drag must be minimized. Propeller efficiency is inextricably linked to the lift-to-drag ratio of the blades. At non-infinite Reynolds numbers the lift-to-drag ratio is necesarily removed from infinity and propeller efficiency cannot approach 1. Typical recreational motor boat propeller efficiencies are about 50-65% whereas the very best marine propellers have efficiencies of about 70-80% (Krueger, 2005). Note that for propellers drag is essentially dissipative whereas for oars it is an essential part of the propulsion.

The essential difference between propellers and oars is that propellers are stuck in the water and oars are not. The fair comparison would be to require oars to have the recovery portion of the stroke entirely submerged. In this case, of course, we would find that oars were generally not very efficient as the oar drag during the recovery would be a significant loss. However, in real rowing, as opposed to real propeller propulsion, we have the recovery phase of the stroke out of the water, in the thin air, and thus incuring small (negligible in our calculations here) cost for the motion of the blade in the direction of the boat.

4.2 Comparison With Previous Calculations of Efficiency

We now discuss the efficiency calculations of previous authors.

Alexander (1927) defines efficiency using Eq. (1) where he calculates W_r as the work done at the oar handles and $D_{\rm b}$ includes dissipation due to both hydrodynamic and aerodynamic drag. He calculates $D_{\rm b}$ using the equation:

$$D_{\rm b} = R\bar{v}_b\tau, \tag{23}$$

where $\tau = 1$ min, v_b is average boat speed, and $R = 0.557\bar{v}_b^{2.012}$ is the resistance when $v_b = \bar{v}_b$. Alexander does not state the equation he uses to calculate W_r. Instead, he offers an ambiguous description of this calculation. He calculates $D_\mathrm{b} = 91,800$ ft lb and $W_r = 148,400$ ft lb using data gathered for an eight, resulting in an efficiency of $\eta = 0.619$. However, this efficiency calculation is faulty for two reasons. First, in his calculation of D_b, Alexander ignores the variation of boat velocity. He uses

$$D_\mathrm{b} = 0.557\left(\frac{1}{\tau}\int_0^\tau v_b\ dt\right)^{3.012}\tau, \tag{24}$$

instead of the proper equation:

$$D_\mathrm{b} = 0.557\int_0^\tau v_b^{3.012}\ dt. \tag{25}$$

Second, Alexander does not take into account the work done by the rower at the footstretcher and he only accounts for the displacement of the oar relative to the boat (not the absolute displacement) in his calculation of the work done at the oar handles.

Wellicome (1967) defines efficiency as:

$$\eta = \frac{W_u}{W_r}, \tag{26}$$

where W_u is "useful work." The equations he uses for calculating W_u and W_r are as follows:

$$W_u = \int_0^T |\mathbf{F}_\mathrm{oar}| v_b \cos(\theta - \alpha)\ dt, \tag{27}$$

$$W_r = \int_0^T |\mathbf{F}_\mathrm{oar}| s\dot{\theta}\cos\alpha\ dt, \tag{28}$$

where α is the angle between the force on the oar blade and $\hat{\mathbf{e}}_\theta$. Using data gathered for an eight, he calculates $\eta = 0.664$ for an eight. Due to his use of a reference frame attached to the boat, Wellicome does not calculate the work done at the footstretcher when calculating W_r and the velocity of the oar handle he uses in Eq. (28) is relative to the boat, not the absolute velocity. Thus, Wellicome miscalculates the actual work.

Celentano et al. (1974) define efficiency as:

$$\eta = \frac{W_r}{W_r + D_o}. \tag{29}$$

They calculate W_r and D_o using the relations:

$$W_r = \bar{F}_{\mathrm{oar}_x}\bar{v}_\mathrm{b}(\tau_2 - \tau_1), \tag{30}$$

$$D_o = \bar{F}_{\mathrm{oar}_x} r, \tag{31}$$

where r is the blade slip distance. Using a previous author's measurement of $\bar{v}_\mathrm{b}$ and r, they calculate $\eta = 0.7$. The boat type for this calculation is not stated. Again, like Alexander and Wellicome, Celentano et al. make the mistake of calculating rower work in a frame of reference attached to the boat. Furthermore, they compute the above work quantities as a product of average force and velocity instead of integrating the time varying product.

Affeld et al. (1993), Kleshnev (1999), and Atkinson (2004) calculate efficiency using a form equivalent to that of Alexander's:

$$\eta = \frac{W_r - D_o}{W_r}. \tag{32}$$

Affeld et al. and Kleshnev calculate W_r as:

$$W_r = \int_0^T F_{\mathrm{hand}_\theta} s\dot{\theta}\ dt. \tag{33}$$

They assume that the resultant force on the oar blade is perpendicular to the blade and break up this force into its lift and drag components (as described above in the Model 2 section). They then calculate D_o using the equation:

$$D_o = \int_{\tau_1}^{\tau_2} F_D|\mathrm{v_O}|\ dt, \tag{34}$$

where F_D and $|\mathbf{v}_\mathrm{O}|$ are determined from the measured oar handle force, boat velocity, and oar angular velocity. Kleshnev does not provide an equation for computing D_o but it is presumably the same as Eq. (34) and Atkinson does not provide any equations for computing W_r and D_o. Using multiple data sets obtained from a single, Affeld et al. compute values of η ranging from 0.7 to 0.75. Kleshnev computes η-values of 0.79, 0.82, 0.84, and 0.85 for a single, pair/double, four/quad, and eight, respectively, using data obtained from these boat types. Atkinson computes $\eta = 0.768$ for a single using his rowing model which is able to well-predict boat velocity. Like other authors, they all fail to consider the work done by the rower on the footstretcher. Due to their use of a non-Newtonian reference frame they incorrectly calculate W_r using the oar handle velocity relative to the boat instead of its absolute velocity.

The problem with calculating rower work in the moving boat reference frame is perhaps clarified with the example of Fig. 7. Let's imagine a crazy rowing stroke, not one that anyone actually uses, but one that illustrates the calculation problem. Imagine a rower moving fore and aft in the boat by periodically extending her legs. If the oars were entirely out of the water the rower would move fore and aft, relative to a fixed reference frame, while the boat moves fore and aft in the opposite direction (approximately out of phase if the water dissipation is small). Now imagine that the rower moves her hands fore and aft in a manner exactly opposite to her leg motion so that the oar is always orthogonal to the boat, and the oar handle has no fore and aft motion relative to the boat. Furthermore, let the rower raise and lower her hands so that the oar is only in the water when the boat would, if there were no oar in the water, be moving backwards. Thus, backwards motions of the boat are more-or-less stopped and the boat moves, on average, forward. The rower is effectively using the oar to make a ratchet. The boat moves forward, there is boat drag and dissipation. The rower does work. Yet, in this way of rowing the work calculated, as calculated by the above authors where the work is calculated by oar handle motion relative to the boat, is zero. That is, for this rowing stroke the moving-reference frame work calculation would give an efficiency of positive infinity. Whereas the definition of efficiency we have chosen is thermodynamically necessarily no greater than 1. If we calculate rower work incorrectly using the methods of Affeld et al. (1993) and Kleshnev (1999) in Method I we get a rower work of 488 J per stroke which is 16 % less than our (hopefully better estimate) here of 583 J.

4.3 How to Improve Oar Efficiency

An obvious conclusion of our calculations here is that oar efficiency can be made greater and greater by increasing the oar drag coefficient C_2. This, in turn, can be increased by increasing the oar blade area. Although this insight might be useful, it has its limitations in terms of incurred costs. For example, there is a cost of lowering and raising a large blade, of learning a new coordination, of carrying the extra weight, of extra air drag, etc. Taking these (neglected here) losses into account, both in terms of their costs and making a design which minimizes these costs, it remains an open question whether the efficiency of real oars can be usefully improved by further increasing thier area.

5 Conclusion

We have proposed a new (rational) measure of oar efficiency and, using on-water data and a simple rowing model, calculated $\eta \approx 0.84$ for a sculling oar, an efficiency higher than that of good nautical propellers. Efficiencies were calculated using two models of the oar blade force and both models were capable of accurately predicting the boat and oar dissipations. In order to obtain good agreement with oar dissipation calculated from the data, we used values of the oar drag coefficients that were larger than the nominal ones chosen. The need for larger drag coefficients may be accounted for by the fact that the transient forces may be larger than those experienced at steady-state. We have also shown that prior definitions of oar efficiency all suffer from at least one deficiency, the most common one being that they fail to account for the work done at the foot stretcher when calculating rower work.

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Table 1: Listed are the values of the variables fixed in the simulations. Note that boat mass is the sum of the fully rigged boat mass and the mass of the data collection equipment.

Variable	Value	Description
T	1.94 s	stroke period
s^*	0.89 m	actual inboard oar length
s	0.83 m	modified inboard oar length ($s^* - 0.06$ m)
ℓ^*	2.02 m	actual outboard oar length
ℓ	1.805 m	modified outboard oar length
		(ℓ^*-(blade length)/2)
m_{O}	2.4 kg	oar mass (for 2 oars)
C_1	3.16 N/(m/s)^2	boat drag coefficient ($F_{\mathrm{boat}} = C_1 v_{\mathrm{b}}^2$)
C_2	58.7 N/(m/s)^2	oar drag coefficient ($F_{\mathrm{oar}} = C_2(\mathbf{v}_{\mathrm{O}} \cdot \hat{\mathbf{e}}_\theta)^2$)
d	0.565 m	distance from oarlock to oar center of mass
I_{G}	1.70 kg m^2	oar moment of inertia ($2m_{\mathrm{O}}(\ell^* + s^*)^2/12$)

Table 2: This table summarizes the calculated values of boat and oar dissipation (W_{b} and W_{O}) and the resulting propulsive efficiencies (η) using three methods. In Method I, calculations are performed on raw data. In Method II.1, calculations are performed on incomplete data which are the results from the best fit of the rowing model to raw data where we use Model 1 of the blade force. In Method II.2, calculations are the same as those of Method II.1 but using Model 2 of the blade force. In Method I we calculate the blade force using the measured oar handle force, oar kinematics, and boat velocity. Thus, we do not report a drag coefficient.

Method	Oar Drag Coeff.	W_{O} (J)	W_{b} (J)	W_{r} (J)	η
I		96	487	583	0.84
II.1	C_2	158	483	641	0.75
II.1	$2.4C_2$	101	493	594	0.84
II.2	$C_L^{\max}$	118	489	607	0.81
II.2	$(4/3)C_L^{\max}$	97	492	589	0.84

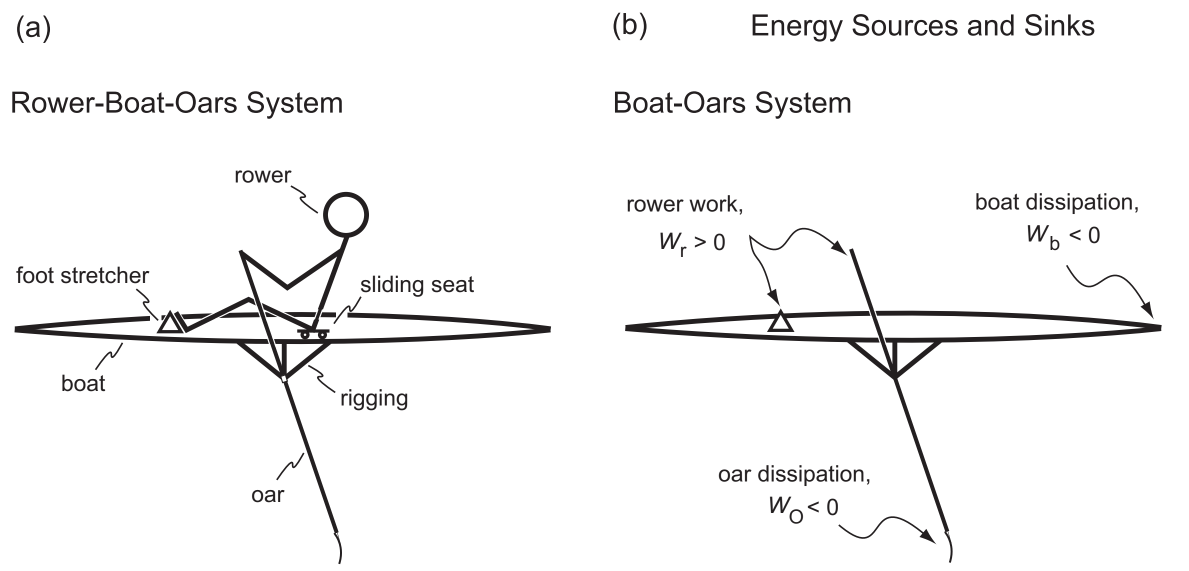

Figure 1: (a) A schematic of the rower-boat-oars system. (b) The boat-oars system, showing the energy sources (rower work) and sinks (boat and oar dissipation) due to forces external to the system.

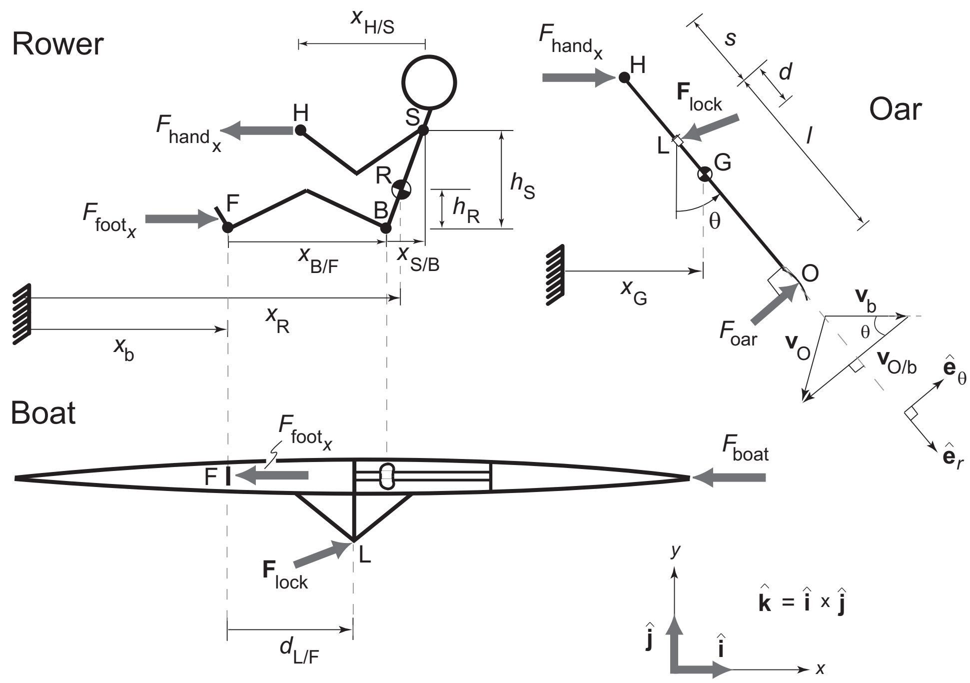

Figure 2: A geometric schematic and free body diagrams of the rower-boat-oars system. Descriptions of variables not mentioned in the text may be found in Cabrera et al. (2006). When calculating efficiency using both the data and the model, we assume that oar rotation occurs in a plane parallel to the water surface, the resultant force on the oar blade is in the $\hat{\mathbf{e}}_\theta$-direction, and boat drag is in the x-direction. In our model we also assume a point mass rower and we only consider the fore-aft motions of the rower's legs, back, and arms. Furthermore, we neglect the variation in the rower's shoulder height and center of mass height from the seat.

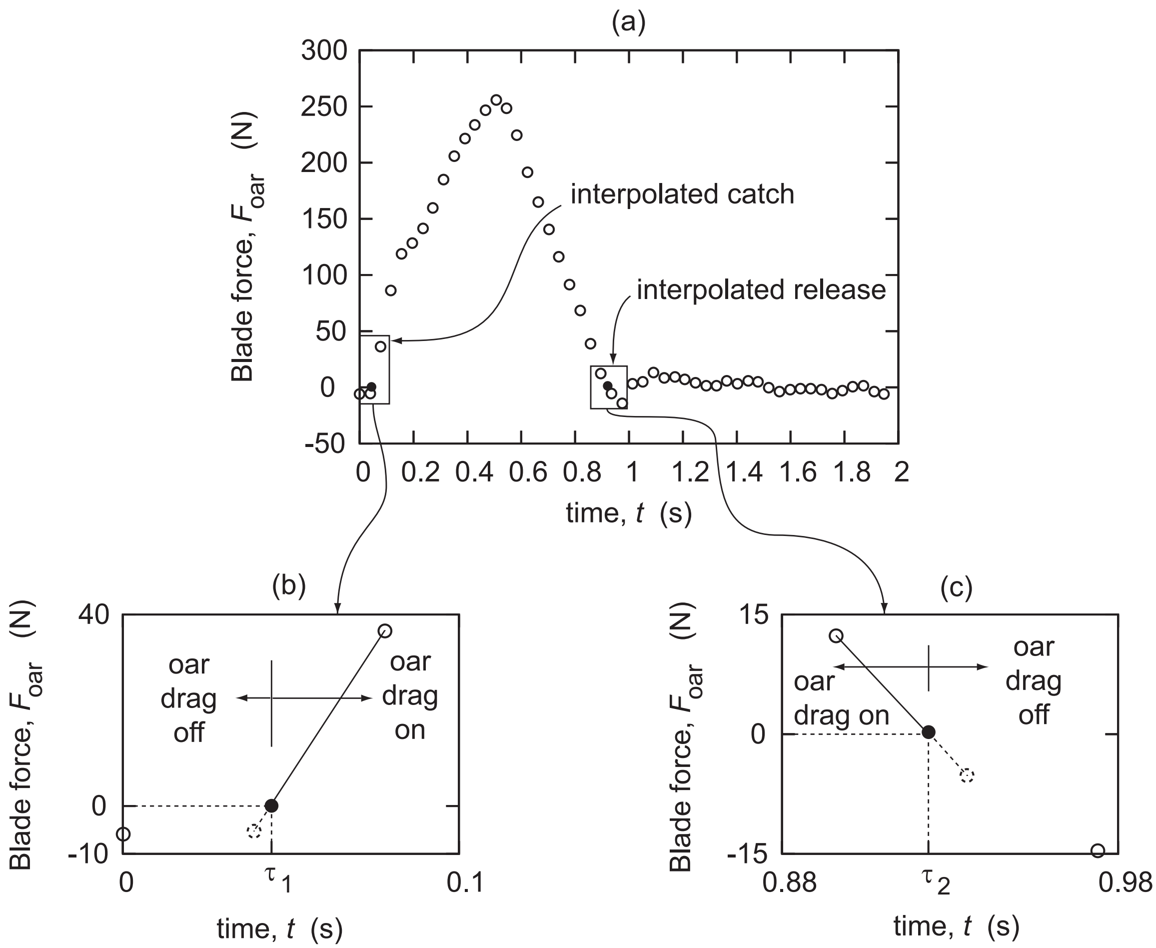

Figure 3: Shown are plots of the blade force (F_{oar}) versus time (t) as determined by Eq. 10 and measured data for F_{hand_θ}, v_{b}, and θ. Post processing of the data (as described in Cabrera et al. (2006)) produced 51 data points. The calculated values of F_{oar} based on these data points are plotted as open circles. The filled circles correspond to the calculated catch and release times, τ_1 and τ_2. By assumption, τ_1 and τ_2 are the times when $F_{\mathrm{oar}} = 0$ N. These times are calculated by linearly interpolating the data when F_{oar} changes sign. Plots (b) and (c), blowups of the regions in plot (a) when the sign changes occur, show the interpolation process. When calculating the oar dissipation, we take the oar force to be zero when $t \leq \tau_1$ and $t \geq \tau_2$.

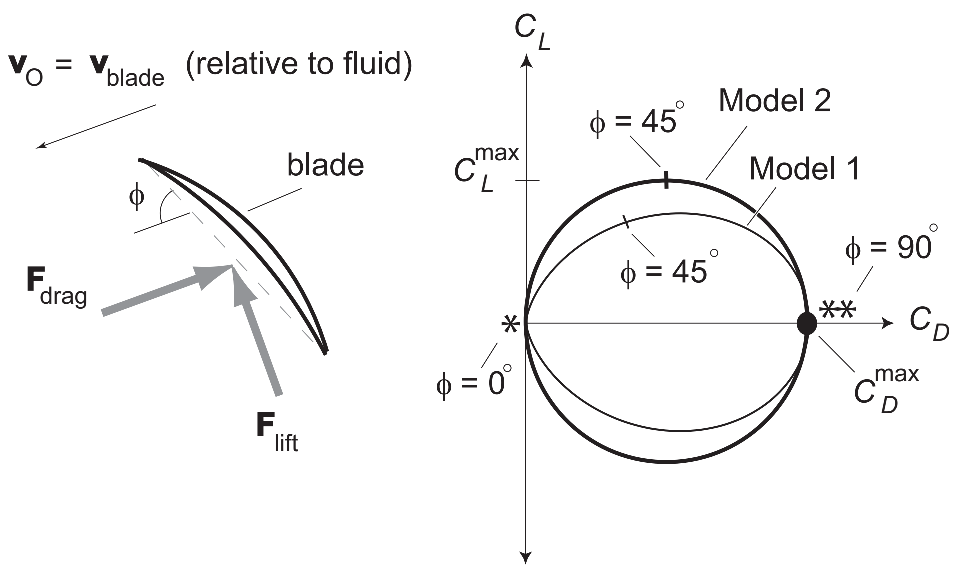

Figure 4: The diagram shows the directions of the lift and drag forces used in Model 2 of the oar blade force as well as the lift-drag polar for both models. Drag opposes the oar velocity, $\mathbf{v}_{\mathrm{O}}$, while lift is perpendicular to drag. Since lift is always perpendicular to $\mathbf{v}_{\mathrm{O}}$, the lift force does no work. Also shown is the angle of attack, ϕ, which is the angle between $\mathbf{v}_{\mathrm{O}}$ and the $\hat{\mathbf{e}}_{\theta}$-direction.

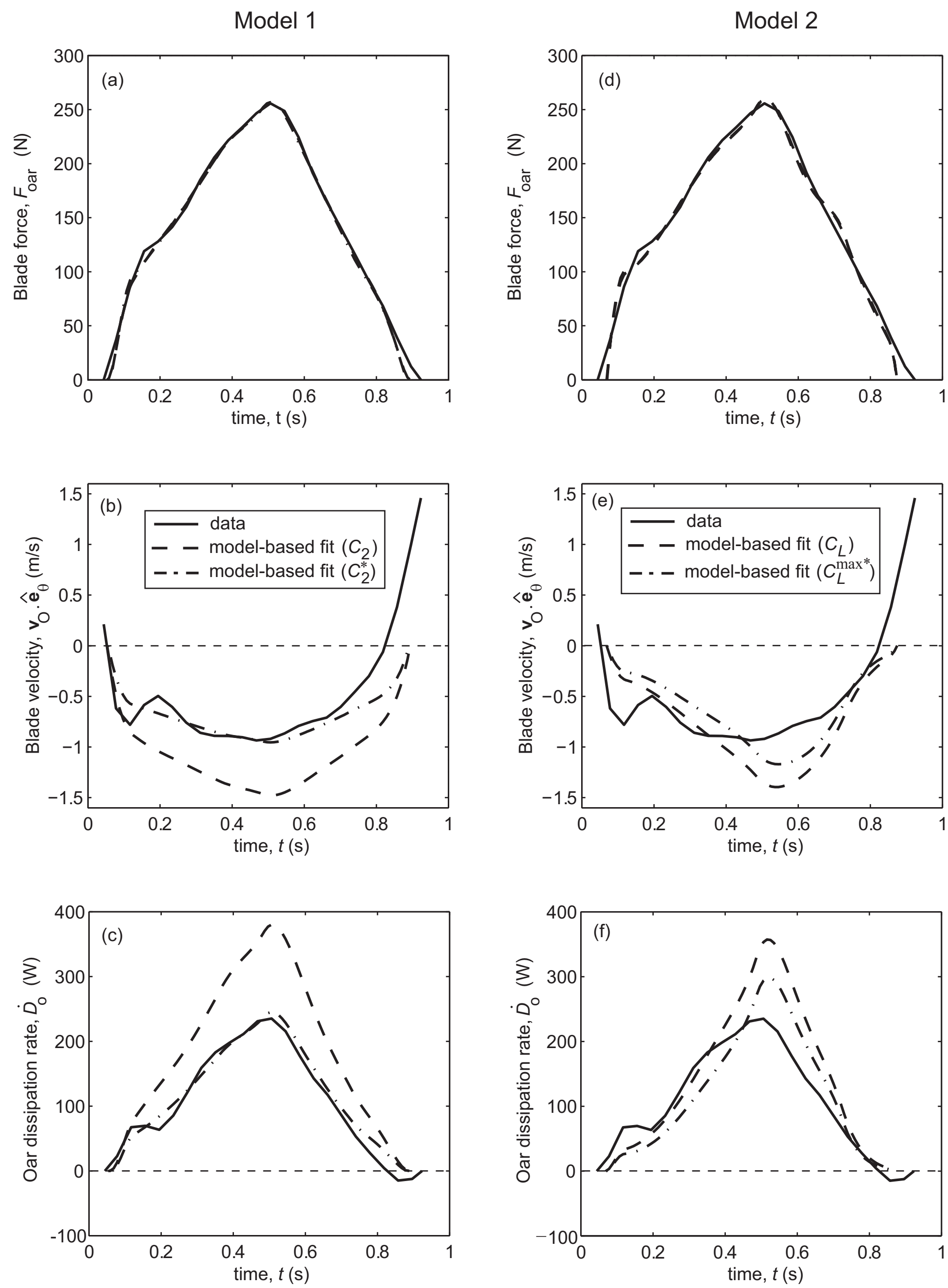

Figure 5: Shown here are plots of F_{oar}, $\mathbf{v} \cdot \hat{\mathbf{e}}_\theta$, and D_o versus time for the drive phase of the rowing stroke. The solid lines correspond to calculations from the data. In the first column of plots (a-c), the dashed lines are the resulting quantities from the best fit simulation of singles using Model 1 and the value of C_2 shown in Table 1 and the dashed-dotted lines are the resulting quantities from the best fit simulation using Model 1 and a value of C_2 that is 2.4 times the nominal value. In the second column of plots (d-f), the dashed lines are the resulting quantities from the best fit simulation of singles using Model 2 and $C_L^{\max} = 2C_2$ (where C_2 is the value shown in Table 1) and the dashed-dotted lines are the resulting quantities from the best fit simulation using Model 2 and a value of $C_L^{\max}$ that is 4/3 times the nominal value.

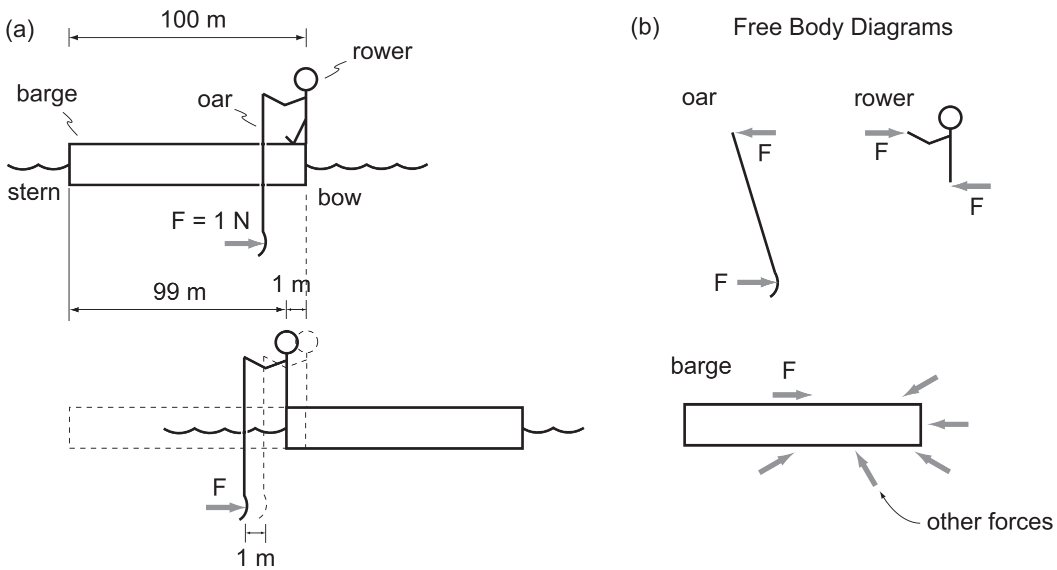

Figure 6: A schematic of the example discussed in the text illustrating how oar efficiency can be made arbitrarily close to 1. In the example, a rower stands on a barge moving steadily to the right. The rower sticks an oar in the water and walks from the bow to the stern a shade faster than the rate at which the barge is moving. When the rower reaches the stern, the boat has moved 99 m to the right and the rower has moved 1 m to the left as indicated in (a). We assume that the force of the water on the oar blade is $F = 1$ N. The rower's acceleration is small enough that we consider both the rower and oar to be approximately in equilibrium. Thus, as indicated in (b), the force at the oar handle, the force between the rower's feet and the barge, and the oar blade force are $F = 1$ N.

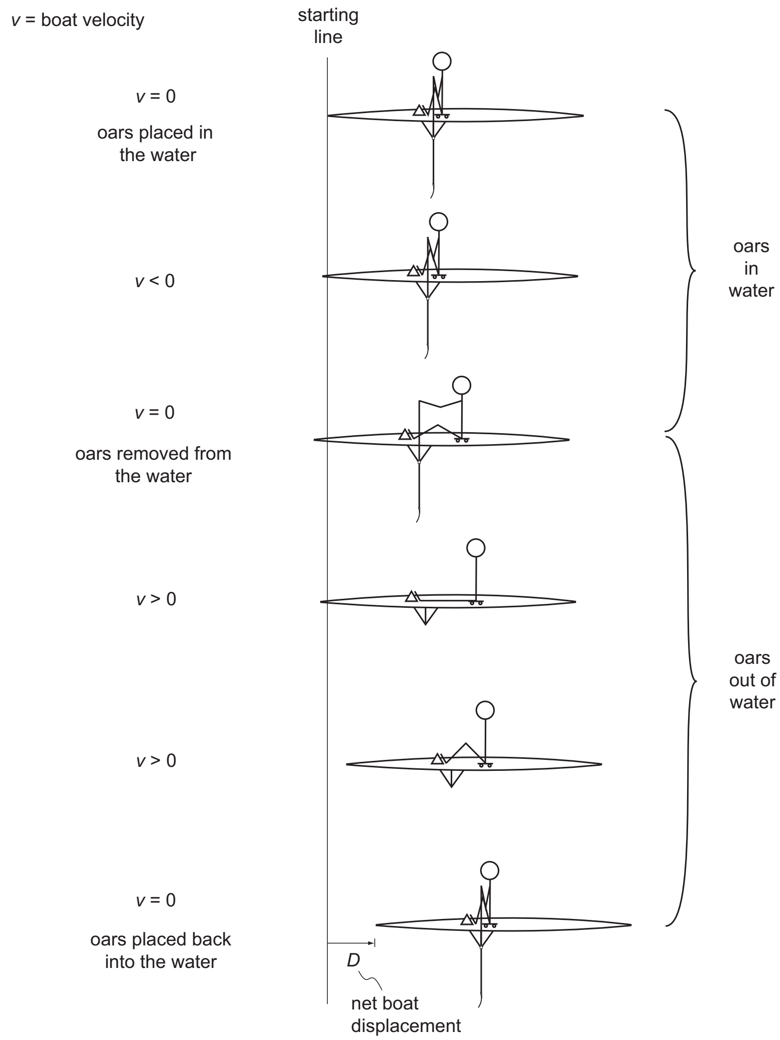

Figure 7: A schematic of the example discussed in the text illustrating the incorrectness of calculating rower work by multiplying oar torque and oar angular displacement. In the example, the rower oscillates relative to the boat and places the oar in the water when the boat is moving to the left ($v < 0$ where v is boat velocity) while keeping the oar perpendicular to the boat's axis. The net boat displacement, D, is positive but the work done by the rower is zero as calculated by some previous authors. This leads to an error of about 16 % when applied to more common coordinations.