Some Results In Passive-Dynamic Walking *

Mariano Garcia Andy Ruina † Michael Coleman

Anindya Chatterjee ‡

Department of Theoretical and Applied Mechanics

Cornell University, Ithaca, NY 14853 USA

e-mail: garcia@tam.cornell.edu, ruina.cornell.edu, coleman@tam.cornell.edu

Abstract

Human walking can be approximated as a mechanical process governed by Newton's laws of motion, and not controlled. Tad McGeer first demonstrated, and we have confirmed, that a two-dimensional legged mechanism with four moving parts can exhibit stable, human-like walking on a range of shallow slopes with *no* actuation and *no* control (energy lost in friction and collisions is recovered from gravity). More recently, we have found a simple walking mechanism that also balances from side to side. There is much that might be understood about walking by considering it as a natural motion of a simple uncontrolled and unpowered dynamical system, or a *passive-dynamic* system. As seen from a control perspective, our work largely involves investigation of control parameters which are physical properties rather than the traditional active-control parameters (such as feedback gains, neural net parameters, genetic algorithm reward schemes, etc.). We are testing the hypothesis that human walking is based on an uncontrolled mechanical process by designing, building, and studying uncontrolled or minimally-controlled walking devices and seeing how well they mimic human motion.

1 Introduction

Coordinated motion, locomotion, and walking in particular, are central aspects of human behavior. So furthering our understanding of them has a wide range of applications. Because human motion is controlled by the nervous system and powered by muscles, the role of nerves and muscles is of central interest. One way to understand the role of nerves and muscles is to learn how much can be done without them. Human walking, for example, might be modeled for some purposes as an uncontrolled mechanical process. The role of the nerves and muscles in walking might be one of gentle guidance rather than imposing control.

The approach here was originally pioneered by McGeer (1989-1993) [13, 14, 16, 15]. McGeer demonstrated that a somewhat anthropomorphic, two-dimensional, four-link mechanism is capable of stable, human-like gait down a shallow slope with no activation (besides gravity) and no control. McGeer's passive-dynamic theory of bipedal locomotion describes gait as a natural repetitive motion of a dynamical system or, in the language of nonlinear dynamics, a *limit cycle*. The simulated stick figure in figure 3f on page 5 (or a video of the unpowered robot) shows the similarity between McGeer-like passive mechanisms and bipeds. This likeness suggests that a good way to learn about human and robot walking may be to learn about passive-dynamic walking.

*This Euromech conference paper is based largely on a funding proposal submitted to the National Science Foundation in December 1997 [18].

1.1 Passive-Dynamic Walking Models

Pure passive-dynamic models are built (theoretically or physically) from passive elements (rigid bodies, springs, dashpots, hinges, frictional and rolling contact) with power coming only from gravity. These uncontrolled models can have the following remarkable properties:

1. **Existence of gait.** With no control, they can have periodic motions that resemble bipedal walking.
2. **Efficient gait.** The passive walkers can have remarkably high efficiency (approaching theoretically-perfect efficiency).
3. **Stable gait.** For some parameter combinations, the gait limit cycles are stable with respect to small perturbations.

2 Overview of Results

2.1 Rimless Wheel in 3 Dimensions

McGeer's 2-D rimless wheel pivots and collides with the ground on rigid spokes instead of rolling. It shares with walking the feature that translation occurs by intermittent non-slipping contact. It is the simplest model that yields, in part, the scaling laws that apply to more complex models.

In 2-D, the rimless wheel cannot fall down, but the 3-D rimless wheel is not constrained from falling down sideways. Because rolling coins, wheels, disks, etc. don't fall over in theory, the stability of the rimless wheel might not seem surprising. However, rolling flat disks are only neutrally stable against lean perturbations (perturbations never decay), whereas the 3-D rimless wheel can be asymptotically stable (small perturbations decay). Our discovery that, even with no hip spacing, intermittent contact augments side-to-side stability in a rolling-like motion raises the possibility that a similar passive processes could contribute to human side-to-side balance [4].

2.2 The Simplest Walker in 2 Dimensions

The simplest walking mechanism with swinging legs that can fall down, and thus uses a balance mechanism of some sort, is the simplified point-foot straight-leg 2-D walker of figure 1a [8]. It is a double pendulum with a big point mass at the 'hip' and much smaller point masses at the 'feet'. It is a simpler version of the theoretical model being studied independently by Goswami and others [9, 20, 10] (who have independently found and/or reproduced some of the results discussed below).

The simplest walker is a deterministic generalization of Alexander's non-deterministic theoretical "minimal biped," [1]. For the simplified point-foot walker the stance leg is an inverted pendulum, while the swing leg is a pendulum whose hinge point moves. At heelstrike, angular momentum balance determines the jumps in joint-angle rates. In our simulations we allow a no-impact swing through at the otherwise inevitable foot scuffing of all 2-D straight-legged walkers.

After nondimensionalizing the governing equations, this walking model has *no* free parameters other than the ground slope γ. This drastic simplification of walking has surprising properties that carry over to the kneed theoretical models and perhaps to human gait [1, 8, 7]. We found two gaits (period-one limit cycles) at every small slope. One of these gaits is unstable and one is stable at shallow slopes ($\gamma < 0.015$). For both of these gaits the stance angle, the angular velocities of the legs, and average transit velocity, scale as the cube root of the slope $\gamma^{1/3}$. Swing period τ scales as a constant plus $\gamma^{2/3}$.

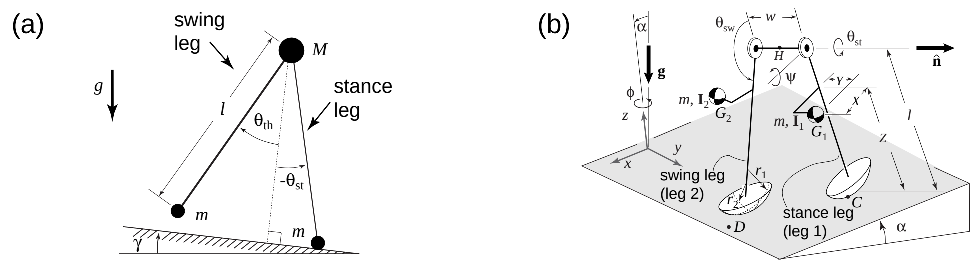

Figure 1: **(a) The 2-D point-foot theoretical walking model** from [8]. Hip mass dominates foot mass. **(b) A general 3-D knee-less theoretical model**: from [5]. This theoretical model may be sufficient to explain the working physical tinkertoy walker.

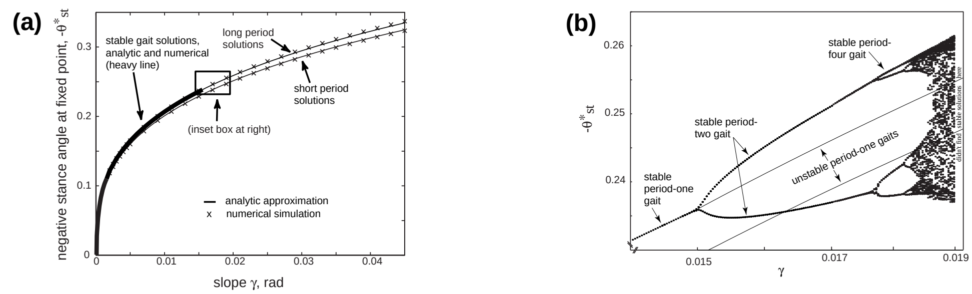

Figure 2: (a) Point-foot stance angle at fixed point as a function of slope, both numerical and analytic predictions are shown, from [8]. At zero slope, the gait is perfectly efficient and infinitesimally slow. The short period semi-analytic solution is $-\theta^*{}_{st} \approx 0.943976\gamma^{1/3} - 0.244101\gamma$. The long period semi-analytic solution is $-\theta^*{}_{st} \approx 0.970956\gamma^{1/3} - 0.234372\gamma$. (b) Period doubling of stable walking motions, inset from previous figure. Unstable period-one cycles are shown for reference. (dotted lines represent stable cycles while solid lines represent unstable ones.) No *stable* walking was found at slopes above ≈ 0.019.

Figure 2a shows stance angles for the short and long period-one gaits, plotted as a function of γ. The region of stable period-one gait bifurcates into a stable period-two gait as the period-one motion becomes unstable. As γ is varied from 0.017 to 0.019, we observed the standard period-doubling route to chaos [19], as shown expanded in figure 2b.

2.2.1 Efficiency of the 2-D point-foot walker

The standard measure of transport cost or transport inefficiency is energy used per unit distance traveled per unit weight carried (where a value of zero is perfectly efficient). This measure is the slope γ for passive downhill machines on small slopes. If gait cycles existed on level ground ($\gamma = 0$), they would be perfectly efficient. In fact, stable gait cycles *do* exist at arbitrarily small slopes, but only at correspondingly slow speeds.

After redimensionalizing the equations, the small-slope scaling rule governing gravitational power usage is [8],

$$\text{Power} = C \cdot m \cdot g^{-1/2} \cdot \ell^{-3/2} \cdot v^4 \tag{1}$$

where C is, for example, $\pi^3/8$ for the short-period gait. For a 50 kg person with a leg length of 1 m walking at 1 m/s, this predicts a reasonable 60 watts.

2.2.2 Straight legged walker in 3-D

McGeer [16] and Fowble and Kuo [6] were unable to find stable walking motions for a 3-D passive walker. Although the asymptotic stability of the rimless wheel (above) inspires some hope, it has gyroscopic terms to help with stabilization that theoretical and physical walking models cannot access. However, work on skateboards by [12] as well as our previous work with bicycles [11] and boats [2] shows that passive balance stability does not necessarily depend on gyroscopic terms. Moreover, the nonholonomic nature of walking may contribute to its stability as well [17] [5].

Thus informed, we have begun investigation of a point-foot walker in three dimensions. The special mass distribution of the simplest 2-D point-foot walker has singular equations of motion in 3-D, so we have used a more general mass distribution. For certain mass distributions that are planar or have planar symmetry, the 3-D walker is known from 2-D analysis to have 2-D walking solutions that are stable against in-plane perturbations. Although our numerical attempts to find stable 3-D walking have failed thus far [3], they led to some insight into stabilizing techniques, which in turn led to a simple successful physical 3-D passive-dynamic walker [5]. This is the only known (to us) passive-dynamic walker that can stably walk, but that cannot stand still in any configuration. Although the mass distribution in this physical model is not anthropomorphic, its success hints at a possible role for passive-dynamics in side-to-side balance as well as fore-aft balance.

2.3 Passive-Dynamic Walking With Knees

Our 2-D kneed walking theoretical and physical models [7], based closely on McGeer's models, are shown in figure 3a-c on page 5. Figure 3d shows one of our dynamic simulations using the theoretical model for just over one step.

The physical 2-D kneed walker of figure 3c exhibits stable limit cycle motions which somewhat resemble human gait. Each simulated solution corresponds to a point on the locus-of-solutions diagram in figure 3e. This plot shows stance angle at a fixed point plotted as a function of slope. For comparison, 3e also shows the locus of solutions for the corresponding kneeless walker.

As inspired by the straight-legged walker results, we found how to theoretically predict perfectly efficient, walking at slope $\gamma = 0+$, kneed and straight-leg walkers [7]. Simulations verify that such mass distributions do lead to perfectly efficient walking. The efficient mass distribution requires colinearity of the nominal contact point, the center of mass, and the hip-joint with the ground normal. Further, the shank center of mass must lie on a vertical line through the knee.

Figure 5 demonstrates the effect of tuning a generic walker. As the center of mass locations and the moments of inertia are varied, the curve of figure 3e turns into a cube-root curve like that of figure 2a. The cusps seem to represent a sudden shift in the dynamics of the walkers. At low enough slopes, the knees have little effect on the gait, but at a certain critical slope, their effect becomes noticeable. Also note a similar result in figure 3e, which compares the un-tuned kneed walker to the same walker with locked knees.

Like the point-foot walker, the kneed walker can also exhibit complex motions. If parameters are appropriately adjusted (but symmetrically), period-two gait cycle motions (limping) can be found as well as chaotic (stumbling) gait [7].

3 Procedure of Study

Most of our work is based on using simulation, nonlinear dynamics theory, and simple physical experiments with a variety of physical models. Simple analytic approaches have also been used. An outline of our procedure (based on McGeer's) is as follows:

1. **Write equations of motion for an appropriate theoretical model.** The motion is determined by the differential equations and jump conditions of classical mechanics. Equations are generated

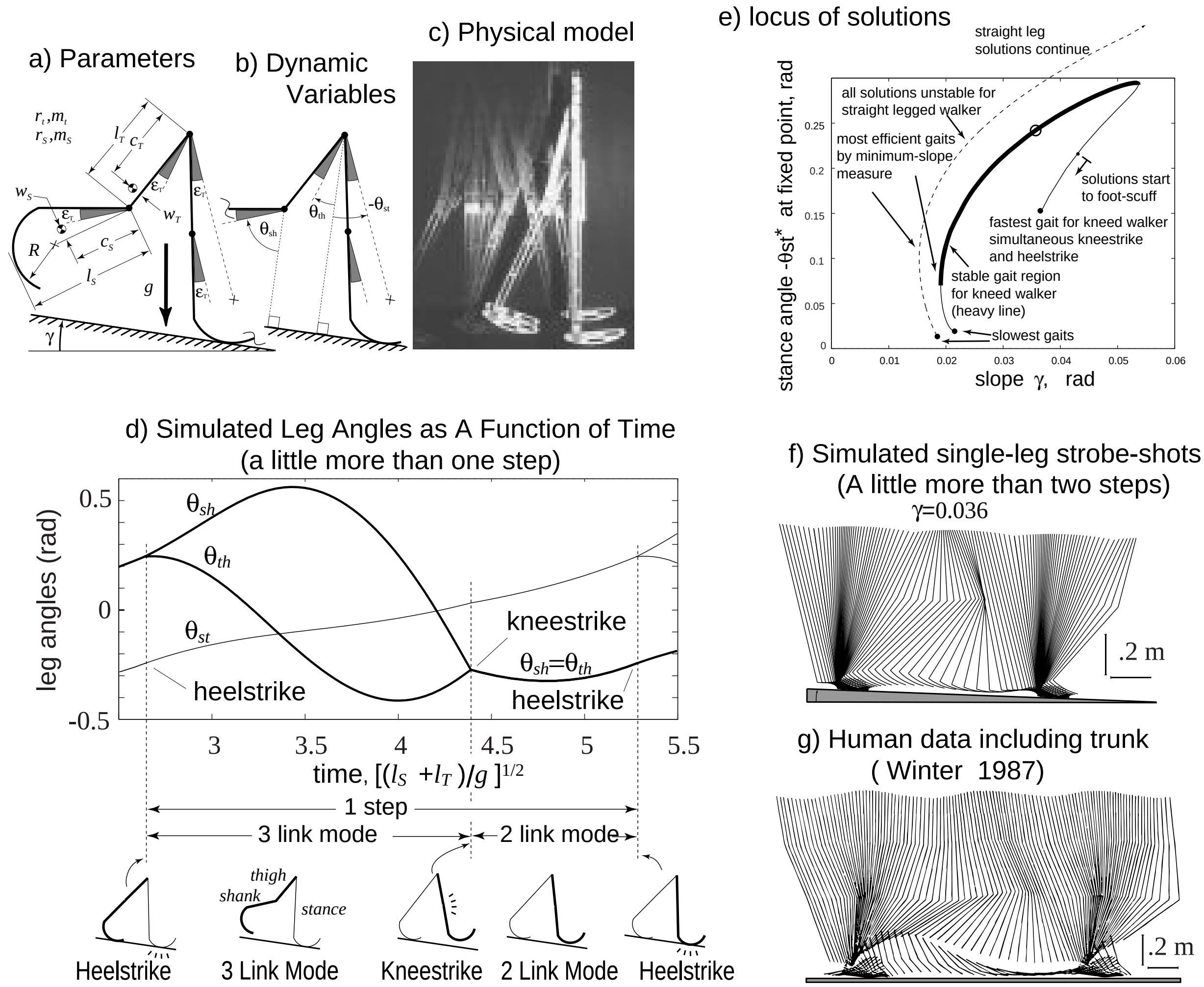

Figure 3: **Kneed 2-D Passive-Dynamic Walker.** **(a) Theoretical model parameters**, not drawn to scale, include radii of gyration and masses of thigh and shank, denoted by r_T, m_T, r_S, and m_S, respectively. The circular foot is centered at the '+'. ϵ_T is the angle between the stance thigh and the line connecting the hip to the foot center. **(b) Dynamic variables** are θ_{st}, θ_{th}, and θ_{sh} which are measured from the ground-normal to lines offset by ϵ_T from their respective segments. **(c) Our physical model** walking down a shallow ramp with strobe exposure (approximately one step). The visible double leg-set constrains the physical model to 2-dimensional motion ($\ell_t = 0.35\text{m}, w_t = 0\text{m}, m_t = 2.354\text{kg}, r_t = 0.099\text{m}, c_t = 0.091\text{m}, \ell_S = 0.46\text{m}, w_S = 0.025\text{m}, m_S = 1.013\text{kg}, r_S = 0.197\text{m}, c_S = 0.17\text{m}, R = 0.2\text{m}, \gamma = 0.036\text{rad}, g = 9.81\text{m/s}^2, \epsilon_T = 0.197\text{rad}$). **d) Computer simulated steady gait cycle** (from [7]). Angles of leg segments are shown from before a heelstrike to after the next heelstrike in a stable gait. The heavy line on the upper graph corresponds to the motion of the heavy-line leg on the 5-frame cartoon under the graph. The angular velocities have discontinuities at kneestrike and heelstrike, which appear as (barely visible) kinks in the curves. The parameters for the simulation correspond to measured values from the physical model in (c). **e) Locus of solutions** for a kneed walker (solid) and the same walker with knees locked (dashed). Both stable (heavy line) and unstable (light line) periodic motions are shown. The solution in (d) is marked with an open circle. **f) strobe line drawing** of the positions of one leg, spaced evenly in time, over a little more than two steps of the simulation in (d). **g) Human subject data** from [21] (taking bigger steps, and shown at a smaller scale).

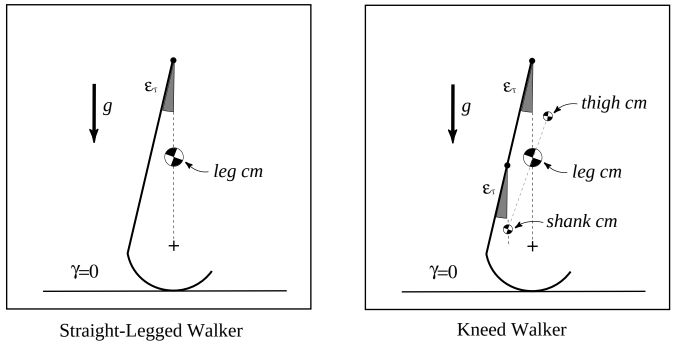

Figure 4: To approach perfect efficiency, a walker must allow a one-legged standing solution at zero slope. In the kneed walker, the cm of the shank must be on a vertical line which passes through the knee. For both kneed and straight-leg walkers, the cm of the locked leg must be on a line connecting the hip and the foot center. The locations of the mass centers are misleading in the figure; in most of the more anthropomorphic cases we study, the walker's mass center is close to the hip joint.

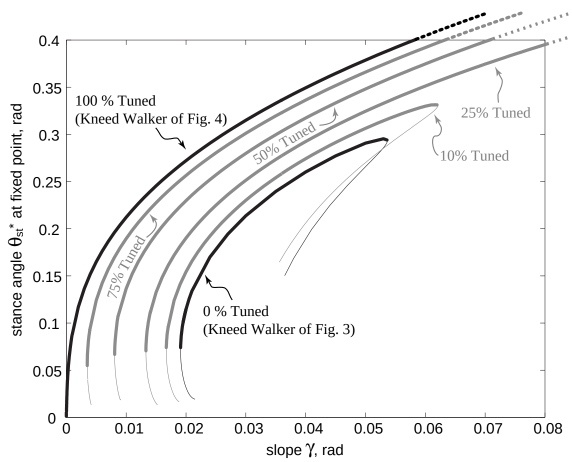

Figure 5: Numerically-generated solution loci which show the effect of tuning the kneed walker of figure 3. The center of mass locations and the moments of inertia were varied to produce walkers with varying degrees of tuning. Not all of the unstable solutions are shown. Thick lines denote stable solutions.

either by hand, using symbolic algebra (e.g.,. Maple or Mathematica), or with a special purpose dynamics-equation generator such as AUTOLEV.

2. **Treat a step as a function.** The solution of the equations, from the state at one step to state at the next step, can be thought of as a function $\mathbf{f}$, termed the "stride function" by McGeer. All of our theoretical and physical models, even to the extent that we add power and control, will be autonomous processes. Thus much information about a step will be encoded in the function $\mathbf{f}$. This function will take as input the list of values of the various angles and rates (the state variable vector $\boldsymbol{\theta}$) just after ground collision (or any other well defined point in the motion) and will return the values of $\boldsymbol{\theta}$ after the next ground collision.

   For a given set of initial conditions, the solution of the governing differential and algebraic equations over the period of time corresponding to one step yields one evaluation of the function $\mathbf{f}(\boldsymbol{\theta})$. In the language of dynamical systems, the stride function is a Poincaré map. Many of our questions about the dynamics of a given theoretical walking model will then be reduced to questions about the function $\mathbf{f}(\boldsymbol{\theta})$.

3. **Find gait cycles** A simple (period-one) *gait cycle*, if it exists, corresponds to a set of initial values for the angles and rates which lead back to the same angles and rates after one step. This set of angles and rates $\boldsymbol{\theta}^*$ is a fixed point of the Poincaré map $\mathbf{f}(\boldsymbol{\theta})$, i.e., $\mathbf{f}(\boldsymbol{\theta}^*) = \boldsymbol{\theta}^*$. This cycle corresponds to a zero of the difference function $\mathbf{g}(\boldsymbol{\theta}) \equiv \mathbf{f}(\boldsymbol{\theta}) - \boldsymbol{\theta}$. A *period-two* gait cycle returns the same variable values after *two* steps, and so on. Period-one motions are our central interest because they correspond to the important task of steady walking.

   There is no guarantee that we will find gait cycles (roots of $\mathbf{g}$) for any given theoretical model and set of parameters. Although finding the limit cycle involves solving n equations for n unknowns, not all parameter combinations lead to solutions. Also, for parameter combinations which do produce gait cycles, there is no guarantee of a numerical routine finding them. The root finding aspects of the work involve a mixture of intuitively based theoretical model definition, on starting new searches on known solutions, and on various numerical methods.

4. **Evaluate performance.** For each steady motion we evaluate the stability and other performance indices (e.g.,. measures of speed or efficiency) using analytically guided numerical methods. A simple and useful measure of stability comes from the eigenvalues of the derivative matrix $\mathbf{J}$ of the map $\mathbf{f}$

$$\mathbf{J} = \frac{\partial \mathbf{f}}{\partial \boldsymbol{\theta}} \qquad \text{with components } J_{ij} = \frac{\partial f_i}{\partial \theta_j} \tag{2}$$

   The *linearization* of $\mathbf{J}$ generally characterizes the dynamics when motion is close to a periodic walking cycle. Small perturbations $\hat{\boldsymbol{\theta}}$ to the limit cycle state vector $\boldsymbol{\theta}^*$ at the start of a step will grow or decay from the kth step to the $k+1$th step approximately according to $\hat{\boldsymbol{\theta}}^{k+1} \approx \mathbf{J}^k \hat{\boldsymbol{\theta}}$. We evaluate $\mathbf{J}$ by numerically evaluating $\mathbf{f}$ a number of times in a small neighborhood of $\boldsymbol{\theta}^*$. We then numerically evaluate the eigenvalues λ_i of the linearization of $\mathbf{J}$. If all of the eigenvalues are small enough, $|\lambda_i| < 1$ all sufficiently small perturbations will decay to $\hat{\boldsymbol{\theta}} = \mathbf{0}$ and the system will asymptotically approach its limit cycle. If the Jacobian has any eigenvalues outside the unit circle, any perturbation with a component along the corresponding eigenvector will bump the system divergently off the limit cycle — the cycle is unstable and can not be realized in an uncontrolled physical model. Inevitably, neutrally stable eigenvalues of magnitude 1 (to first order) generally appear and do not affect balance stability. For example, the indifference of most of the 3-D devices to direction of travel generates an eigenvalue of 1 in the map.

4 Summary

We have reproduced much of the experimental and theoretical work of McGeer on 2-D bipeds with and without knees. We have examined in more detail an irreducibly simple model. We have discovered the possibility of period doubling and chaos for models with and without knees. We have found walking at near-zero slopes and scaling rules for such walkers. We have numerically studied the relationship between zero-slope-tuned walkers and non-tuned walkers. Finally, we have found an experimental 3-D walker that has *no* stable configuration, yet walks stably.

References

[1] R. M. Alexander. Simple models of human motion. *Applied Mechanics Review*, 48:461–469, 1995.

[2] Rob Bennet and Tim Cardanha. Dynamic keel project. Report on a dynamic keel to stabilize an otherwise unstable boat. Project advisor: Andy Ruina., 1991.

[3] M. J. Coleman. *A Stability Study of a Three-dimensional Passive-dynamic Model of Human Gait*. PhD thesis, Cornell University, Ithaca, NY, 1997. Defended and being revised.

[4] Michael Coleman, Anindya Chatterjee, and Andy Ruina. Motions of a rimless spoked wheel: A simple 3D system with impacts. *Dynamics and Stability of Systems*, 12(3):139–160, 1997.

[5] Michael Coleman and Andy Ruina. An uncontrolled toy that can walk but cannot stand still. Accepted for *Physical Review Letters*, 1997.

[6] J. Victoria Fowble and Arthur D. Kuo. Stability and control of passive locomotion in 3D. *Proceedings of the Conference on Biomechanics and Neural Control of Movement*, pages 28–29, 1996.

[7] Mariano Garcia, Anindya Chatterjee, and Andy Ruina. Speed, efficiency, and stability of small-slope 2-d passive dynamic bipedal walking. submitted for ICRA98, expanded version in preparation for *International Journal of Robotics Research*, 1998.

[8] Mariano Garcia, Anindya Chatterjee, Andy Ruina, and Michael Coleman. The simplest walking model: Stability, complexity, and scaling. *ASME Journal of Biomechanical Engineering*, 1997. in press for April 1998.

[9] Ambarish Goswami, Bernard Espiau, and Ahmed Keramane. Limit cycles and their stability in a passive bipedal gait. *International Conference on Robotics and Automation*, 1996.

[10] Ambarish Goswami, Bernard Espiau, and Ahmed Keramane. Limit cycles in a passive compass gait biped and passivity-mimicking control laws. *Journal of Autonomous Robots*, 1997. In Press.

[11] R. Scott Hand. Comparisons and stability analysis of linearized equations of motion for a basic bicycle model. Master's thesis, Cornell University, Ithaca, NY, 1988. Advisors: Andy Ruina & James Papadopoulos.

[12] Mont Hubbard. Lateral dynamics and stability of the skateboard. *Journal of Applied Mechanics*, 46:931–936, 1979.

[13] Tad McGeer. Passive dynamic walking. *International Journal of Robotics Research*, 9:62–82, 1990.

[14] Tad McGeer. Passive walking with knees. *Proceedings of the IEEE Conference on Robotics and Automation*, 2:1640–1645, 1990.

[15] Tad McGeer. Passive dynamic biped catalogue. In *Proceedings of the 2nd International Symposium of Experimental Robotics*. Springer-Verlag, New York, 1991.

[16] Tad McGeer. Dynamics and control of bipedal locomotion. *Journal of Theoretical Biology*, 163:277–314, 1993.

[17] Andy Ruina. Non-holonomic stability aspects of piecewise holonomic systems. submitted to *Reports on Mathematical Physics*, 1997.

[18] Andy Ruina and Mariano Garcia. Passive, nonlinear-dynamic study of walking: Simulation, analysis, and experiment. A research proposal submitted to the National Science Foundation, 1997.

[19] Steven Strogatz. *Nonlinear Dynamics and Chaos*. Addison-Wesley, Reading, MA, 1994.

[20] Benoit Thuilot, Ambarish Goswami, and Bernard Espiau. Bifurcation and chaos in a simple passive bipedal gait. *IEEE International Conference on Robotics and Automation*, 1997.

[21] David A. Winter. *The Biomechanics and Motor Control of Human Gait*. University of Waterloo Press, 1987.