STABILITY, SCALING, AND CHAOS IN PASSIVE-DYNAMIC GAIT MODELS

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In this work, we study computer simulations of simple biped models with no actuation except gravity, and no control. These so-called *passive-dynamic* models of human gait were first studied by McGeer (1989). Computer simulations were also used to construct two kneed walkers for demonstration purposes.

We begin our study with a simple one-parameter walking model, in 2-D, then we move to more general 2-D models with and without knees, and finally we study a 3-D model with no knees. In 2-D, we are most interested in gait efficiency, while in 3-D, we focus on gait stability. We find general rules for the one-parameter model which can be extended to understand the behavior of the more complicated models.

A summary of the main points is as follows:

1. The "simplest" walking model with only a point-mass at the hip exhibits two gait cycles, one of which is stable at small slopes. Both gait cycles extend to arbitrarily small slopes, and are therefore "perfectly efficient." This model has a step length proportional to the cube root of the slope; power usage scales

with (velocity)^4. An asymptotic analysis agrees with numerical simulation results at small slopes. The long-step gait exhibits period doubling bifurcations to chaotic gait as the slope is varied.

2. More general models with and without knees can also have up to two gait cycles, one of which can be stable. In general, these models will not be able to walk at arbitrarily small slopes. We present mass distribution conditions for perfect walking efficiency. These "tuned" walkers retain one of the cube-root-scaling gaits, but the other gait, which is always unstable, has a step length proportional to the slope at very small slopes. A period-doubling route to chaos is also numerically-demonstrated for a tuned kneed walker. Some data is also presented from a working physical walker.

3. In 3-D, planar 2-D gaits still exist but are unstable. A torsional spring at the hip of a 3-D model improves its stability somewhat. Automated gradient-based parameter searches to minimize the maximum eigenvalue terminate at local minima; no stable 3-D walking gaits were found for our model. We conclude that this model is not sufficient to explain the stability of the walker of Coleman and Ruina (1998).

Biographical Sketch

Mariano was born on November 11, 1971 to Alfredo Mariano Garcia, M.D. Ph.D., and Natalia Gabrusewycz Garcia, Ph.D., at Crouse-Irving Memorial Hospital in Syracuse, NY. During nearly all of his childhood, he lived on Loomis Hill Road (since renamed to Stevens Road), at the top of Vesper Hill in the town of Tully, New York, where he developed a love for trees, tractors, snowplows, blizzards, and country life in general.

After schooling at Tully Elementary and Christian Brothers Academy in Syracuse, Mariano attended the Sibley School of Mechanical and Aerospace Engineering at Cornell University in Ithaca, NY, and graduated in June 1993 with a BS in Mechanical Engineering, and remained as a graduate student to pursue his PhD. After his advisor, Jeff Koechling, left Cornell, Mariano transferred to the Department of Theoretical and Applied Mechanics, and continued to pursue his PhD under the guidance of Andy Ruina in the Human Power, Biomechanics, and Robotics Lab at Cornell.

Mariano defended his thesis in November of 1998, and plans to pursue post-doctoral work in the Department of Integrative Biology at the University of California, Berkeley, starting in January 1999. While there, he plans to pursue his interests in multibody dynamics and simulations of biomechanical systems.

For the memory of my late grandfather Oleh, who never cared much for protocol and who felt that the best thing one could do with their life was to be a scientist,

and

for my father Alfredo Mariano; without his thoroughly irritating encouragement and advice, I would not have subjected myself to the insecurity of graduate school,

and

for my mother Natalia, who always placed my needs above her own,

and

for my wife Ellen, who binds my heart and head together and makes me complete.

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There is almost nothing in this thesis that I can claim as uniquely mine; rather, the results and miniscule scientific contributions contained within are largely the products of collaborations with other people. For the most part, this means that other people supplied the theories and good ideas, while I sweated the details.

The most notable source of vision was my advisor, Andy Ruina. I owe him a great deal of thanks for taking me under his wing and allowing me to soak up some of his knowledge and insight.

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